Stellar Evolution 2016 Q1

Ugeseddel 2 (week 36)

Updated 8 September

In the lectures Monday 5 September I completed stability considerations (*Kippenhahn, Weigert & Weiss*, Chapter 6) and discussed convective energy transport (Chapter 7). Thursday 8 September I shall make a few additional comments on convection, and then Günter Houdek will take over the lectures, dealing with the treatment of chemical evolution and mass loss (Chapters 8 and 9). On 12 September Günter provides an overview of the equations of stellar evolution (Chapter 10) and covers boundary conditions and numerical procedures (Chapters 11 and 12; we skip section 12.6). The following lectures (15 and 19 September) will deal with the detailed treatment of the equation of state (Chapters 13 – 16).

The exercise class on 14 September will consider:

i) Briefly go through the steps in the verification of energy conservation (neglecting the kinetic energy) in *Kippenhahn, Weigert & Weiss*, Section 4.5

ii) Show, for a general equation of state, that

$$\left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{\rm ad} = \frac{1}{\alpha - \delta \, \nabla_{\rm ad}}$$

iii) Exercise U2.1 below.

Additional material:

The web page (http://astro.phys.au.dk/~jcd/stel-struc) contains links to additional material on convection, including extensive numerical results obtained by Bob Stein. As noted in the lectures, Remo Collet is also heavily involved in these simulations. We may get back to more details on the treatment of convection in the second part of the course in Q2.

Corrections to ***Kippenhahn, Weigert & Weiss*****:**

- **p. 81, Eq. (8.28)**: This equation is not correct as it stands, even on dimensional grounds. A correct form would be to add to the diffusion equation the term

$$\frac{\partial}{\partial r}\left[D_c \frac{\partial X_i}{\partial r}\right] = \frac{\partial}{\partial r}\left[\left(\frac{1}{3} v_{\rm m} \ell_{\rm m}\right) \frac{\partial X_i}{\partial r}\right]$$

- **p. 108, line below Eq. (12.4)**: Here the text talks about 'linearization' of the differential equations. Presumably 'discretization' is intended; this constitutes the approximation to the original equations. Linearization follows later, in the iterative solution of the nonlinear difference equations; this can in principle be done to any specified numerical accuracy (although in practice limited by round-off errors etc.).
- **p. 109, Fig. 12.1**: For consistency with the text, 'i' should have been used as subscript on 'A_i^1' etc. The double use of 'j' in the figure is potentially confusing.
- **p. 113, l. 4**: The size of the Henyey matrix is $(4K-2)\times(4K-2)$ (for $K=4$ as in Fig. 12.3 the matrix is 14×14), rather than $K\times K$.
- **p. 113, l. -4**: Here the equation should be $X_i^{n+1} = X_i^n + \Delta t \dot{X}_i^{n+1} = X_i^n + \Delta X_i^{n+1}$ and in the line below ΔX_i^n should correspondingly be replaced by ΔX_i^{n+1}.
- **p. 116, Eq. (12.20)**: It is perhaps a little confusing that 'M' is used here for the number of grid points, rather than 'K'.

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Exercise U2.1:

Convective stability or overstable oscillations. We consider the motion of a convective element, taking into account heat exchange characterized by the time scale $\tau_{\rm adj}$. The goal is to analyse the conditions for overstable oscillations of the element. The notation follows *Kippenhahn, Weigert & Weiss*.

i) Show that the equation of motion of the element is

$$\frac{\partial^2 \Delta r}{\partial t^2} = g\left(\delta \frac{DT}{T} - \phi H_P^{-1} \nabla_\mu \Delta r\right) . \tag{2.1}$$

ii) Show that DT satisfies

$$\frac{1}{T}\frac{\partial DT}{\partial t} = -\frac{DT}{T\tau_{\rm adj}} - H_P^{-1}(\nabla_{\rm ad} - \nabla)\frac{\partial \Delta r}{\partial t} \tag{2.2}$$

(see *Kippenhahn, Weigert & Weiss*, equation (6.27)).

We now assume that Δr and DT vary with time as $\exp(-\mathrm{i}\,\omega t)$, where the frequency ω in general is complex.

iii) Show that ω satisfies the dispersion relation

$$(\omega^2 - \phi g H_P^{-1} \nabla_\mu) \left(1 + \frac{\mathrm{i}}{\omega \tau_{\mathrm{adj}}}\right) - g\delta H_P^{-1} (\nabla_{\mathrm{ad}} - \nabla) = 0 \; . \tag{2.3}$$

iv) Show that when $\tau_{\mathrm{adj}} \to \infty$, the frequency is given by

$$\omega^2 = \omega_{\mathrm{ad}}^2 \equiv \frac{g\delta}{H_P} \left(\nabla_{\mathrm{ad}} - \nabla + \frac{\phi}{\delta} \nabla_\mu\right) \; . \tag{2.4}$$

Give an interpretation of this result in terms of convective stability or instability.

v) Assume that $\tau_{\mathrm{adj}}^{-1} \ll |\omega_{\mathrm{ad}}|$ and write $\omega = \omega_{\mathrm{ad}} + \eta$. Find an approximate expression for η. Discuss the result both in the case of convective instability and convective stability. Show in particular that overstable oscillations result when $\nabla_{\mathrm{ad}} < \nabla < \nabla_{\mathrm{ad}} + (\phi/\delta)\nabla_\mu$.
