Stellar Evolution 2016 Q1

Ugeseddel 5 (week 39)

In the lecture on Thursday September 29 Günter will discuss zero-age main sequences (*Kippenhahn, Weigert & Weiss*, Chapters 22 and 23), and main-sequence evolution. In the following week Günter is fully occupied by a workshop on convection. Fortunately Steve Kawaler, who is a visitor from Iowa State University, has very kindly agreed to take over the lectures, covering remaining issues of main-sequence evolution and evolution after the main sequence (*Kippenhahn, Weigert & Weiss*, Chapters 31 – 34). The lectures 10 October are cancelled owing to the visit of a committee evaluating the Stellar Astrophysics Centre. Instead we plan to use the exercise class on 12 October for lectures, dealing with the evolution and properties of the Sun (*Kippenhahn, Weigert & Weiss*, Chapter 29). The final lecture of Q1, 13 October, will deal briefly with supernovae (*Kippenhahn, Weigert & Weiss*, Chapters 35 and 36).

The exercise class on 5 October will consider:

i) Finish the analysis of the properties of polytropic models, based on *Lecture Notes on Stellar Structure and Evolution*, Section 4.6. Specifically, consider Exercises 4.8, 4.9 4.11 and 4.12. (Exercise 4.10 is fun and challenging, so you might take a look at that also.)

ii) Solve the Lane-Emden equation numerically for some representative cases of the polytropic index n. This is discussed in *Lecture Notes on Stellar Structure and Evolution*, Exercise 9.1. You can use any programming language and algorithms that you are familiar with to integrate ordinary differential equations. Note that the singularity at the centre requires the use of an expansion around the centre; it would not hurt to derive this expansion. Use the analytical solution for $n = 1.0$ to test the code.

iii) The properties of predominantly convective stars are important in early and late phases of stellar evolution. Go through the analysis in *Lecture Notes on Stellar Structure and Evolution*, Section 7.2, including Exercise 7.3.

iv) In the unlikely event that you have time to spare, Exercise 1 from the Winter Exam 1987–88 in Astronomi A (see below)

Corrections to ***Kippenhahn, Weigert & Weiss*****:**

- **p. 345, Fig. 30.4**: There are problems with the labelling of the ordinate axis: the top three labels (1.0×10^3, 1.5×10^3 and 2.0×10^3) should be changed to 1.0×10^4, 1.5×10^4 and 2.0×10^4. Also, in l. -4 of the caption 'about 10^4 times larger' should be changed to 'about 10^3 times larger'.

- **p. 370, first paragraph**: Here there is a mistake in the Kelvin-Helmholz time quoted for the passage from C to D in Fig. 31.2; as is clear from the figure the appropriate time is more like 3×10^6 yr.

- **p. 380, l. 3 from bottom**: replace 'its maximum $h = 1$' by 'its minimum $h = 1$'.

- **p. 388, Fig.32.2**: It may be a little confusing that the figure shows results for two different evolutionary stages. The lines marked 'X' show the hydrogen profile in models at the end of central hydrogen burning, with the characteristic steep slope left behind by a retreating convective core. The lines marked 'Y' show the helium profile in a model roughly half-way through central helium burning. Here the growing convective core causes the discontinuous increase around $m/M = 0.1$, while the very thin hydrogen-burning shell corresponds to the decrease near $m/M = 0.2$.

- **p. 401, bottom, p. 402, top**: Here the timescale of the helium flash is underestimated. A more reasonable version of this sentence would be: 'The local luminosity l at maximum exceeds $10^{10} L_\odot$, comparable with that of a whole galaxy, but only for about a day'. (However, compared with the overall evolution time scales, the expression "helium flash" remains quite appropriate.)

- **p. 404, caption to Fig. 33.8**: In fact, the letters A – C bear no relation to the labelling in Figs 33.3 and 33.4. (It is only fair to point out that Thomas (1967) did not make this mistake.)

28 September 2016 — Jørgen Christensen-Dalsgaard

ASTRONOMY A — WINTER 1987–88

EXERCISE 1

We consider the contraction of a star before the main sequence. The star is assumed to be in hydrostatic equilibrium and to be on the Hayashi track, so that the energy transport is by convection everywhere. The star is thus approximated by a polytrope of index 3/2.

a) Show that the surface luminosity of the star is

$$L_{\rm s} = -\frac{3}{7}\frac{GM^2}{R}\frac{{\rm d}\ln R}{{\rm d}t} \ ,$$

where M and R are the surface mass and radius of the star and G is the gravitational constant. You may use that the gravitational potential energy of a polytrope of index 3/2 is

$$\Omega = -\frac{6}{7}\frac{GM^2}{R} \ .$$

We assume that the effective temperature of the star is constant during the contraction on the Hayashi track.

b) Show that the radius R at time t is given by

$$\left(\frac{R_0}{R}\right)^3 = 1 + \frac{7L_0R_0}{GM^2}(t - t_0) \ ,$$

where L_0 and R_0 are the surface luminosity and radius of the star at the start of the contraction, at $t = t_0$.

We assume that the release of gravitational potential energy takes place uniformly through the star, such that $L(r)/m(r)$ is constant; here $L(r)$ is the flow of energy through a spherical surface of radius r and $m(r)$ is the mass within this sphere. Matter in the star is completely ionized and satisfies the ideal gas law; radiation pressure can be neglected. The opacity κ is given by the Kramers approximation,

$$\kappa = \kappa_0 \rho T^{-3.5} \ ,$$

where ρ is density, T is temperature and κ_0 is a constant.

The condition that the star is on the Hayashi track is that it is everywhere convective.

c) Show that this condition can be written as

$$\text{constant} \times \frac{L_{\rm s}^{5/4}}{T_{\rm eff} M^{5.5}} > \frac{2}{5} ,$$

where $T_{\rm eff}$ is the effective temperature. You do not need to find the value of the constant.

By inserting the relevant basic constants and the constant κ_0 in the expression for the opacity it can be shown that the above expression becomes

$$790\,{\rm K} \frac{Z(1+X)}{T_{\rm eff}\mu^{7.5}} \left(\frac{L_{\rm s}}{L_\odot}\right)^{5/4} \left(\frac{M}{M_\odot}\right)^{-5.5} > \frac{2}{5} ,$$

where X and Z are the abundances by mass of hydrogen and heavy elements, and μ is the mean molecular weight.

d) Find the minimum luminosity on the Hayashi track for a $1\,M_\odot$ star, with the chemical composition $X = 0.73$, $Z = 0.02$, and $T_{\rm eff} = 4500\,{\rm K}$.

e) Calculate the duration of the contraction on the Hayashi track for a $1\,M_\odot$ star, using the results of questions b) and d). We assume an initial luminosity of $L_{\rm s,0} = 100\,L_\odot$.