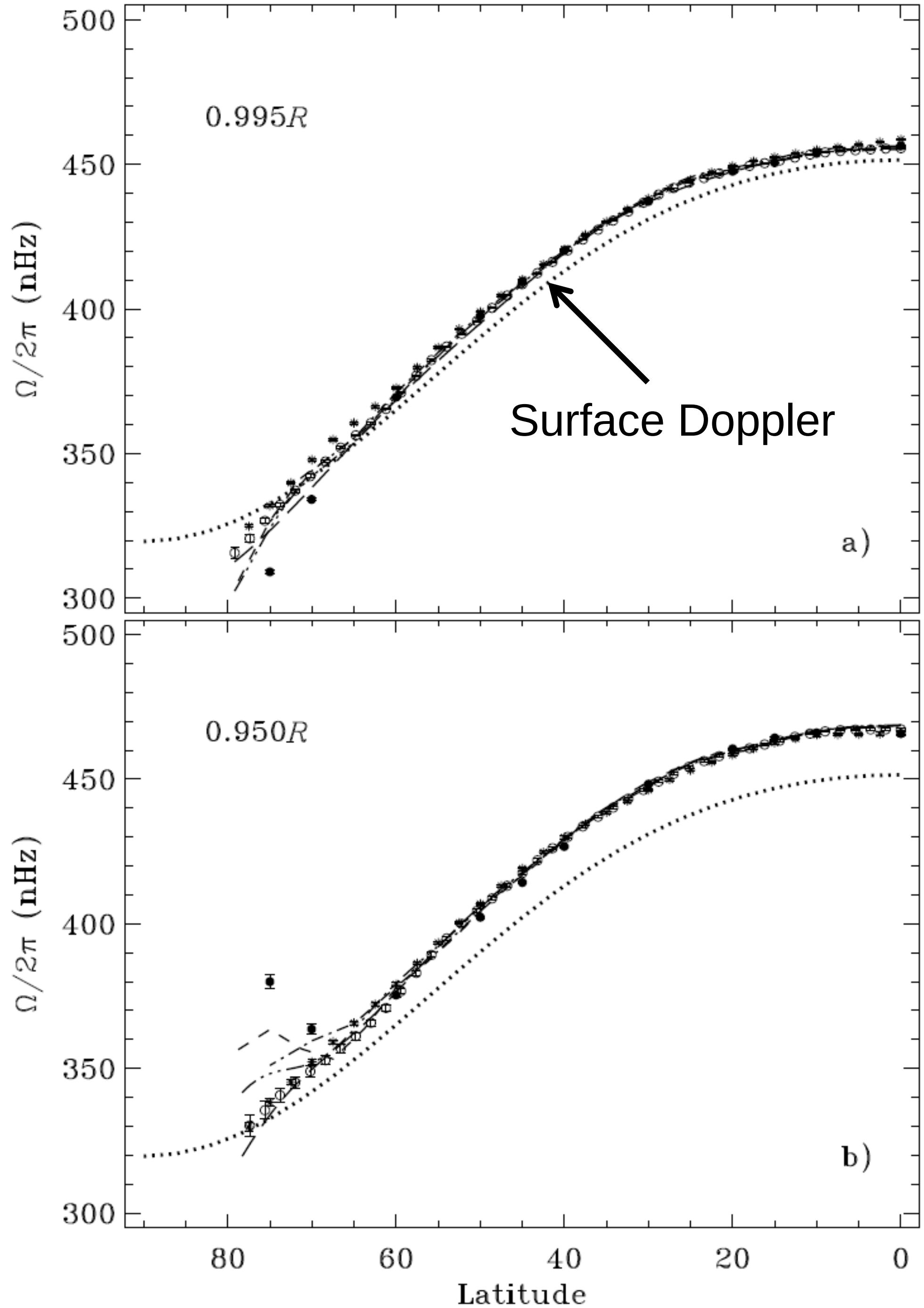
Stellar rotation

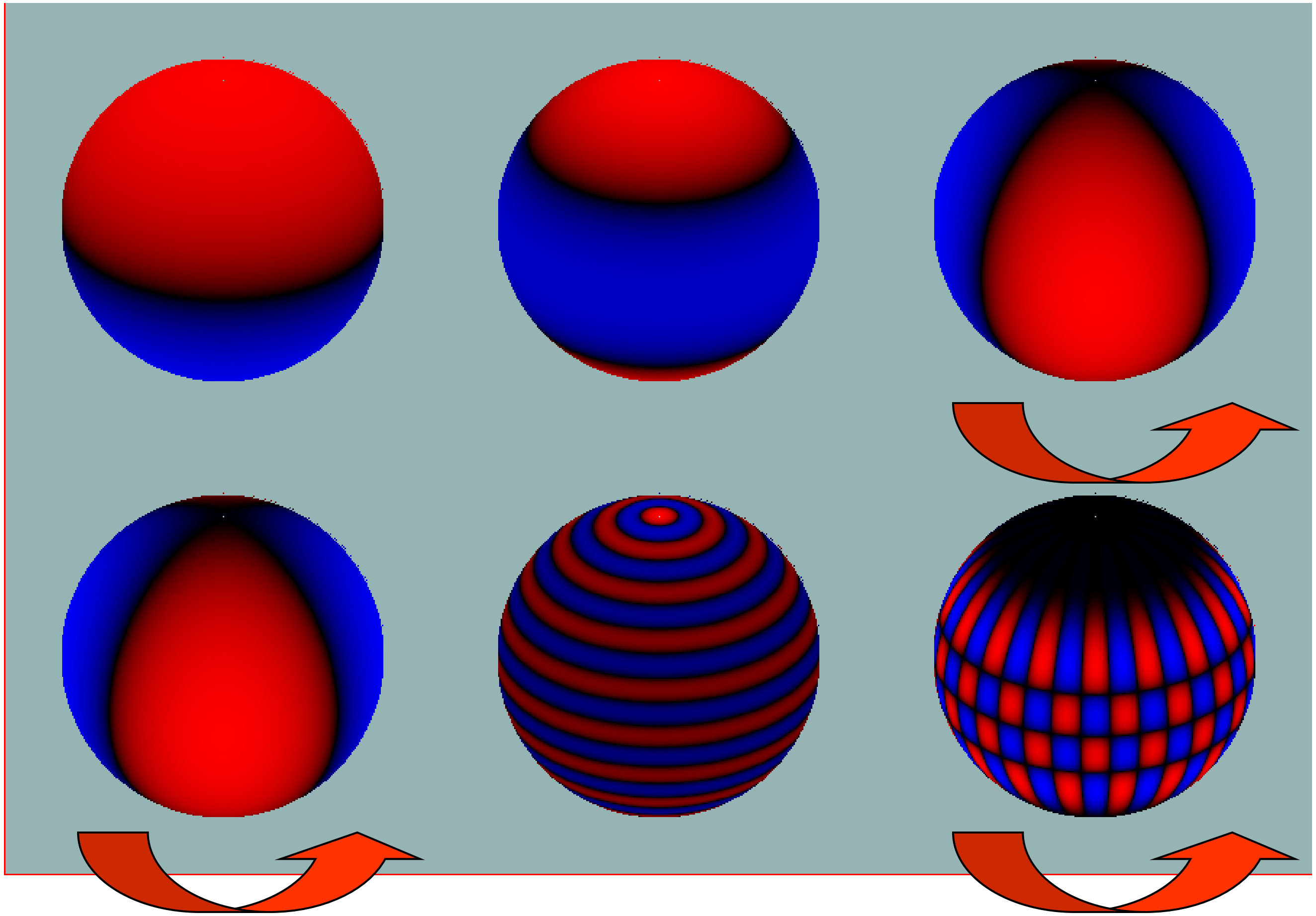
Solar surface rotation

Solar near-surface rotation

Schou et al. (1998; ApJ 505, 390)

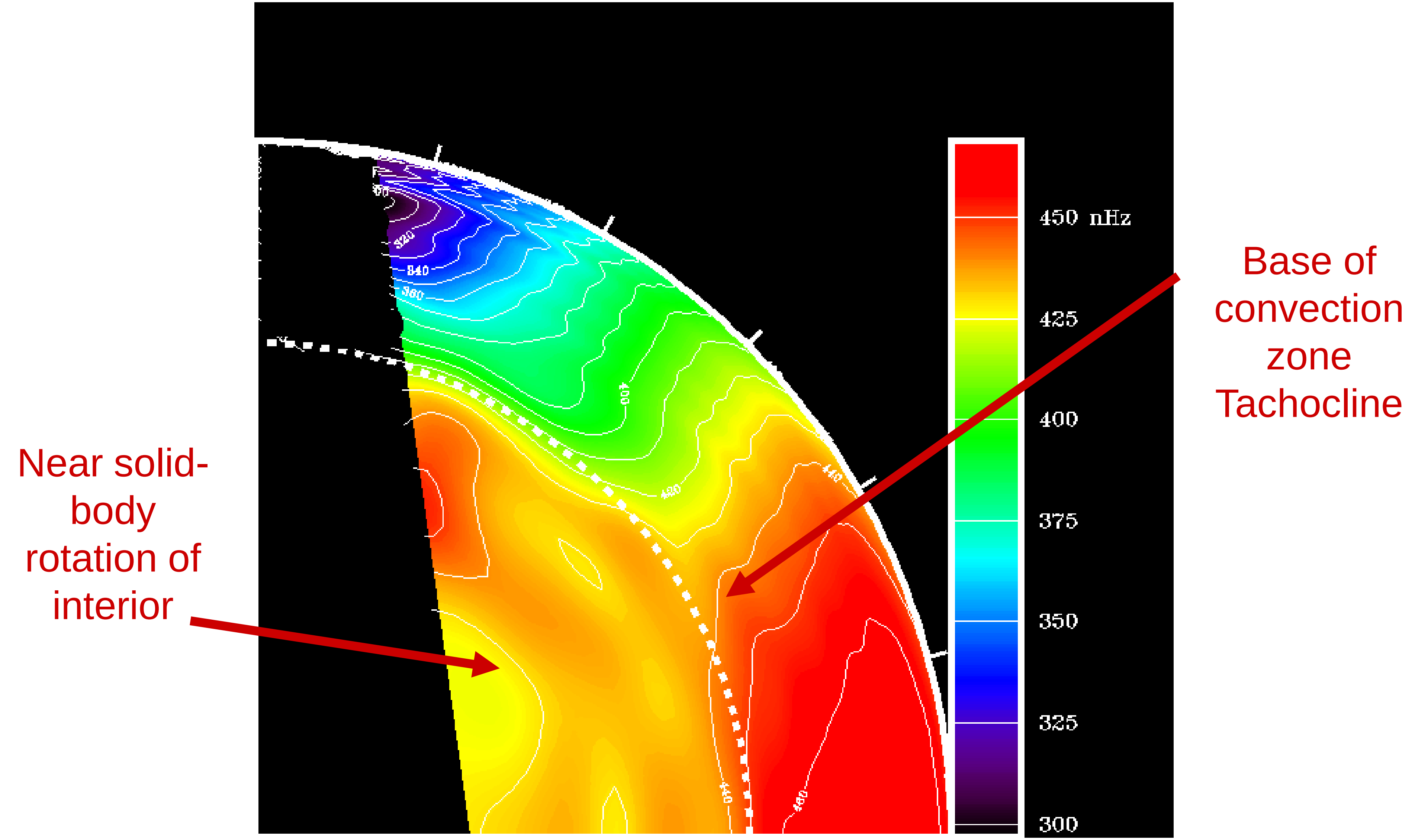


Rotational splitting

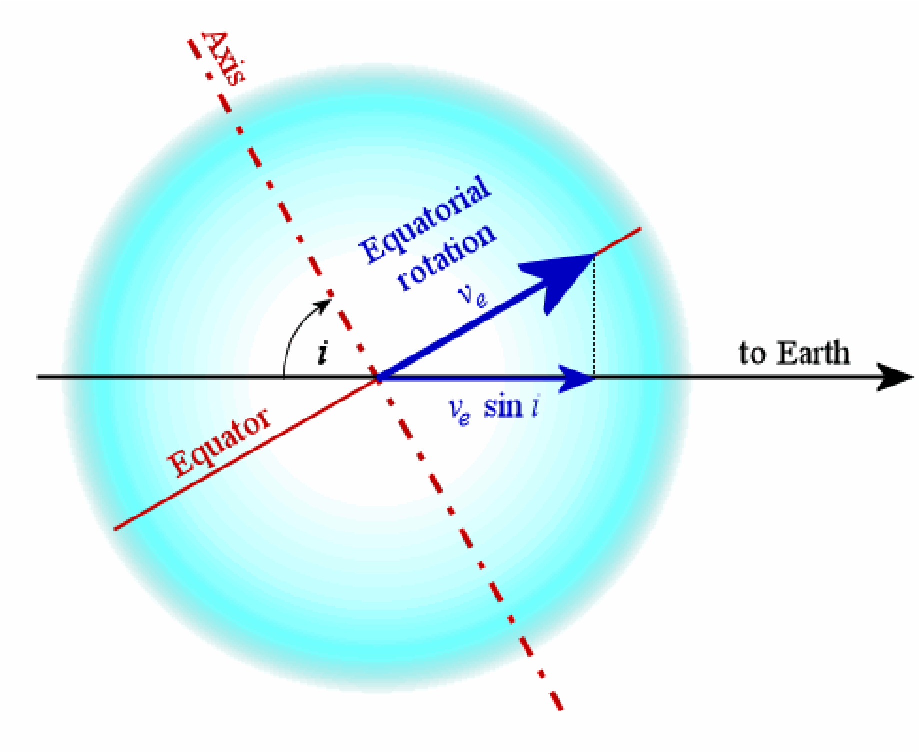


$$\omega_{nlm} = \omega_{nl0} + m\langle\Omega\rangle$$

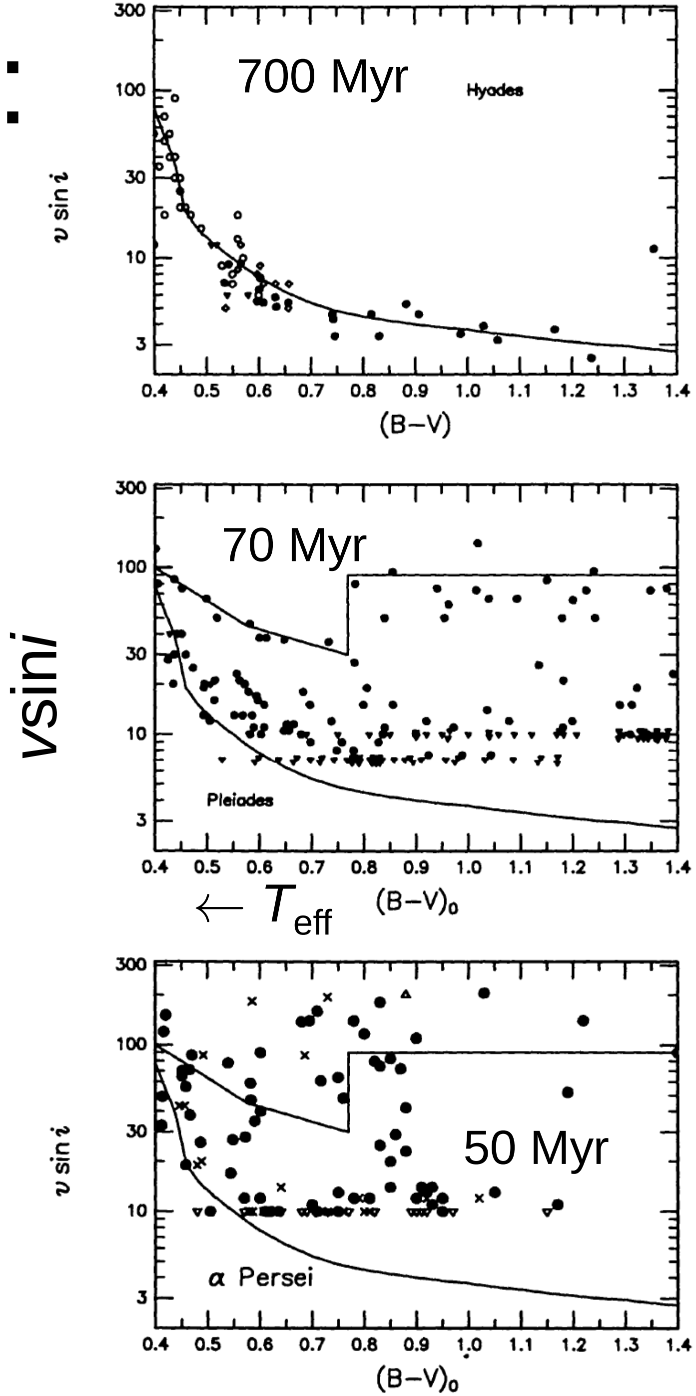
Inferred solar internal rotation



Stellar observations: spectral line broadening

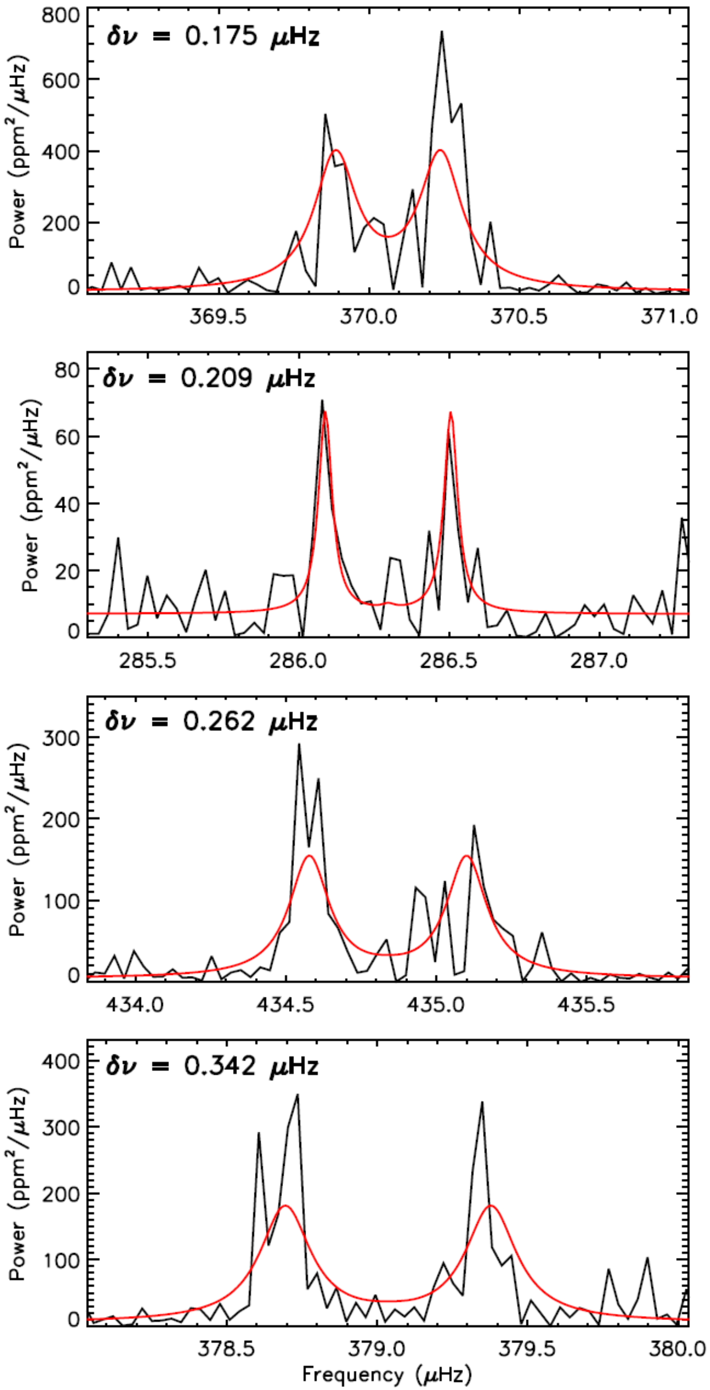


Evolution with age: stellar clusters



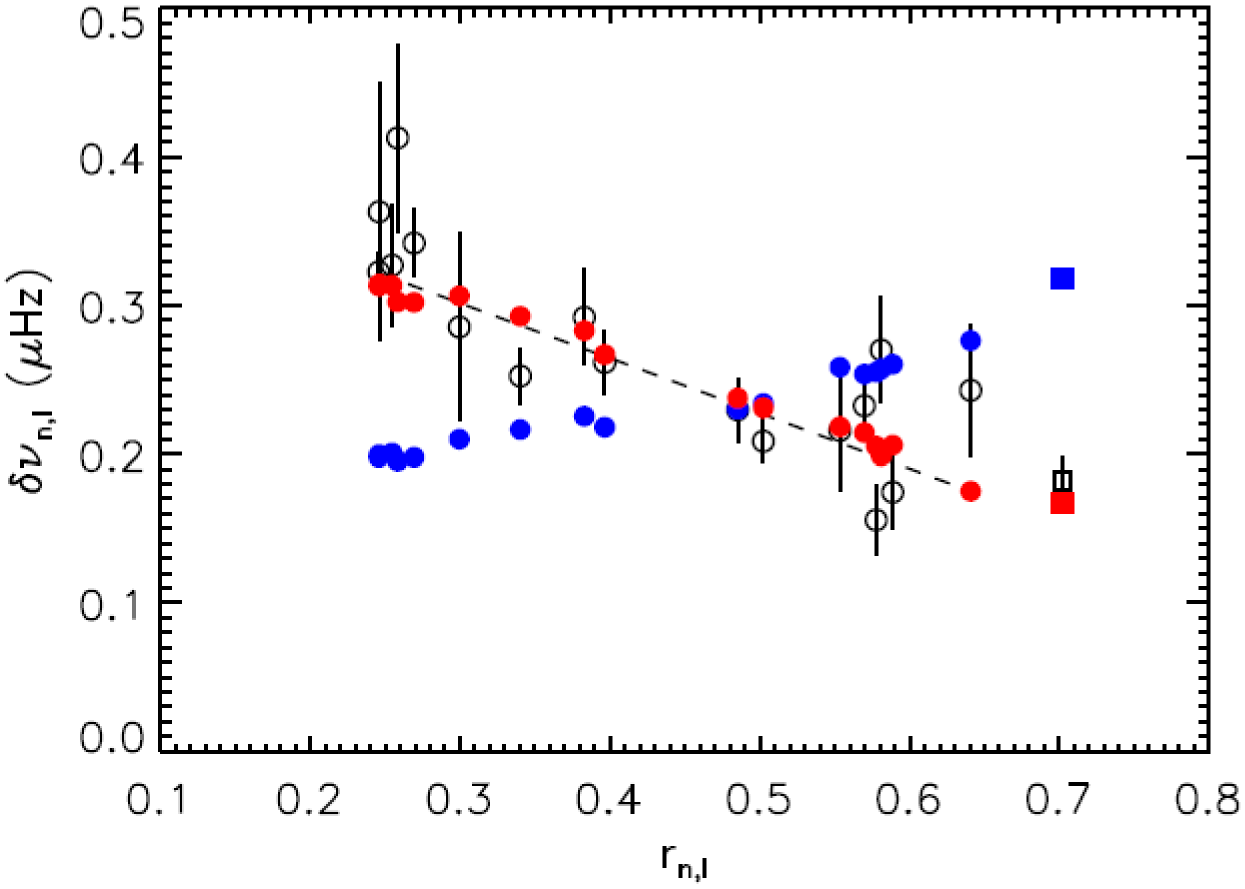
Soderblom et al. (1993; ApJ 409, 629)

Observed rotational splittings
in early red giant



Deheuvels et al. (2012; ApJ 756, 19)

Fitted rotational splittings

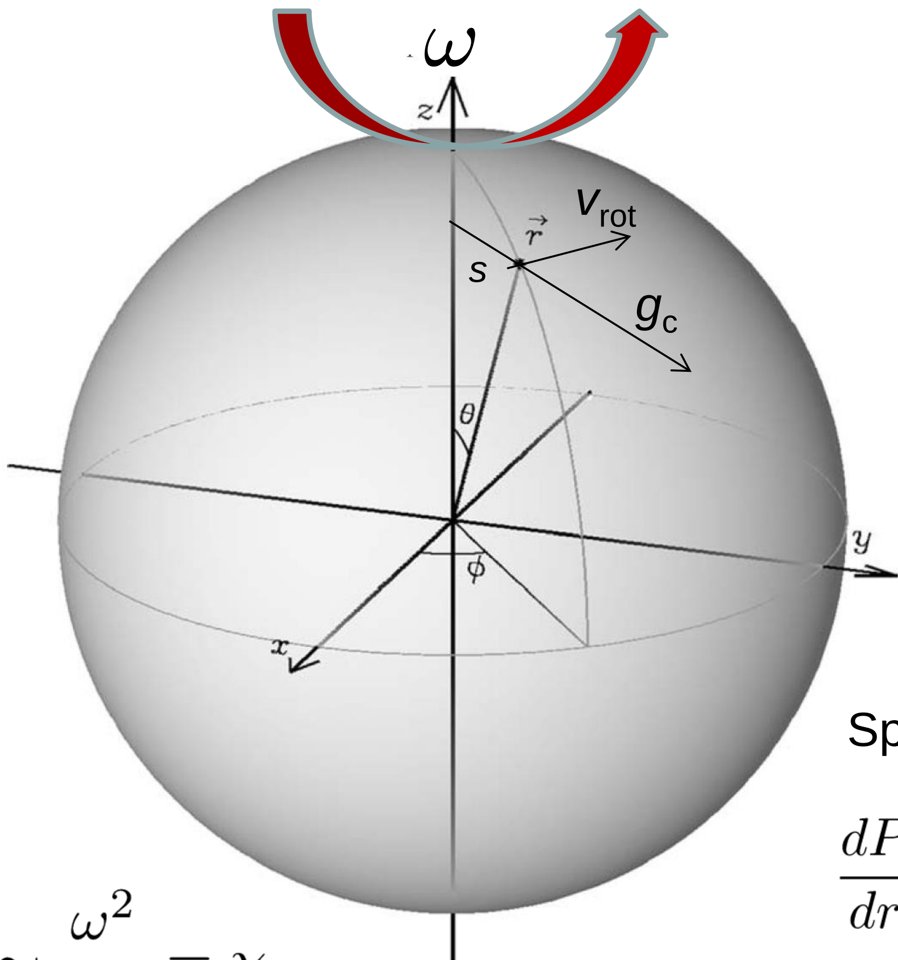


Solid-body rotation
$\Omega/2\pi$= 328 nHz

Two-zone model
Convective envelope, radiative core
$\Omega_c/2\pi$= 696 nHz
$\Omega_e/2\pi$= 51 nHz

Deheuvels et al. (2012; ApJ 756, 19)

Centrifugal acceleration



$$s = r \sin\vartheta$$

$$v_{\rm rot} = \omega r \sin\vartheta$$

$$g_{\rm c} = \frac{v_{\rm rot}^2}{s} = \omega^2 s$$

Importance of centrifugal acceleration:

$$\frac{g_{\rm c}}{g} \sim \frac{\omega^2 R^3}{GM} \sim \frac{\omega^2}{G\bar{\rho}} = \chi$$

Spherical approximation

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} + \frac{2}{3}\rho\omega^2 r$$

Pear-shaped figures of equilibrium

Rotating liquid bodies

Jeans (1929; *Astronomy and Cosmogeny*)

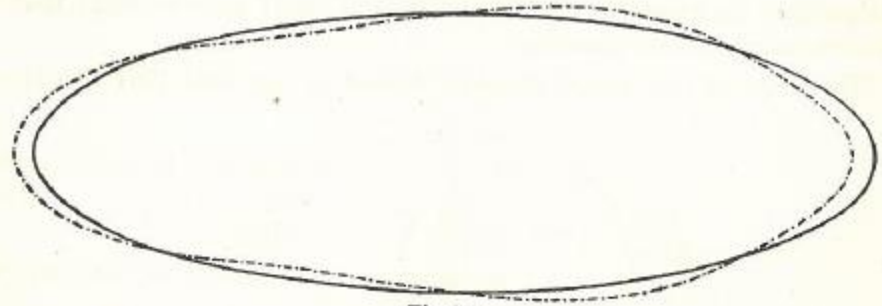


Fig. 27.

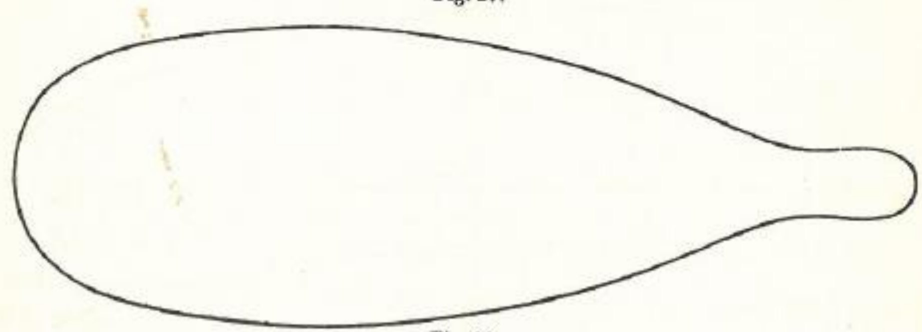


Fig. 28.

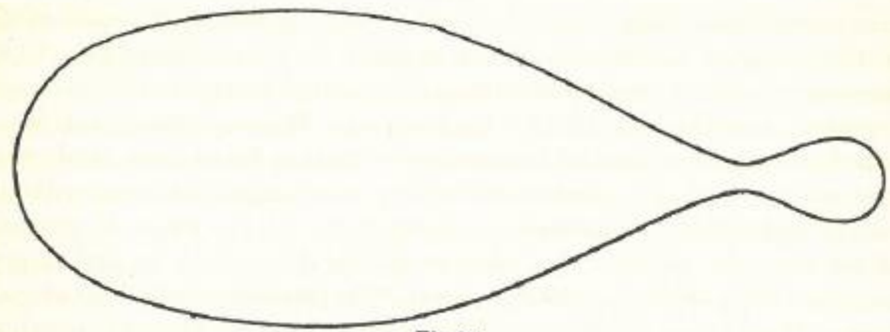


Fig. 29.

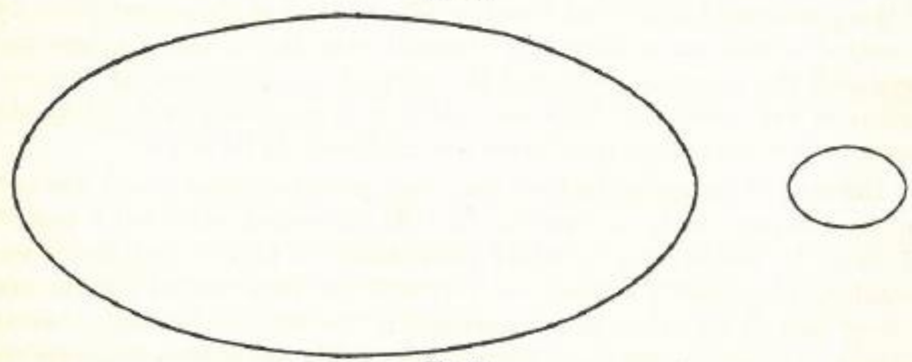


Fig. 30.

Roche model

$$\Phi = -\frac{GM}{r}\ .$$

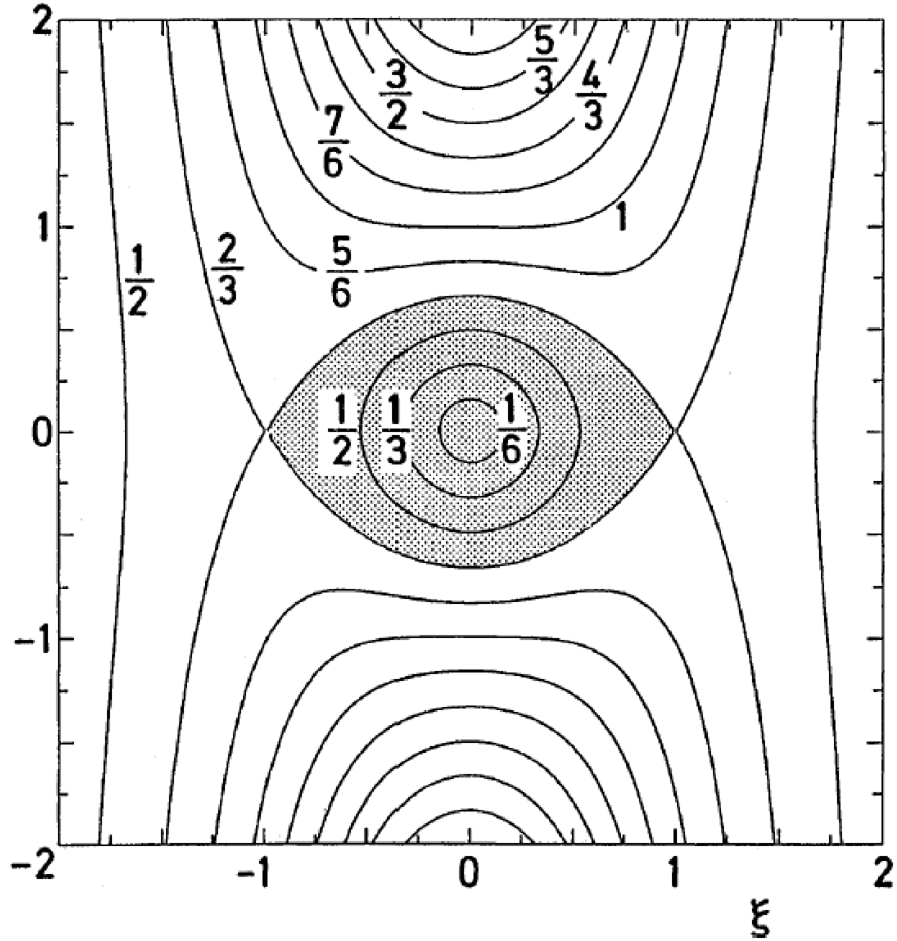
$$V = -\frac{1}{2}s^2\omega^2$$

$$\Psi = \Phi + V$$

$$= -\frac{GM}{(s^2+z^2)^{1/2}} - \frac{1}{2}s^2\omega^2$$

$$-\nabla\Psi \ = \ \boldsymbol{g}_{\rm eff}$$

$$s_{\rm cr}^3 = \frac{GM}{\omega^2}$$



$\xi = s/s_{\rm cr},\ \eta = z/s_{\rm cr}$

Conservative rotation: $\omega = \omega(s)$

$$\Psi := \Phi + V$$

$$V = -\int_0^s \omega^2 s \, ds$$

$$\nabla P = -\varrho \nabla \Psi$$

$$P = P(\Psi)$$

$$\rho = -\frac{\mathrm{d}P}{\mathrm{d}\Psi} = \rho(\Psi)$$

$$\frac{T}{\mu} = \frac{P}{\rho \mathcal{R}} = \left(\frac{T}{\mu}\right)(\Psi)$$

Gravity darkening

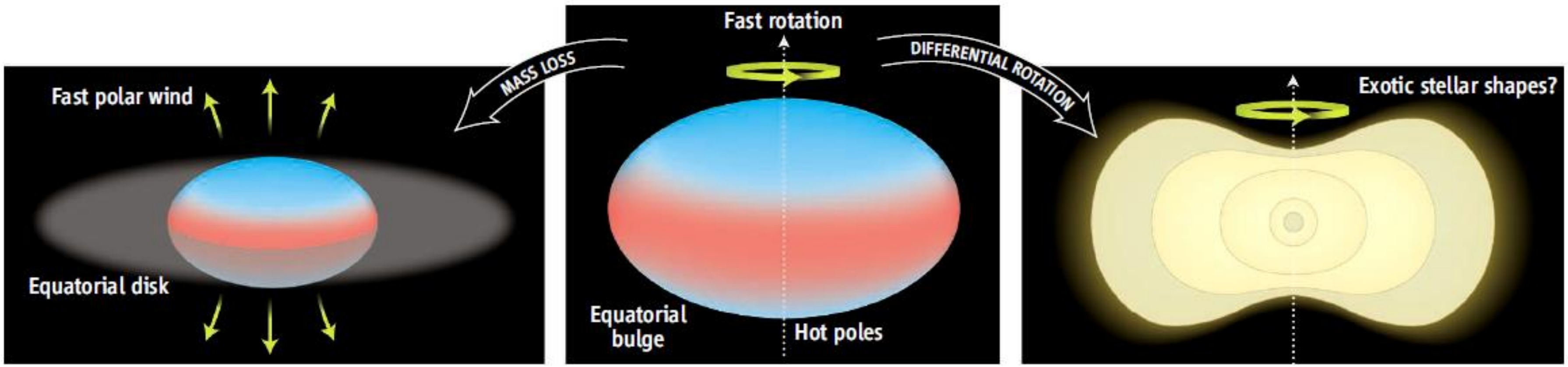
$$\boldsymbol{F} = -\frac{4ac}{3\kappa\varrho} T^3 \nabla T$$

$$T = T(\Psi) \qquad -\nabla\Psi = \boldsymbol{g}_{\mathrm{eff}}$$

$$\boldsymbol{F} = \frac{4ac}{3\kappa\varrho} T^3 \frac{dT}{d\Psi} \boldsymbol{g}_{\mathrm{eff}} = -k(\Psi)\boldsymbol{g}_{\mathrm{eff}}$$

$$g_{\mathrm{eff}} = \frac{GM}{R^2} - \omega^2 R \sin\vartheta$$

Effects of rotation



$T_{eff}^4 \propto F \propto g_{eff}$

Quirrenbach (2007; Science 317, 325)

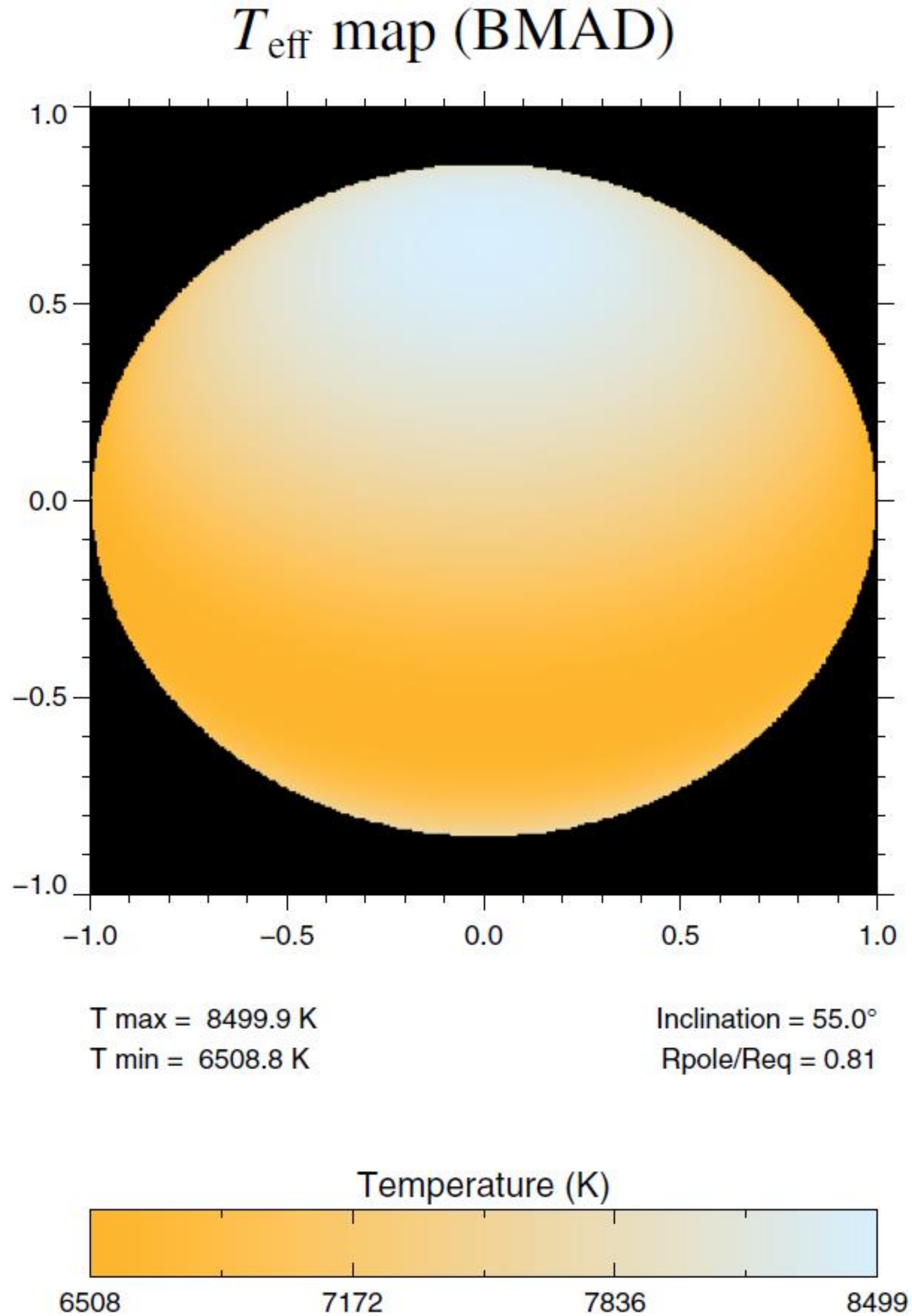
Observed gravity darkening

Altair (α Aquilae)

$v_{eq} \simeq 230$ km/sec

$T_{eff}^4 \propto g_{eff}$

Domiciano de Souza et al. (2005; A&A 442, 567)



Thermal imbalance

$$\boldsymbol{F} = \frac{4ac}{3\kappa\varrho}T^3\frac{dT}{d\Psi}\boldsymbol{g}_{\rm eff} = -k(\Psi)\boldsymbol{g}_{\rm eff}$$

$$\begin{aligned}\nabla\cdot\boldsymbol{F} &= -\frac{dk}{d\Psi}(\nabla\Psi)^2 - k(\Psi)\Delta\Psi \\ &= -\frac{dk}{d\Psi}(\nabla\Psi)^2 - k(\Psi)\left(4\pi G\varrho - \frac{1}{s}\frac{d(s^2\omega^2)}{ds}\right) = \varepsilon\varrho\end{aligned}$$

Not possible in general since $\varepsilon\rho = (\varepsilon\rho)(\Psi)$

Solution: Eddington-Sweet circulation

Meridional circulation

$$\nabla \cdot \boldsymbol{F} = \varepsilon \varrho - \varrho T \frac{d\sigma}{dt}$$

$$T \frac{d\sigma}{dt} = c_P \frac{dT}{dt} - \frac{\delta}{\varrho} \frac{dP}{dt}$$

Meridional circulation

$$\nabla \cdot \boldsymbol{F} = \varepsilon\varrho - \varrho T \frac{d\sigma}{dt}$$

$$T\frac{d\sigma}{dt} = c_P \frac{dT}{dt} - \frac{\delta}{\varrho}\frac{dP}{dt}$$

$$\nabla \cdot \boldsymbol{F} = \varepsilon\varrho - c_P\varrho\frac{\partial T}{\partial t} + \delta\frac{\partial P}{\partial t} - \boldsymbol{v}\cdot[c_P\varrho\nabla T - \delta\nabla P]$$

$$\nabla \cdot \boldsymbol{F} = \varepsilon\varrho - c_P\varrho T\boldsymbol{v}\cdot\left[\frac{1}{T}\nabla T - \frac{\delta}{c_P\varrho T}\nabla P\right]$$

$$\nabla \cdot \boldsymbol{F} = \varepsilon\varrho - \frac{c_P\varrho T}{P}(\nabla - \nabla_{\rm ad})(\boldsymbol{v}\cdot\nabla P)$$

Slow rotation

$$\nabla \cdot \boldsymbol{F} = -\frac{dk}{d\Psi}(\nabla\Psi)^2 - k(\Psi)\left(4\pi G\varrho - \frac{1}{s}\frac{d(s^2\omega^2)}{ds}\right)$$

$$\nabla \cdot \boldsymbol{F} = \varepsilon\varrho - \left[\frac{c_P\varrho^2 T}{P}(\nabla - \nabla_{\mathrm{ad}})g\right]_0 v_r$$

$$\mathrm{div}(\rho\mathbf{v}) = 0$$

$$\frac{1}{r^2}\frac{\partial(\varrho r^2 v_r)}{\partial r} + \frac{1}{r\sin\vartheta}\frac{\partial(\varrho v_\vartheta \sin\vartheta)}{\partial\vartheta} = 0$$

Slow rotation, general case

$$\nabla P = -\rho \nabla \Phi + \rho c$$

$$\nabla \cdot \boldsymbol{F} = \varepsilon \varrho + \left[\frac{c_P \varrho^2 T}{P} (\nabla - \nabla_{\mathrm{ad}}) g \right]_0 v_r$$

$$\boldsymbol{F} = -\frac{4ac}{3\kappa\varrho} T^3 \nabla T \; ,$$

$$\Delta\Phi = 4\pi G \varrho \; .$$

$$c_r = \omega^2 r \sin^2 \vartheta = \frac{2}{3} \omega^2 r (1 - L_2) \; , \quad \omega = \omega(r, \vartheta)$$

$$c_\vartheta = \omega^2 r \sin \vartheta \cos \vartheta = -\frac{1}{3} \omega^2 r \frac{\partial L_2}{\partial \vartheta}$$

$$L_2(\vartheta) = (3 \cos^2 \vartheta - 1)/2 = \mathcal{P}_2(\cos \vartheta)$$

Expansion of solution

$$P(r,\vartheta) = P_0(r) + P_2(r)L_2(\vartheta)\ , \quad T = T_0 + T_2L_2, \quad \Phi = \Phi_0 + \Phi_2L_2$$

$$F_r = F_{r0}(r) + F_{r2}(r)L_2\ , \quad F_\vartheta = F_{\vartheta 2}(r)\frac{dL_2(\vartheta)}{d\vartheta}$$

$$v_r = 0 + v_{r2}(r)L_2(\vartheta)\ , \quad v_\vartheta = v_{\vartheta 2}(r)\frac{dL_2(\vartheta)}{d\vartheta}$$

$$\boxed{\frac{dP_0}{dr} = -\varrho_0\frac{d\Phi_0}{dr} + \frac{2}{3}\varrho_0\omega^2 r}$$

$$\frac{dP_2}{dr} = -\varrho_0\frac{d\Phi_2}{dr} - \varrho_2\frac{d\Phi_0}{dr} - \frac{2}{3}\varrho_0\omega^2 r$$

$$P_2 = -\rho_0\Phi_2 - \frac{1}{3}\rho_0\omega^2 r^2$$

Estimate velocity (results)

$$\chi = \frac{\omega^2}{2\pi G\bar{\rho}}$$

$$v_r \approx \frac{L}{\bar{g}m}\chi \approx \frac{LR^2}{GM^2}\chi$$

$$\tau_{\rm circ} \approx \frac{R}{v_r} \approx \frac{GM^2}{LR}\frac{1}{\chi} \approx \frac{\tau_{\rm KH}}{\chi}$$

More realistic rotation (shellular rotation)

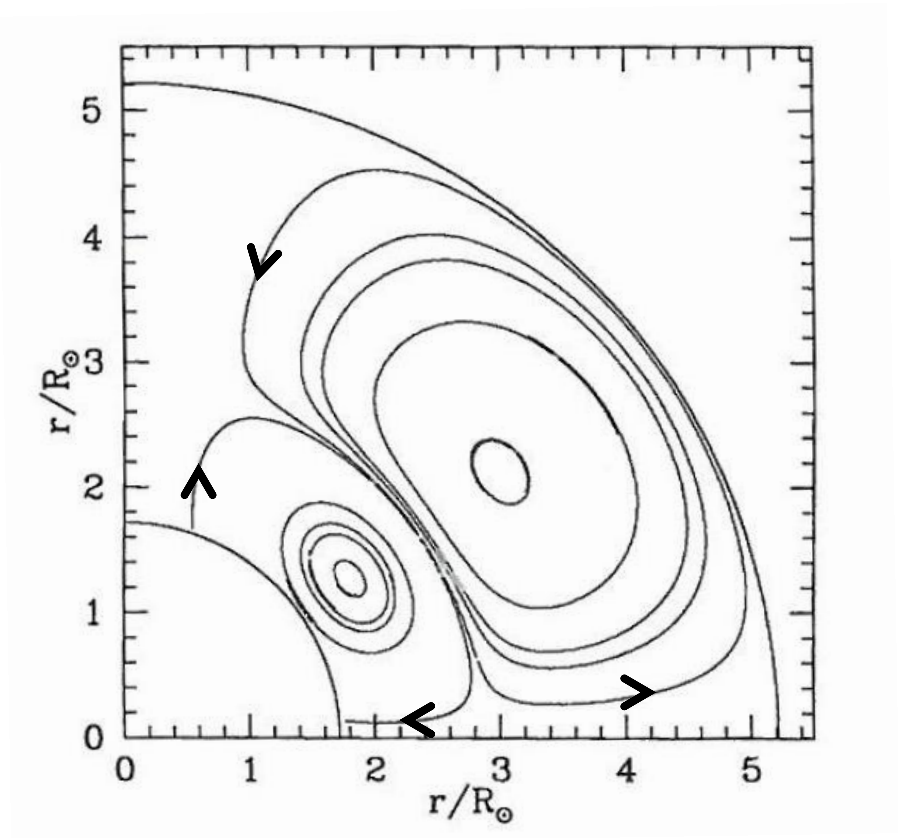
$$\omega \simeq \bar{\omega}(r) + \omega^*(r,\theta)$$

where $\omega^* \sim O(\chi)$.

Including a second term then gives (again very roughly)

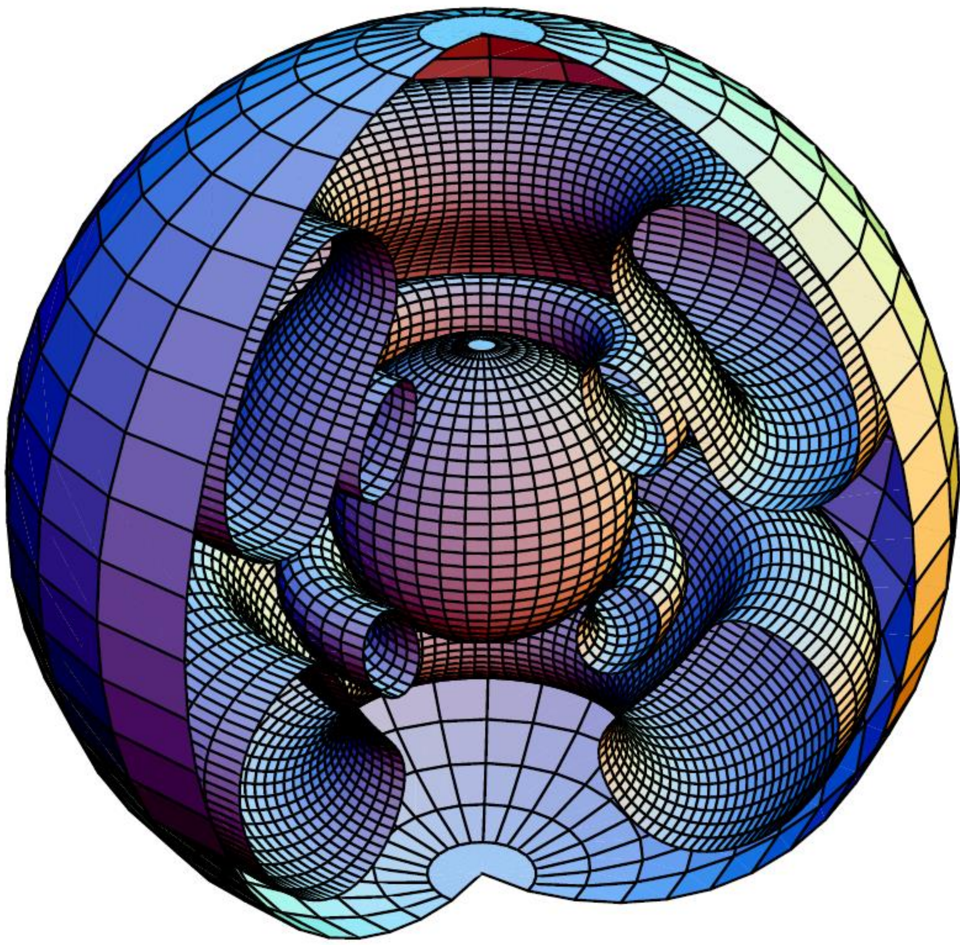
$$v_r \simeq \frac{LR^2}{GM^2}\chi\left(1 - \frac{\bar{\omega}^2}{2\pi G\rho}\right)$$

Two-cell circulation

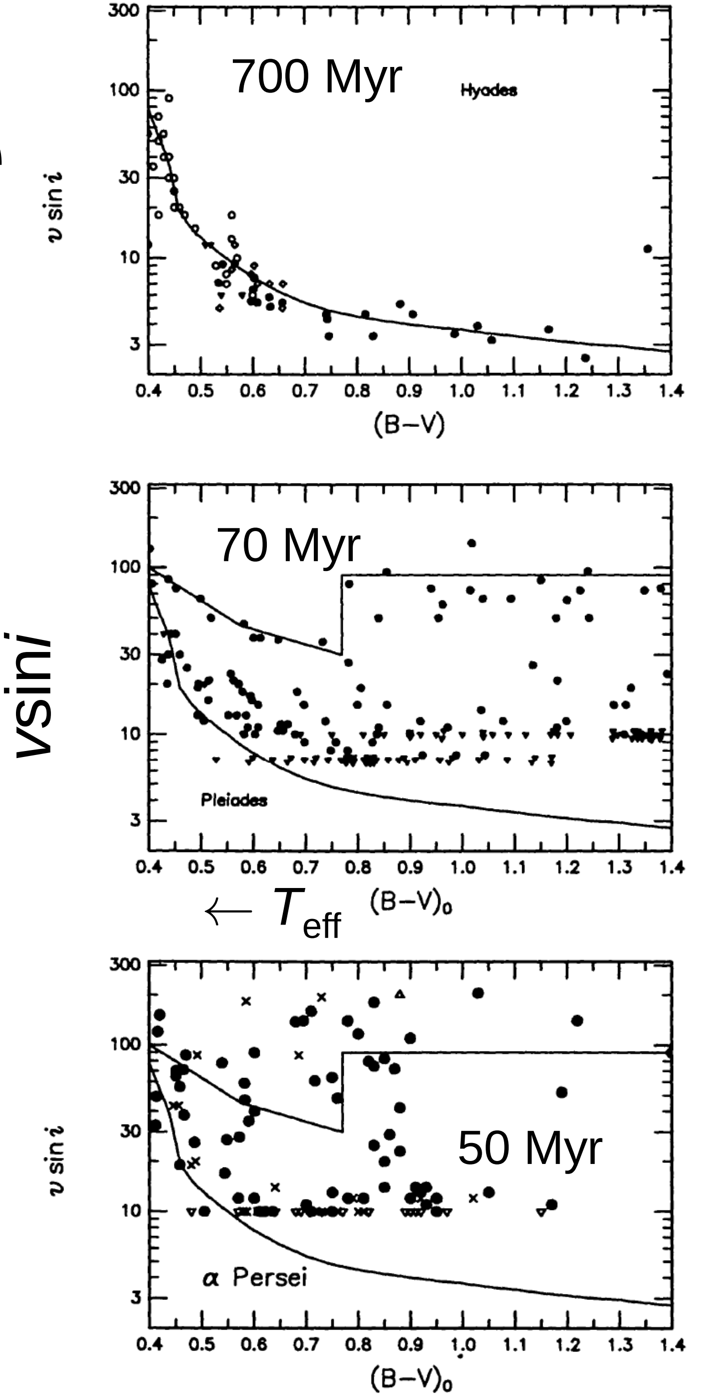


$$M = 20M_{\odot},\ v_{\rm ini} = 300\,{\rm km\,s^{-1}}$$

Meynet & Maeder (2002; A&A 390, 561)

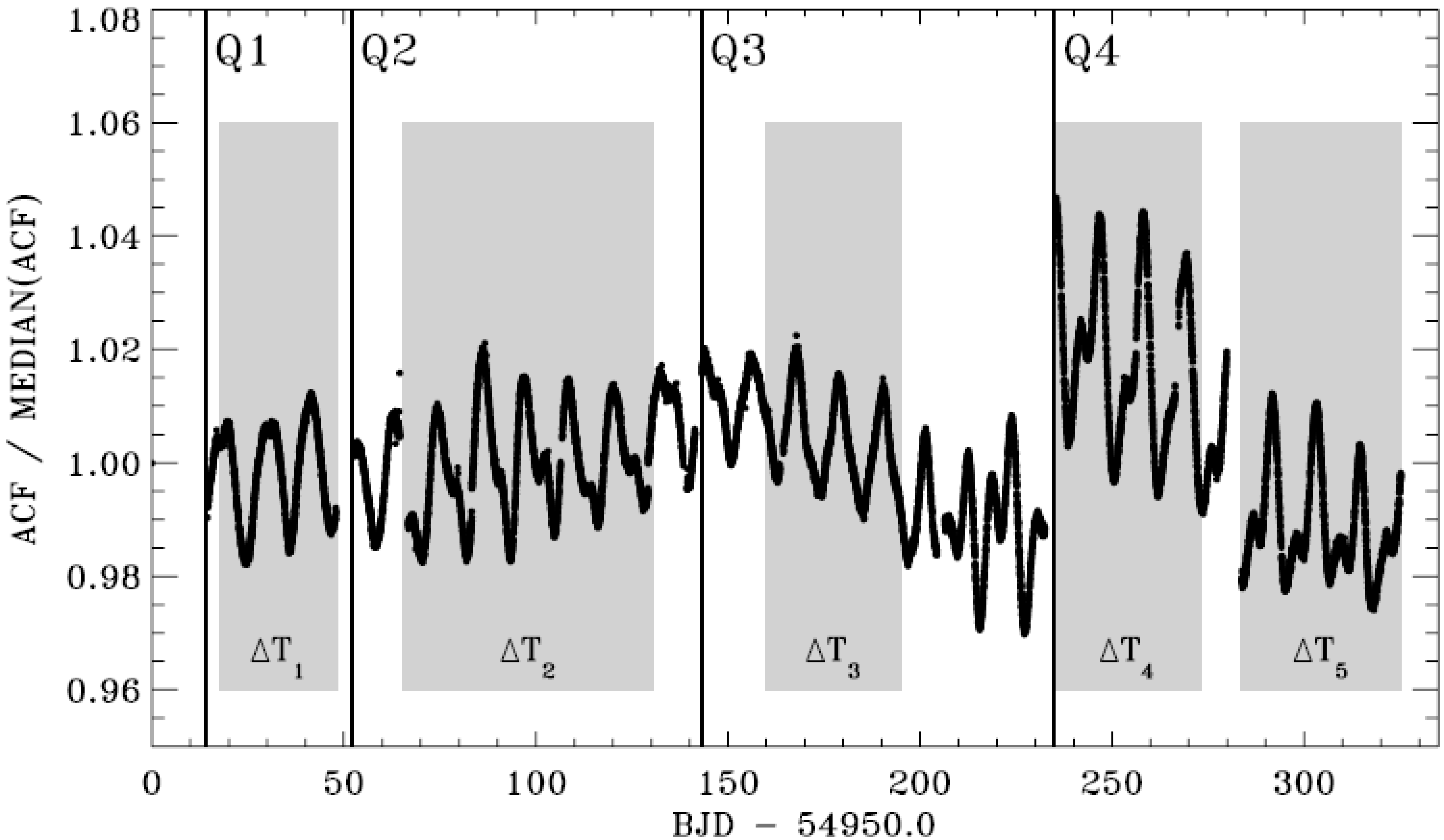


Evolution with age



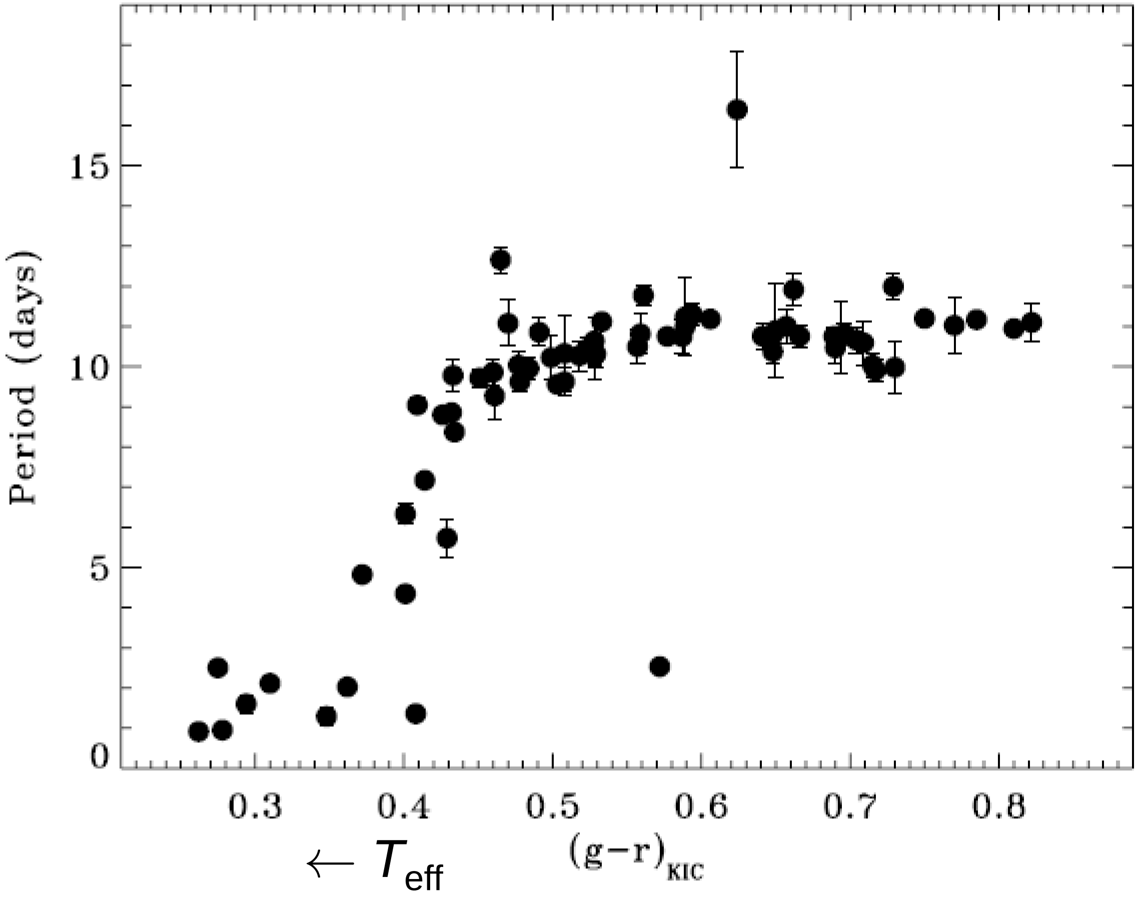
Soderblom et al. (1993; ApJ 409, 629)

Kepler rotation measurement in NGC 6811



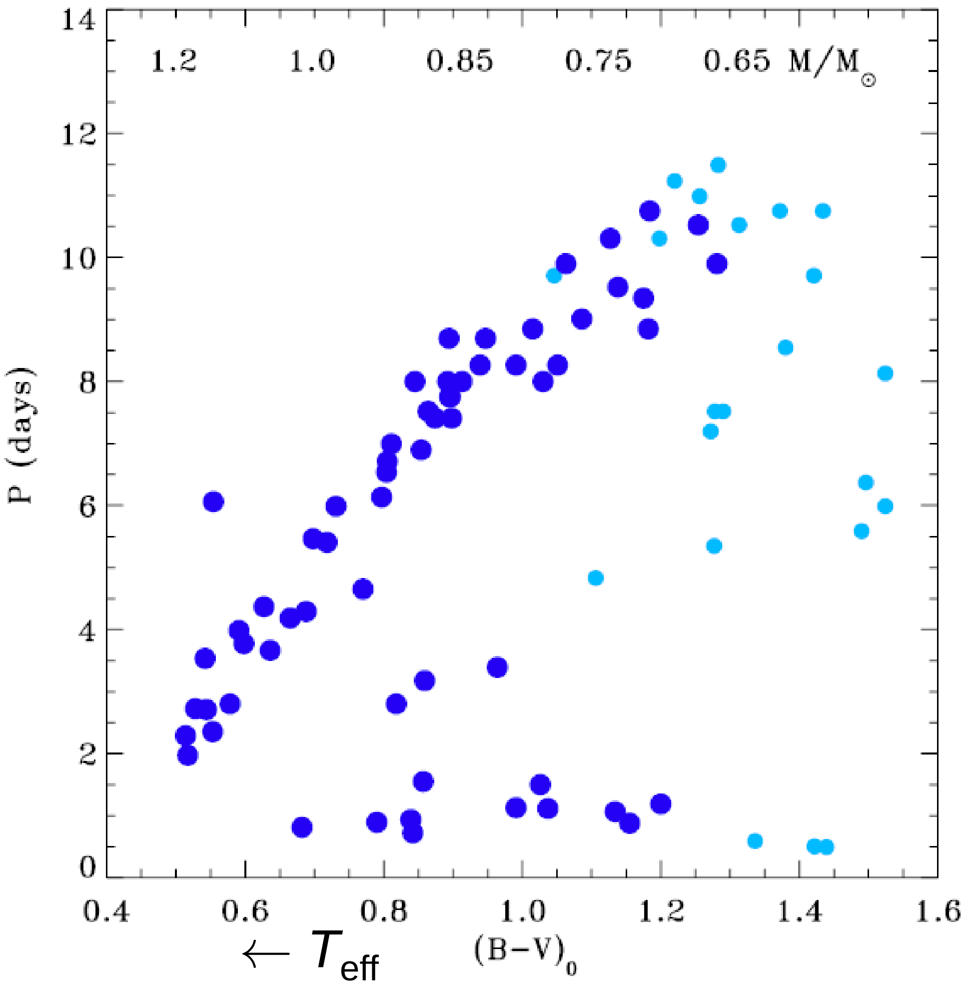
Meibom et al. (2011; ApJ 733, L9)

Kepler rotation measurement in NGC 6811



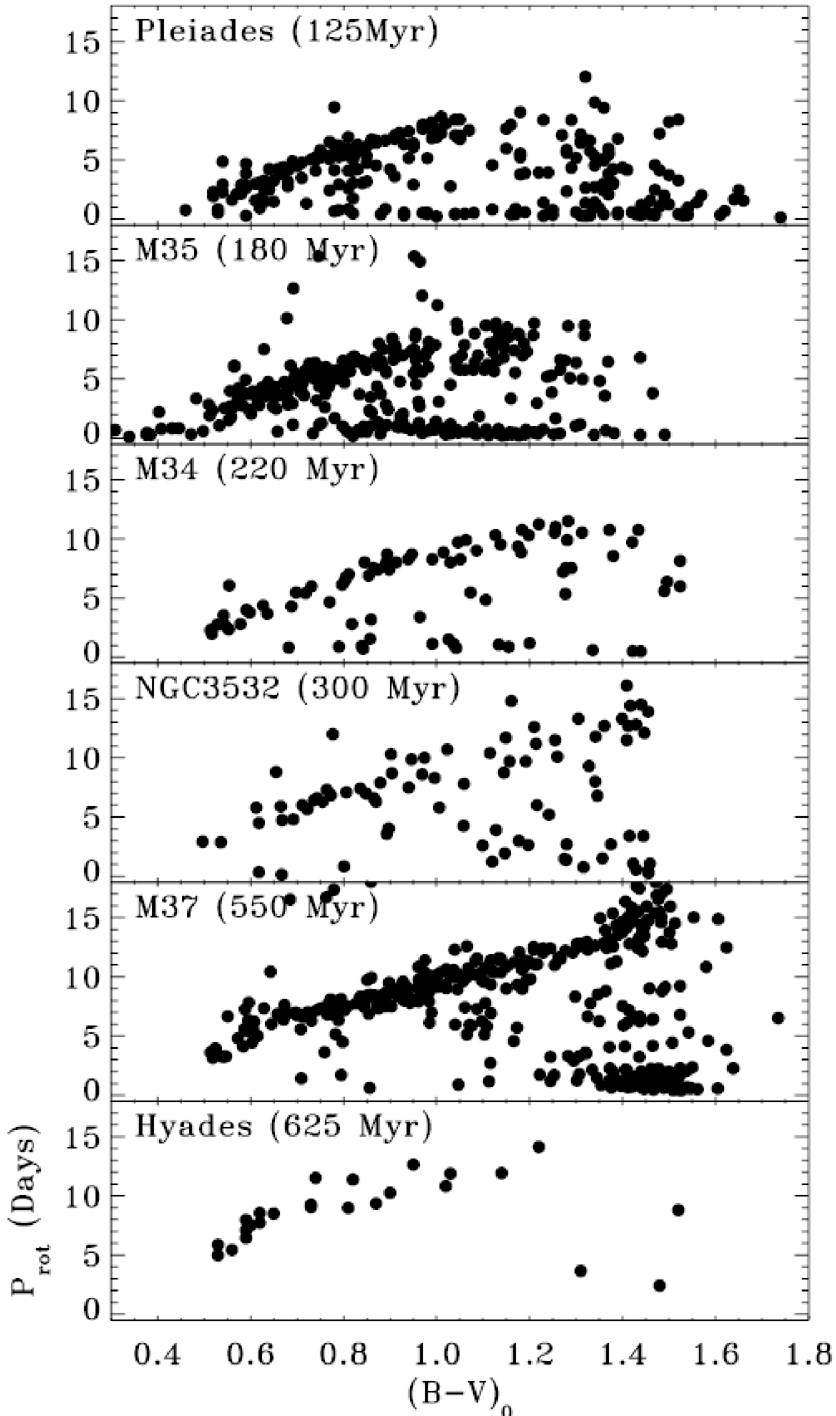
Meibom et al. (2011; ApJ 733, L9)

Rotation in M34



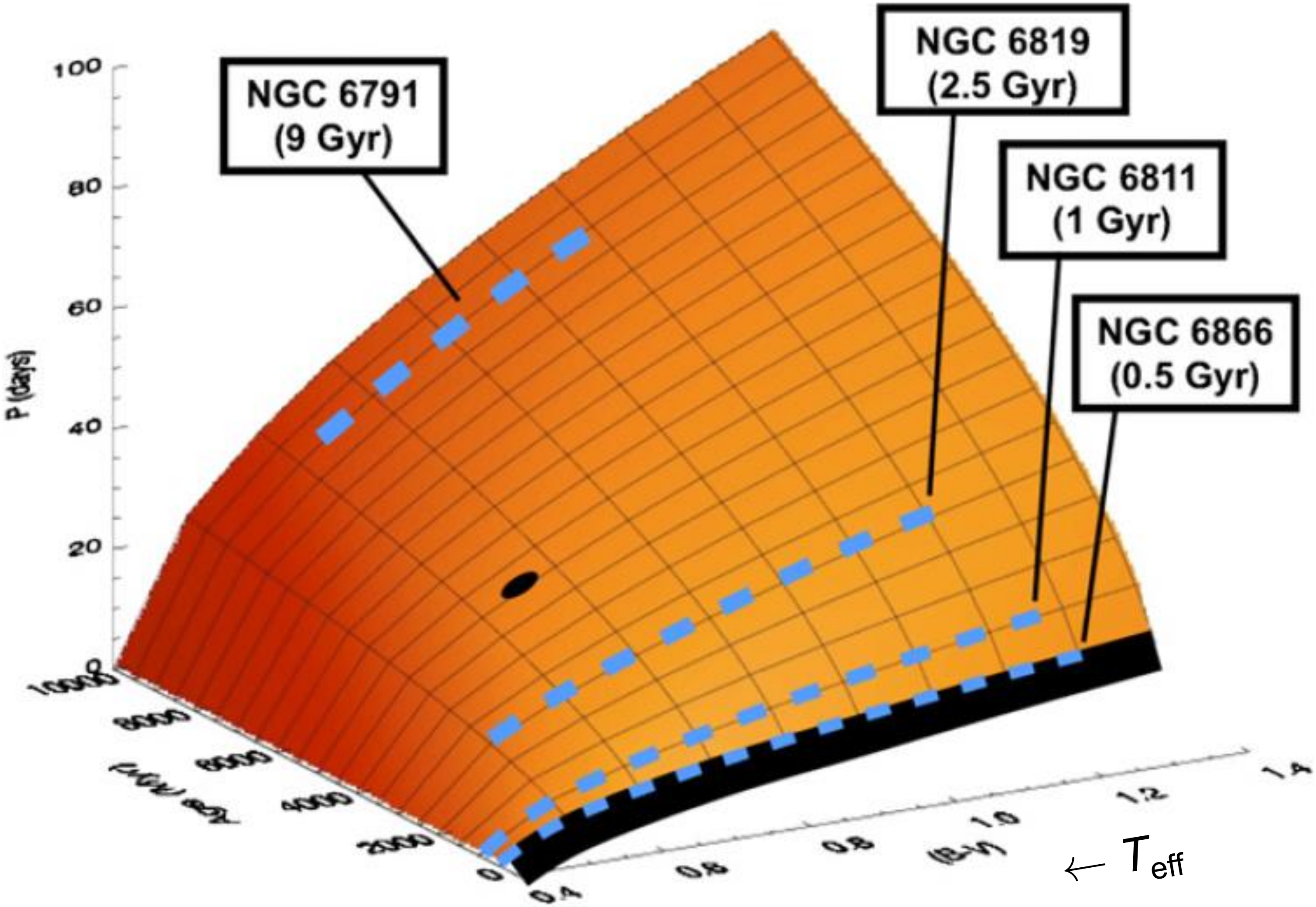
Meibom et al. (2011; ApJ 733, 115)

Comparison of clusters



Meibom et al. (2011; ApJ 733, 115)

Gyrochromochronology



Meibom et al. (2011; ApJ 733, L9)

Evolution of internal rotation

- Loss of angular momentum?
- Redistribution of angular momentum

$$J = \int_V \rho v_\phi s \mathrm{d}V = \int_V \rho \omega s^2 \mathrm{d}V$$

Constant ω:

$$J = \omega I$$

$$I = 2\pi \int_0^R \int_0^\pi \rho r^2 \sin^2 \vartheta r^2 \sin \vartheta \mathrm{d}r \mathrm{d}\vartheta = \frac{8\pi}{3} \int_0^R \rho r^4 \mathrm{d}r$$

Scaling relations

$$I = \frac{8\pi}{3}\int_0^R \rho r^4 \mathrm{d}r \propto MR^2$$

At constant J,

$$\omega \propto R^{-2}$$

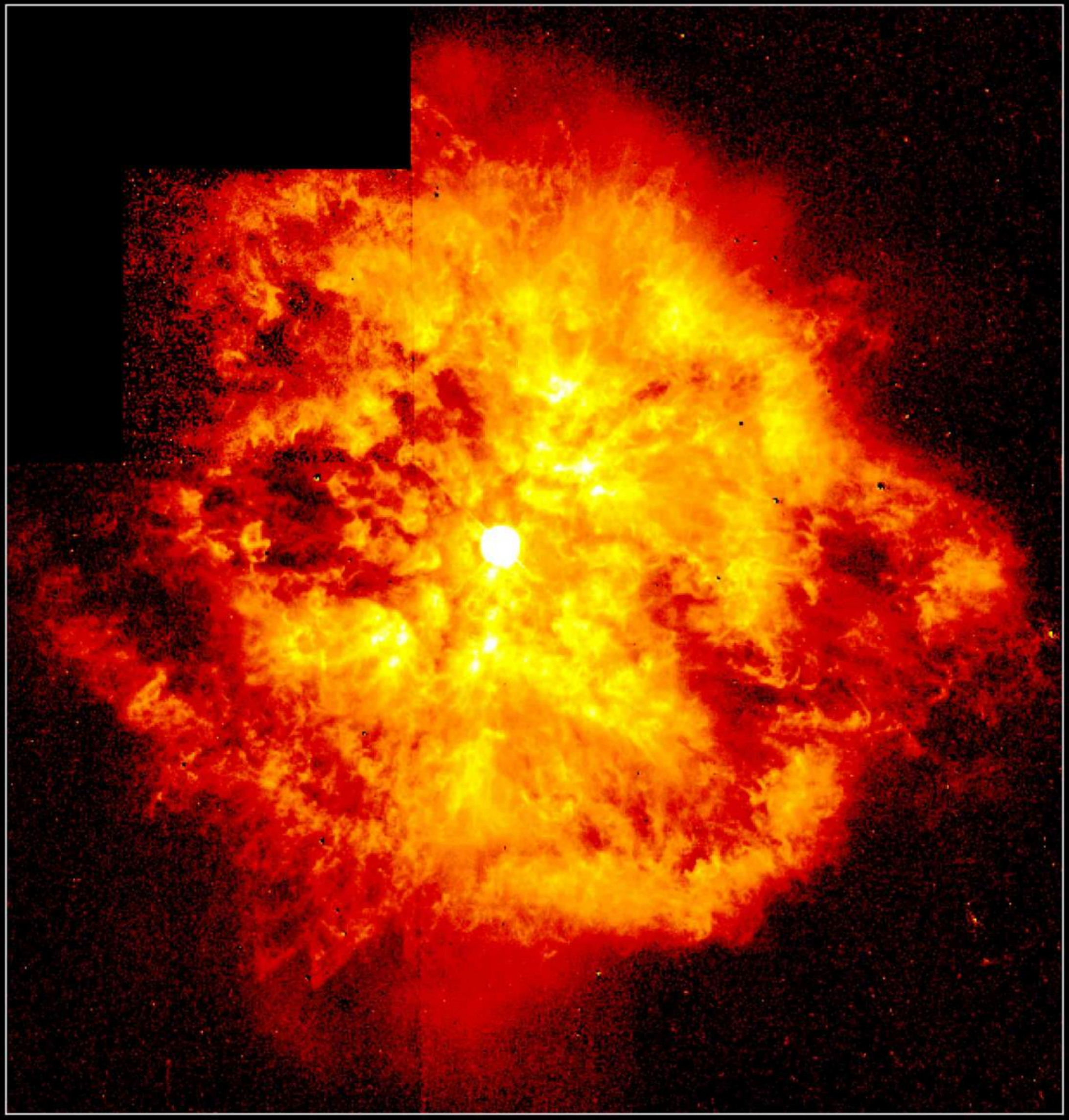
Local conservation in core:

$$\omega_{\mathrm{c}} \propto r_{\mathrm{c}}^{-2}$$

$$\omega_{\mathrm{c}}/\omega_{\mathrm{s}} \propto (R/r_{\mathrm{c}})^2 \propto (\rho_{\mathrm{c}}/\bar{\rho})^{2/3}$$

Mass loss – general features

- **Causes loss of angular momentum**
  - In massive stars, mass loss is chiefly a consequence of radiation pressure on grains and atoms. In quite massive stars, shocks and turbulence may be very important.
  - In low-mass stars, magnetically dominated stellar winds



Nebula M1-67 around Star WR224
Hubble Space Telescope • WFPC2

PRC98-38 • STScI OPO • Y. Grosdidier and A. Moffat (University of Montreal) • NASA

The Wolf-Rayet star WR224 is found in the nebula M1-67 which has a diameter of about 1000 AU

The wind is clearly very clump and filamentary.

Mass loss in massive stars

- Radiation pressure on spectral lines
- Depends on radiative flux F and line spectrum in wind

$$\dot{M} \sim A(T_{\rm eff})F \sim A(T_{\rm eff})g_{\rm eff}$$

note that

$$F \propto T_{\rm eff}^4 \propto g_{\rm eff}$$

A depends on details of line spectrum.

STELLAR WINDS & ROTATION

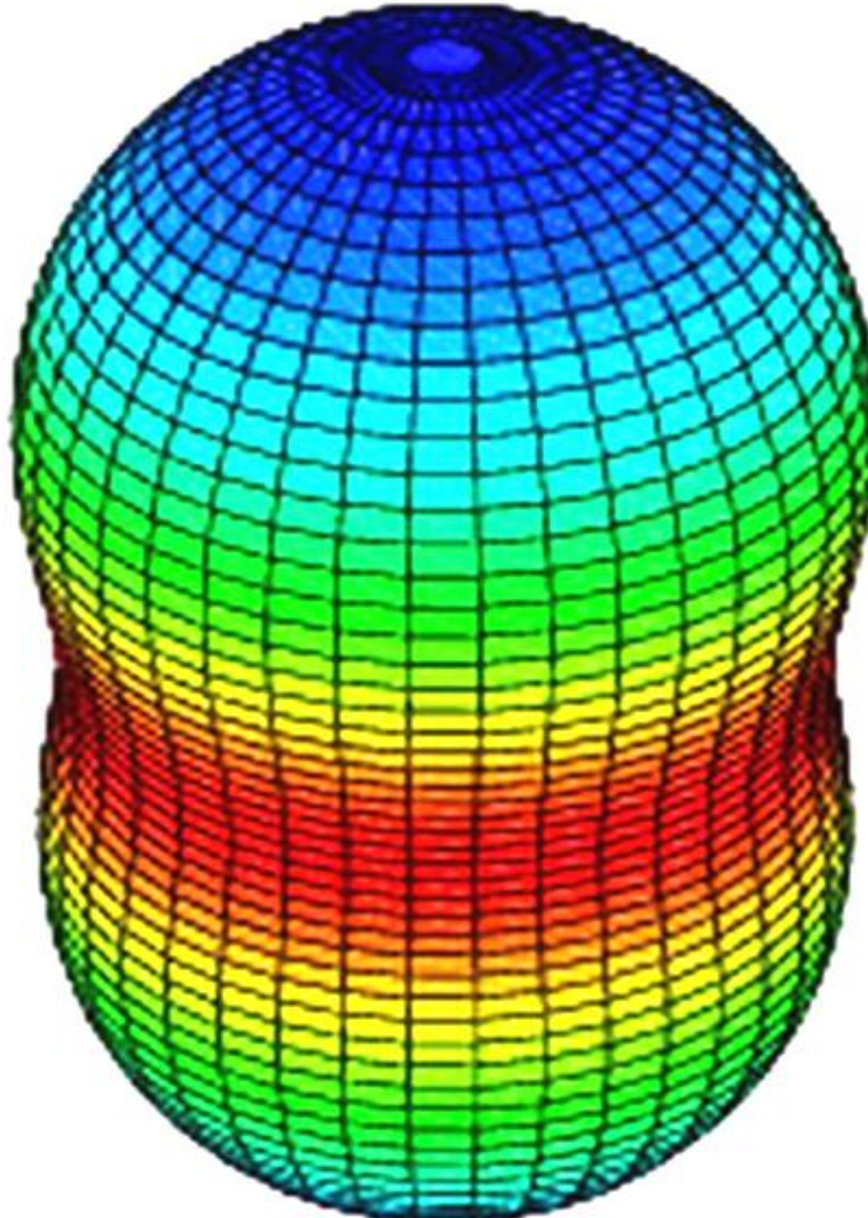
Meynet & Maeder
(1999; A&A 372, L9)

$$30\,000\,K$$

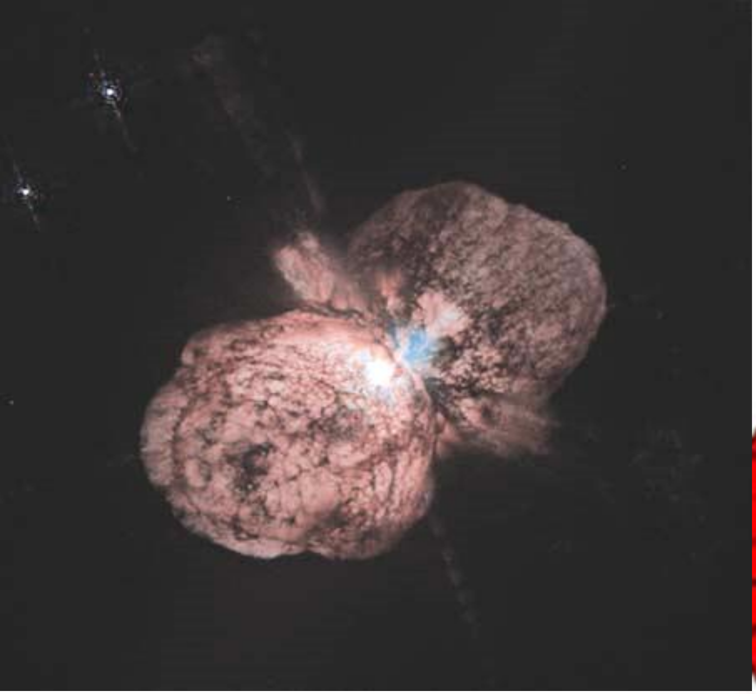
$$L = 10^6\, L_0$$

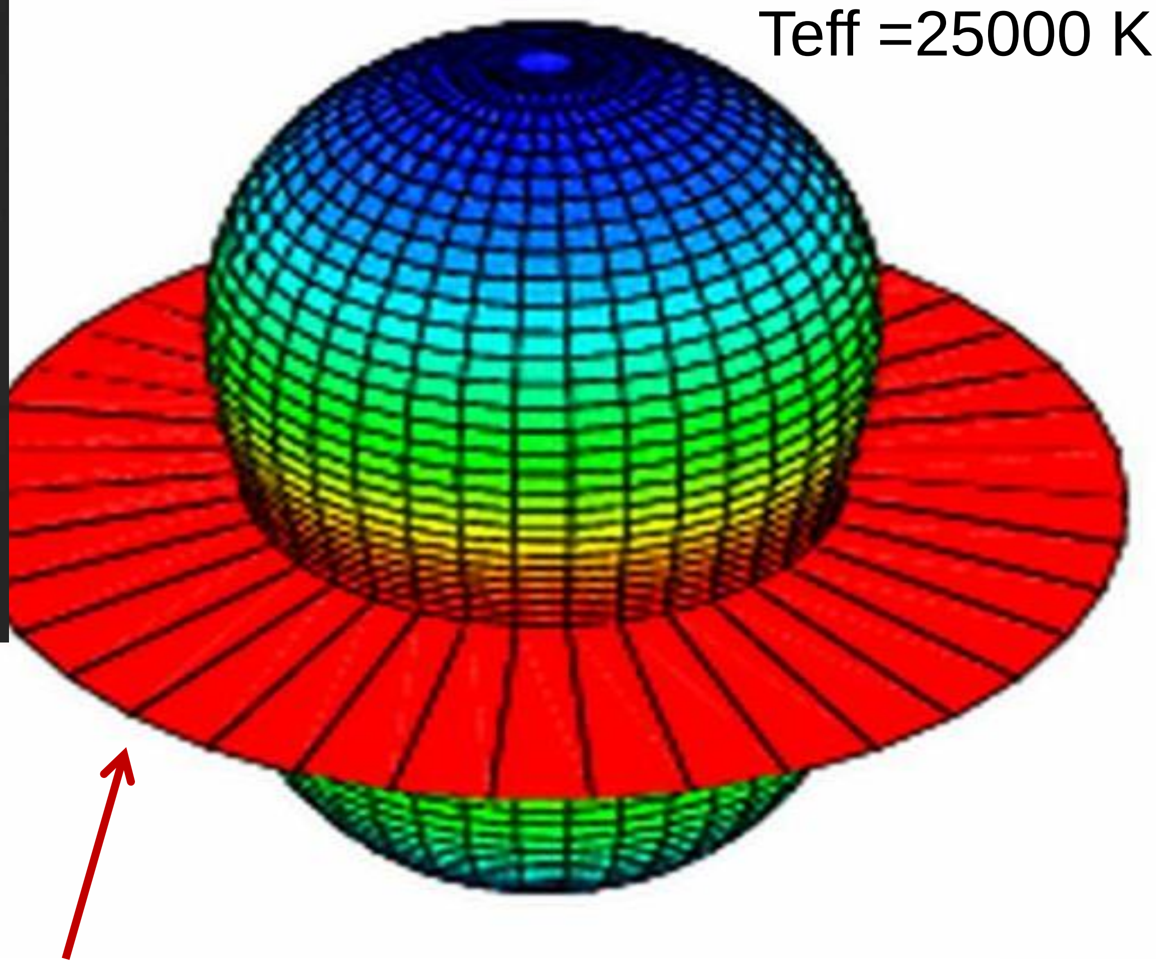
$$\omega^2 = 0.64\,\omega^2_{\text{crit}}$$

Enables a massive star to lose lots of mass and little angular momentum
→ GRBs



iso mass loss André Maeder



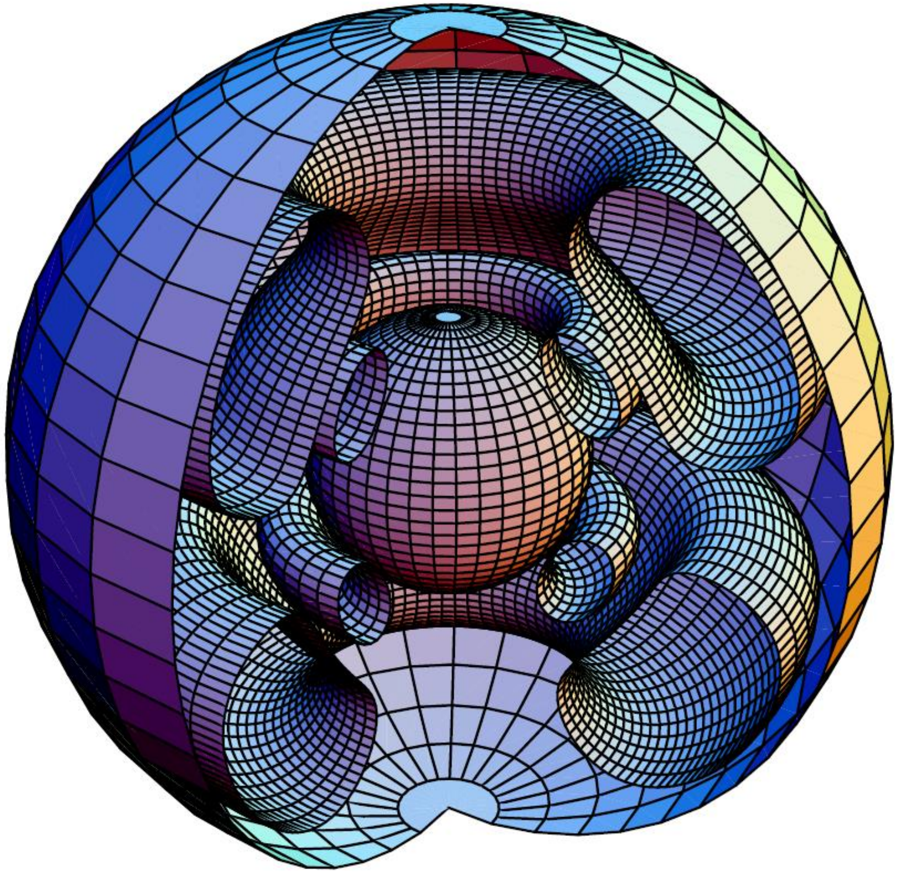


LARGE
ENHANCEMENTS FROM A!

André Maeder

Eta Carina

Effects on evolution



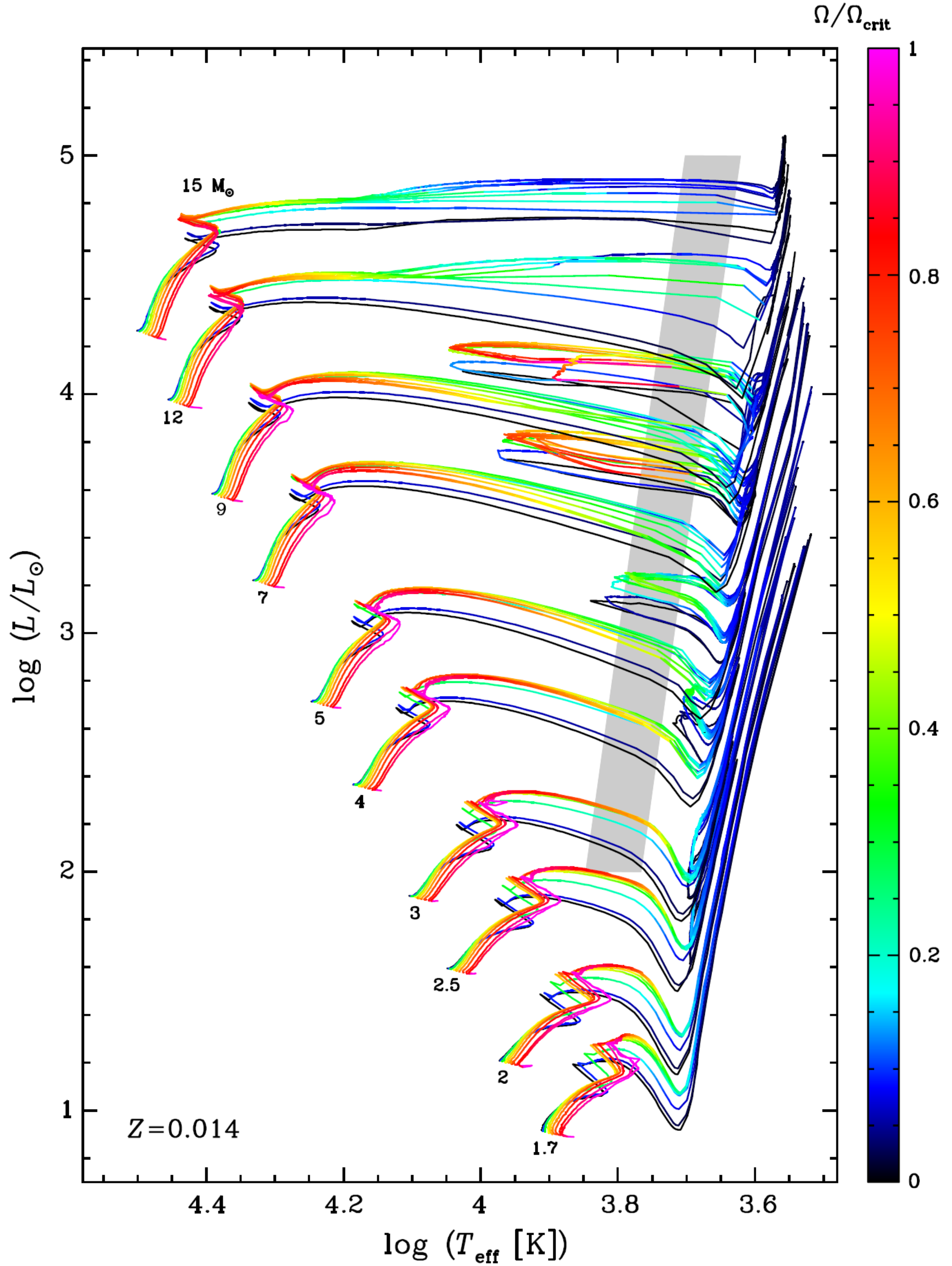
STRUCTURE
- Oblateness (interior, surface)
- New structure equations
- Shellular rotation

MASS LOSS
- Stellar winds
- Anisotropic losses of mass and angular momentum

MIXING
- **Meridional circulation**
- **Shear instabilities + diffusion**
- **Horizontal turbulence**
- Advection + diffusion of angular momentum
- Transport + diffusion of elements

André Maeder

Effect of rotation



Georgy et al. (2013; A&A 553, A24)

Results of rotational mixing in massive stars (I)

- Fragile elements like Li, Be, B destroyed to a greater extent when rotational mixing is included. More rotation, more destruction.
- Higher mass loss
- Initially luminosities are lower (because g is lower) in rotating models. Later luminosity is higher because He-core is larger
- Broadening of the main sequence; longer main sequence lifetime

Results of rotational mixing in massive stars (II)

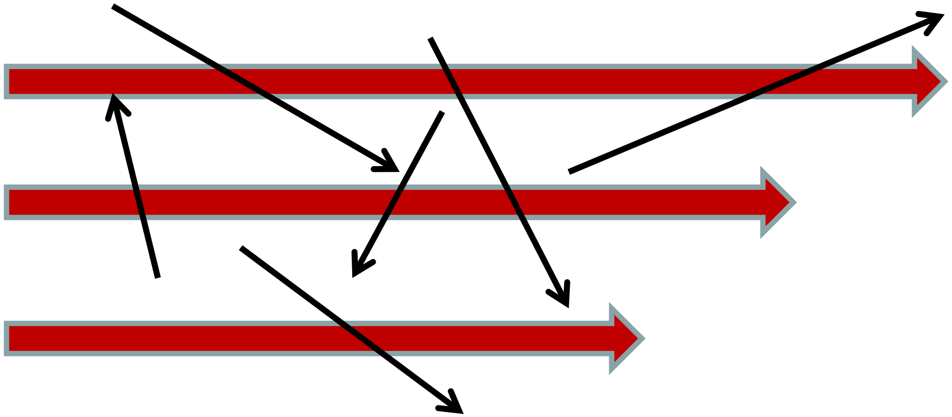
- More evidence of CN processing in rotating models. He, ^{13}C, ^{14}N, ^{17}O, ^{23}Na, and ^{26}Al are enhanced in rapidly rotating stars while ^{12}C, ^{15}N, $^{16,18}O$, and ^{19}F are depleted.
- Decrease in minimum mass for WR star formation.

These predictions are in some accord with what is observed.

Transport of angular momentum

- Reflects loss of angular momentum to stellar wind
- Controls evolution of angular momentum profile $\Omega(r)$
- Depends on Eddington-Sweet circulation and hence on $\Omega(r)$ and composition profile
- Depends on (highly uncertain) instabilities and turbulence which again depend on $\Omega(r)$
- Turbulence also controls composition profile

Momentum transport through viscosity



Viscosity $\nu \sim v_{\rm part}\ell_{\rm part}$

$$\frac{{\rm d}v}{{\rm d}t} = \frac{\partial v}{\partial t} + (v \cdot \nabla)v = g - \frac{1}{\rho}\nabla P + \nu\nabla^2 v$$

Viscosity probably dominated by turbulent viscosity.

Turbulent viscosity

- Relevant only in radiative region
- Caused by a variety of instabilities
- Motion in vertical direction suppressed by buoyancy
- Hence $v_h \gg v_v$
- Nearly uniform composition on spherical surfaces
- Nearly uniform Ω on spherical surfaces: shellular rotation

Transport of angular momentum

$$\frac{\partial}{\partial t}[\rho r^2 \,\overline{\Omega}] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4\,\overline{\Omega}\,U\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho \nu_v\, r^4 \frac{\partial \overline{\Omega}}{\partial r}\right]$$

Uniform rotation, uniform composition

$$U(r) = 2\,\frac{L}{Mg}\left(\frac{P}{C_P \rho T}\right)\frac{1}{\nabla_{ad} - \nabla}\left[1 - \frac{\varepsilon}{\varepsilon_m} - \frac{\Omega^2}{2\pi G\rho}\right]\frac{\tilde{g}}{g}$$

Zahn (1992; A&A 265, 115)

Transport of angular momentum

$$\frac{\partial}{\partial t}[\rho r^2 \overline{\Omega}] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \overline{\Omega}\, U\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho \nu_v r^4 \frac{\partial \overline{\Omega}}{\partial r}\right]$$

General case

NB!!

$$U(r) = \frac{L}{Mg}\left(\frac{P}{C_P \rho T}\right)\frac{1}{\nabla_{ad} - \nabla}(E_\Omega + E_\mu)$$

$$\begin{aligned} E_\Omega = {} & 2\left[1 - \frac{\Omega^2}{2\pi G\rho} - \frac{\varepsilon}{\varepsilon_m}\right]\frac{\tilde{g}}{g} \\ & - \frac{\rho_m}{\rho}\left[\frac{r}{3}\frac{\mathrm{d}}{\mathrm{d}r}\left(H_\mathrm{T}\frac{\mathrm{d}\Theta}{\mathrm{d}r} - \chi_\mathrm{T}\,\Theta\right) - 2\frac{H_\mathrm{T}}{r}\Theta + \frac{2}{3}\Theta\right] \\ & - \frac{\varepsilon}{\varepsilon_m}\left[H_\mathrm{T}\frac{\mathrm{d}\Theta}{\mathrm{d}r} + (\varepsilon_\mathrm{T} - \chi_\mathrm{T} - 1)\,\Theta\right] - \Theta\,, \end{aligned}$$

$$\Theta = \frac{1}{3}\frac{r^2}{g}\frac{\mathrm{d}\Omega^2}{\mathrm{d}r} = \frac{2}{3}\left(\frac{\Omega^2 r}{g}\right)\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\ln r}$$

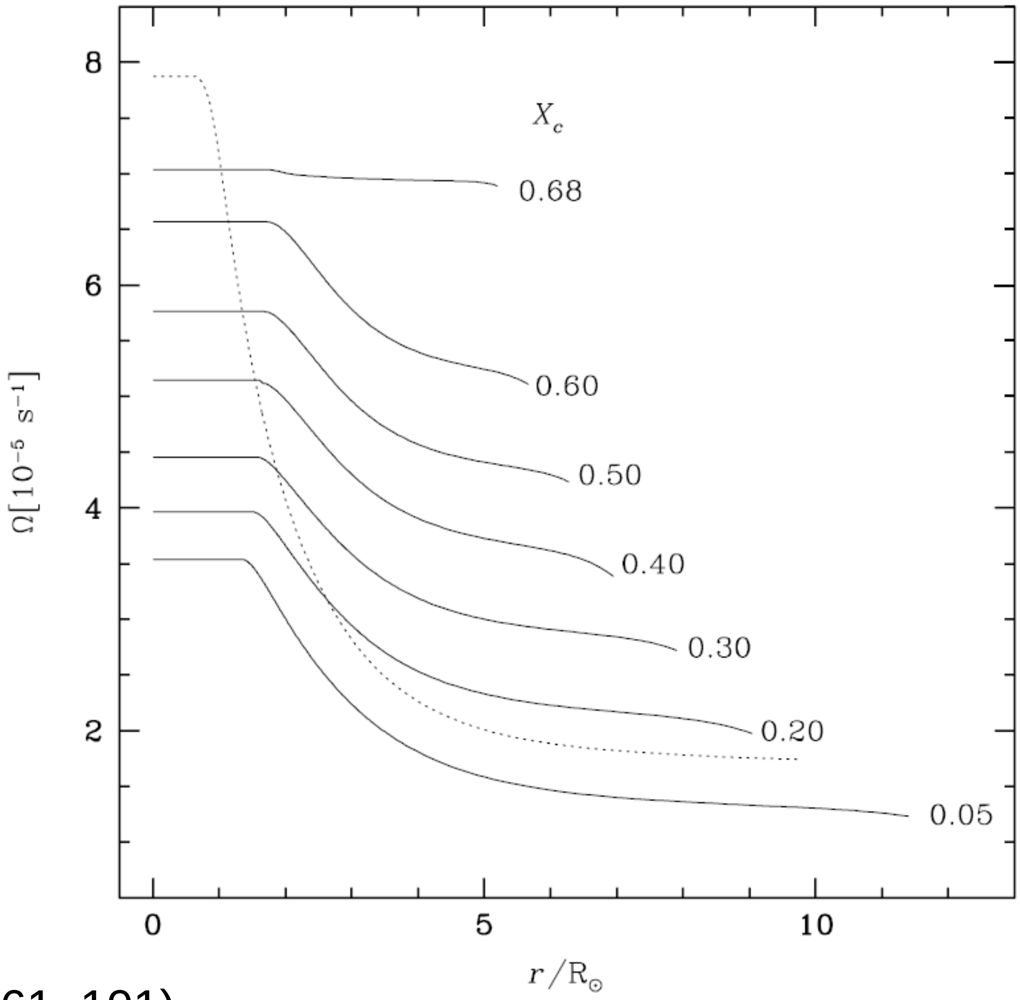
$$\frac{\tilde{g}}{g} \approx \frac{4}{3}\left(\frac{\Omega^2 r^3}{GM}\right)$$

$$\begin{aligned} E_\mu = {} & \frac{\rho_m}{\rho}\left[\frac{r}{3}\frac{\mathrm{d}}{\mathrm{d}r}\left(H_\mathrm{T}\frac{\mathrm{d}\Lambda}{\mathrm{d}r} - (\chi_\mu + \chi_\mathrm{T} + 1)\,\Lambda\right) - 2\frac{H_\mathrm{T}}{r}\Lambda\right] \\ & + \frac{\varepsilon}{\varepsilon_m}\left[H_\mathrm{T}\frac{\mathrm{d}\Lambda}{\mathrm{d}r} + (\varepsilon_\mu + \varepsilon_\mathrm{T} - \chi_\mu - \chi_\mathrm{T} - 1)\,\Lambda\right], \end{aligned}$$

$$\Lambda = \tilde{\mu}/\mu$$

$$\mu(r,\vartheta) = \bar{\mu} + \tilde{\mu}(r) P_2(\cos\vartheta)$$

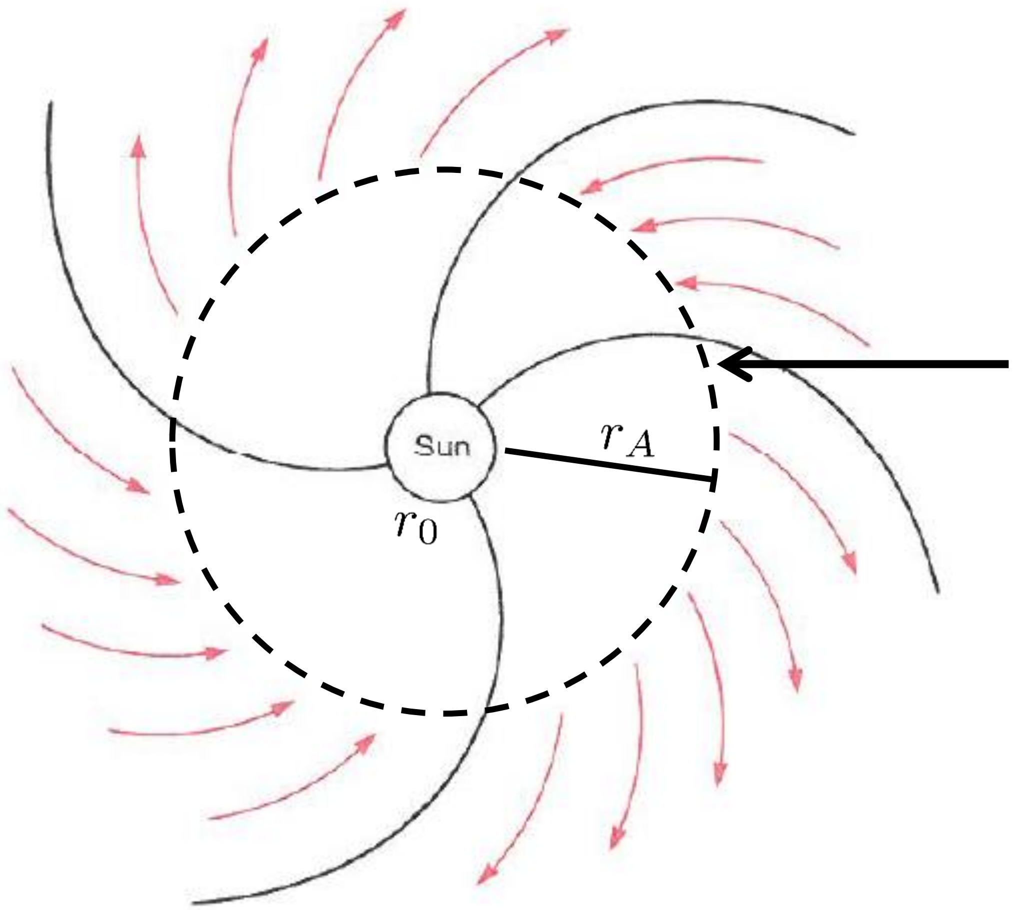
Evolution of angular velocity



$$M = 20 M_\odot,\ v_{\rm ini} = 300\,{\rm km\,s^{-1}}$$

Meynet & Maeder (2000; A&A 361, 101)

Magnetic wind in solar-like stars



Decoupling where

$$v = v_A = \frac{B_r^2}{4\pi\rho}$$

Simple model

$$\frac{\mathrm{d}J}{\mathrm{d}t} \propto -\rho_A r_A^2 v_A (\Omega r_A^2)$$

$$r_0^2 B_0 = r_A^2 B_A \qquad B_A^2 \propto \rho_A v_A^2$$

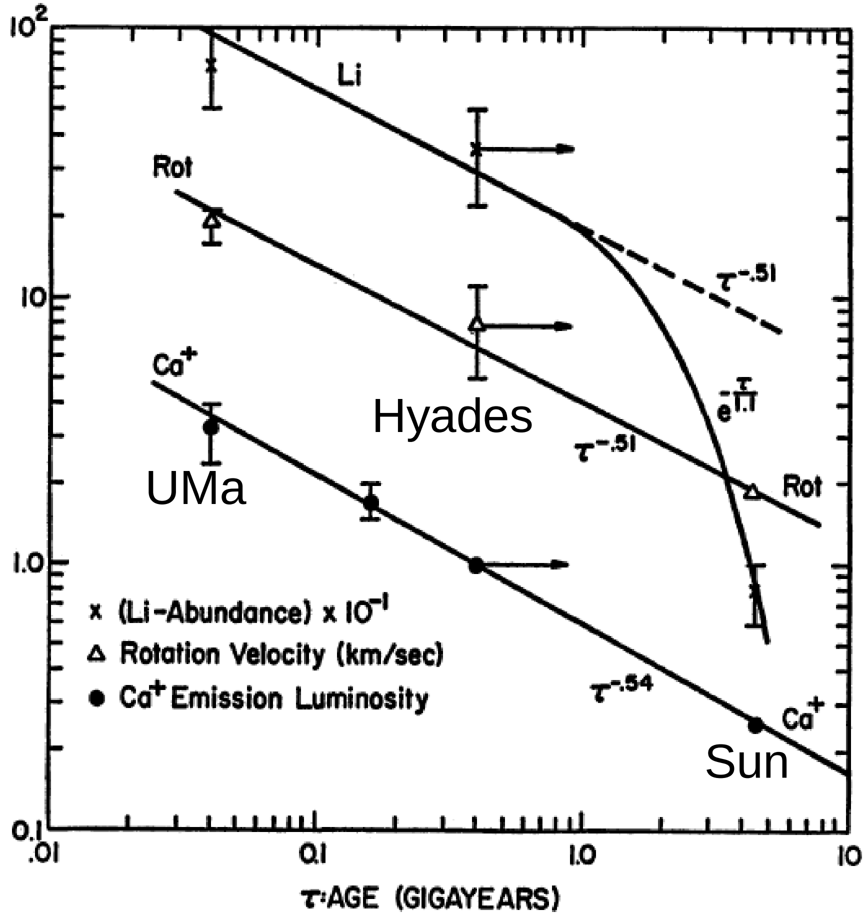
$$\frac{\mathrm{d}J}{\mathrm{d}t} \propto -B_0^2 \frac{\Omega}{v_A}$$

From generation of magnetic field, take $B_0 \propto \Omega$.
Take $v = c_s$ to be constant.

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} \propto -\Omega^3$$

$$\Omega \propto t^{-1/2}$$

The Skumanich law

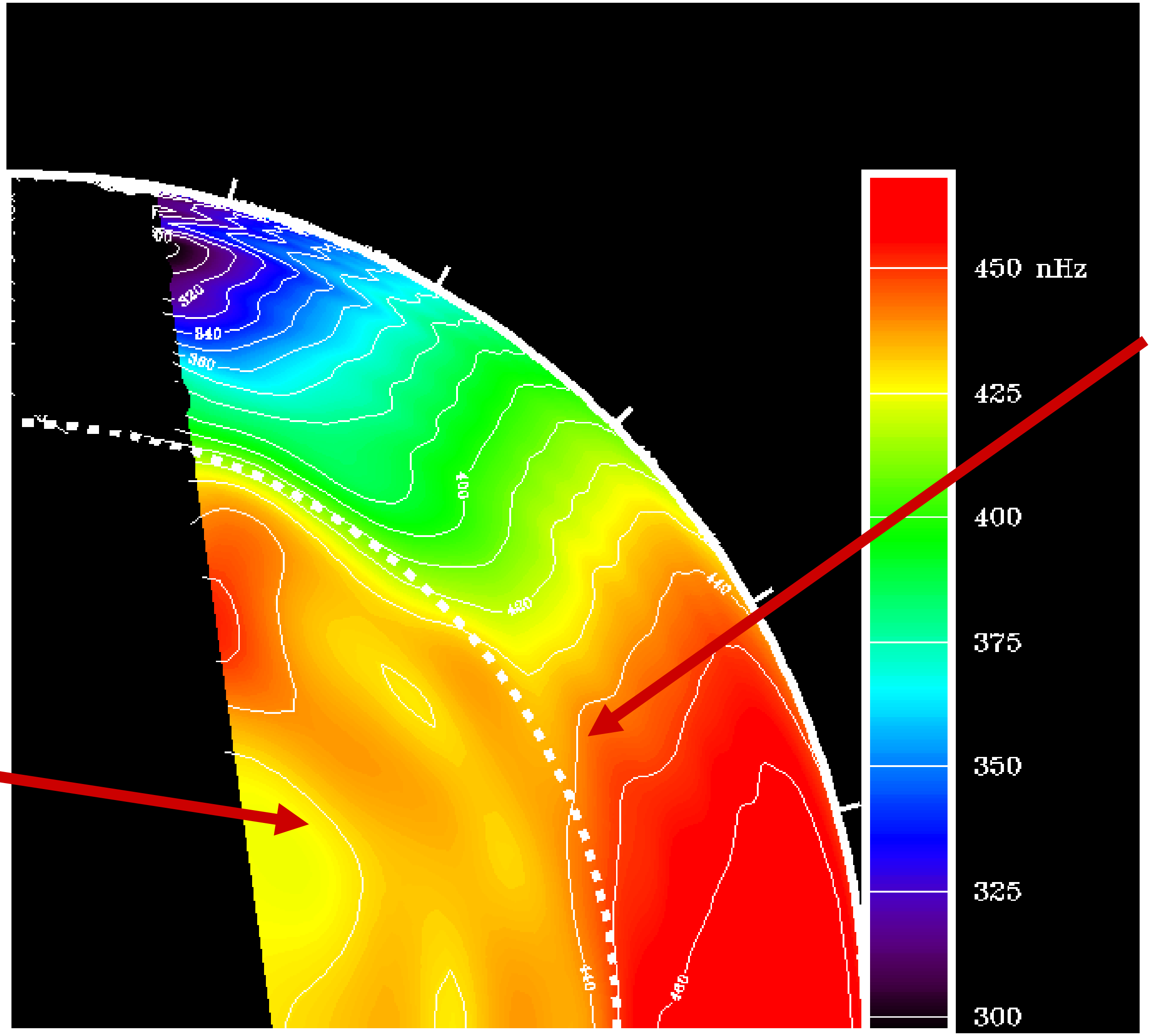


Skumanich (1972; ApJ 171, 565-567)

Lithium destruction

- Destroyed at $T \simeq 2.5\ 10^6$ K
- Strongly depleted in many stars relative to BBNS (a factor 140 in the Sun relative to meteorites)
- Requires extra mixing beneath convective envelope
- Related to rotational instabilities?
  Or convective overshoot?
  Or …..

Inferred solar internal rotation



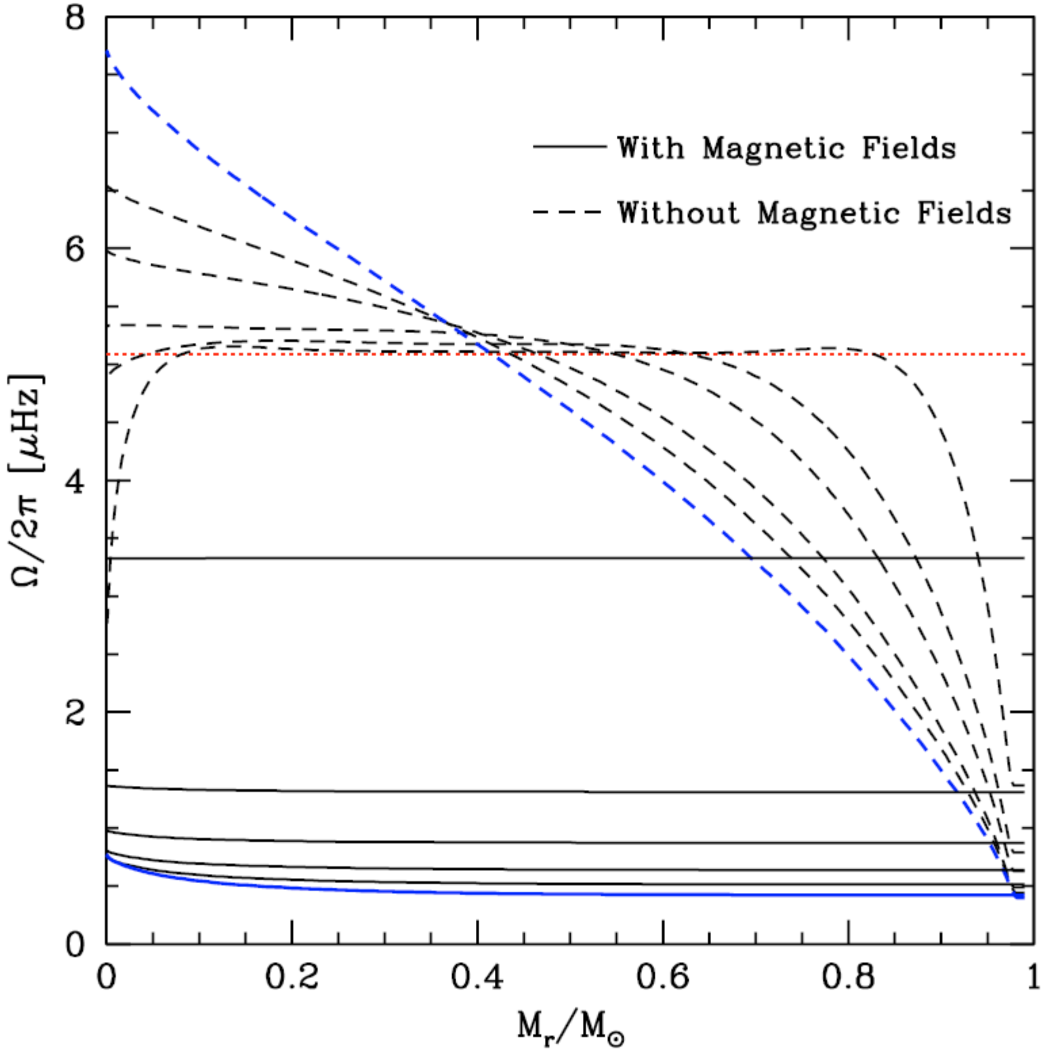
Base of convection zone Tachocline

Near solid-body rotation of interior

The solar rotation problem

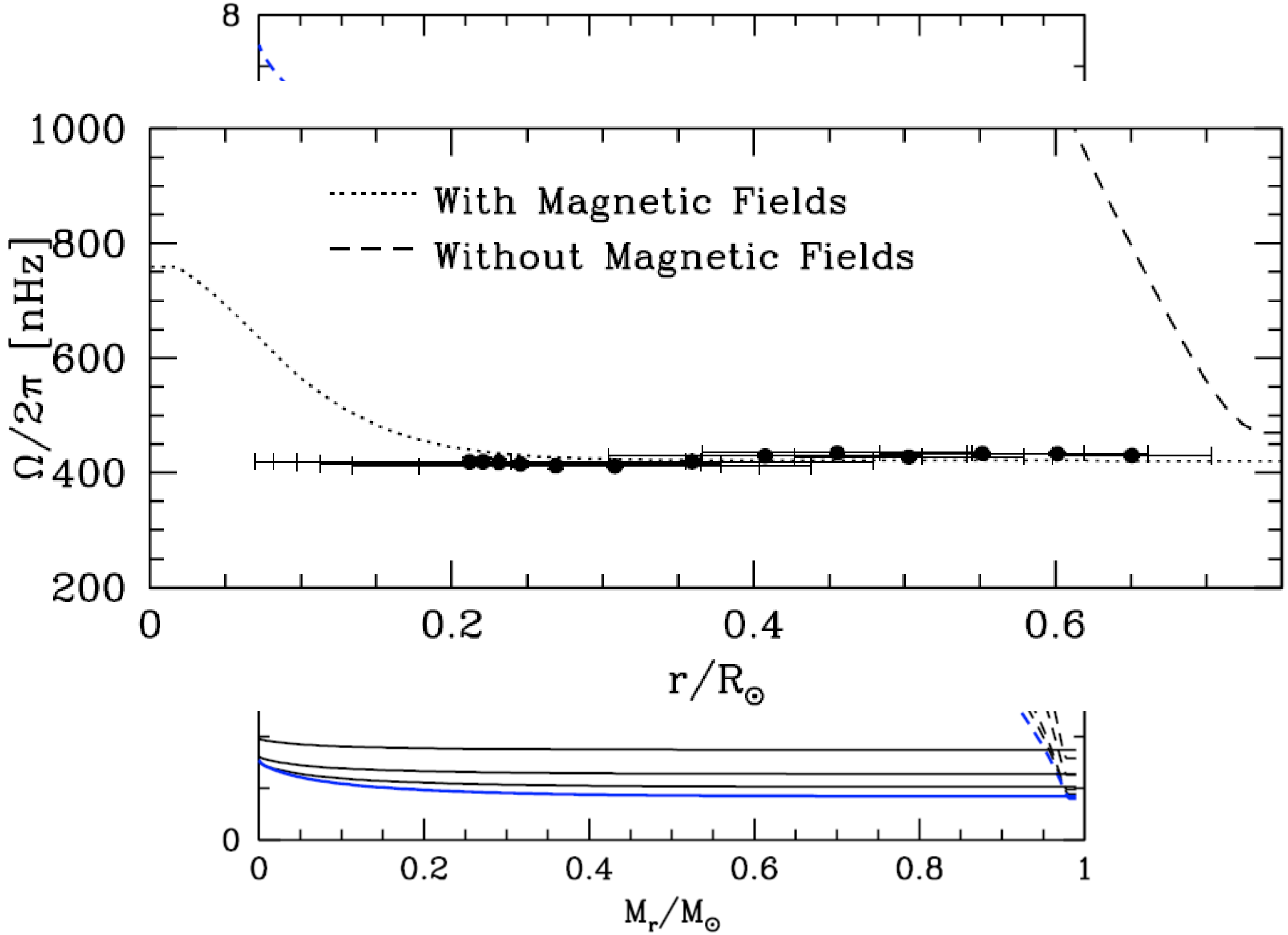
- Cannot be reproduced by simple hydrodynamic models
- How is the tachocline established and maintained?
- Gravity-wave transport?
- Magnetic fields?
- Relation to lithium depletion??

Modelling solar rotation



Eggenberger et al. (2005; A&A 440, L9)

Modelling solar rotation

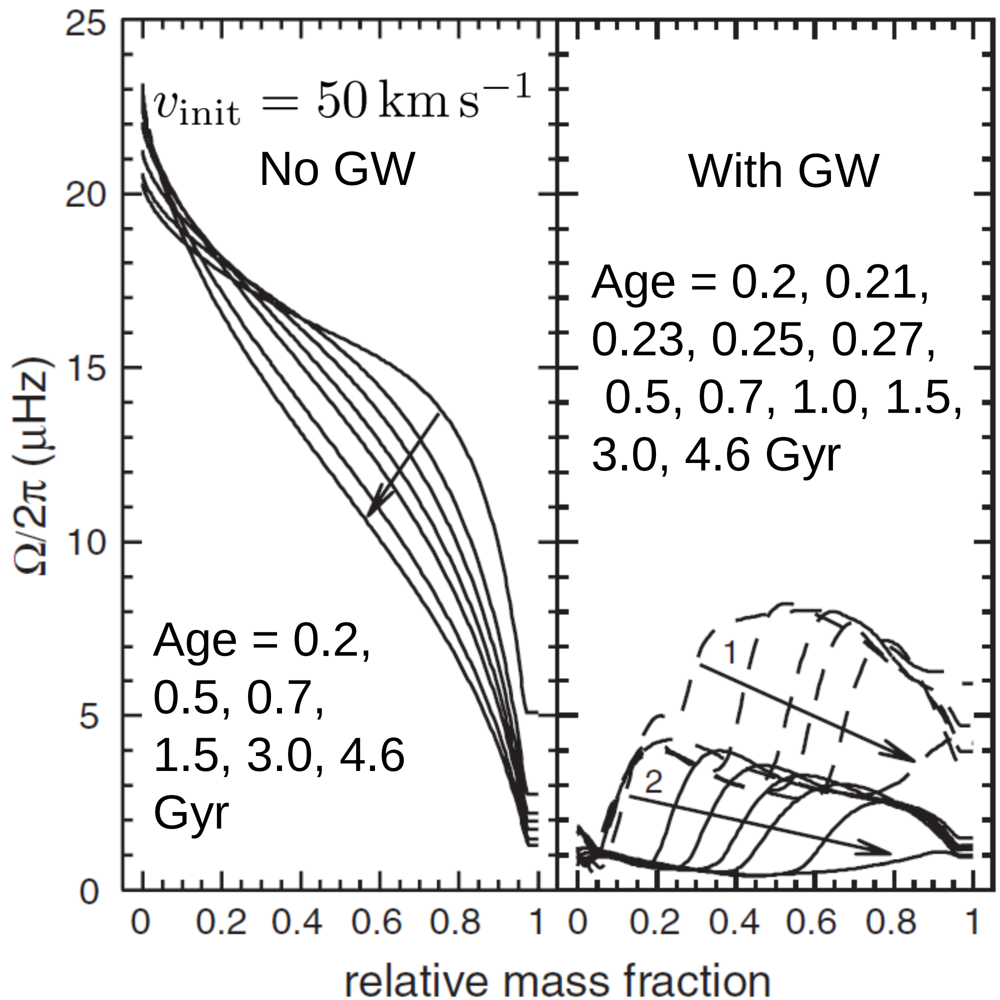


Eggenberger et al. (2005; A&A 440, L9)

Plumb & McEwan experiment

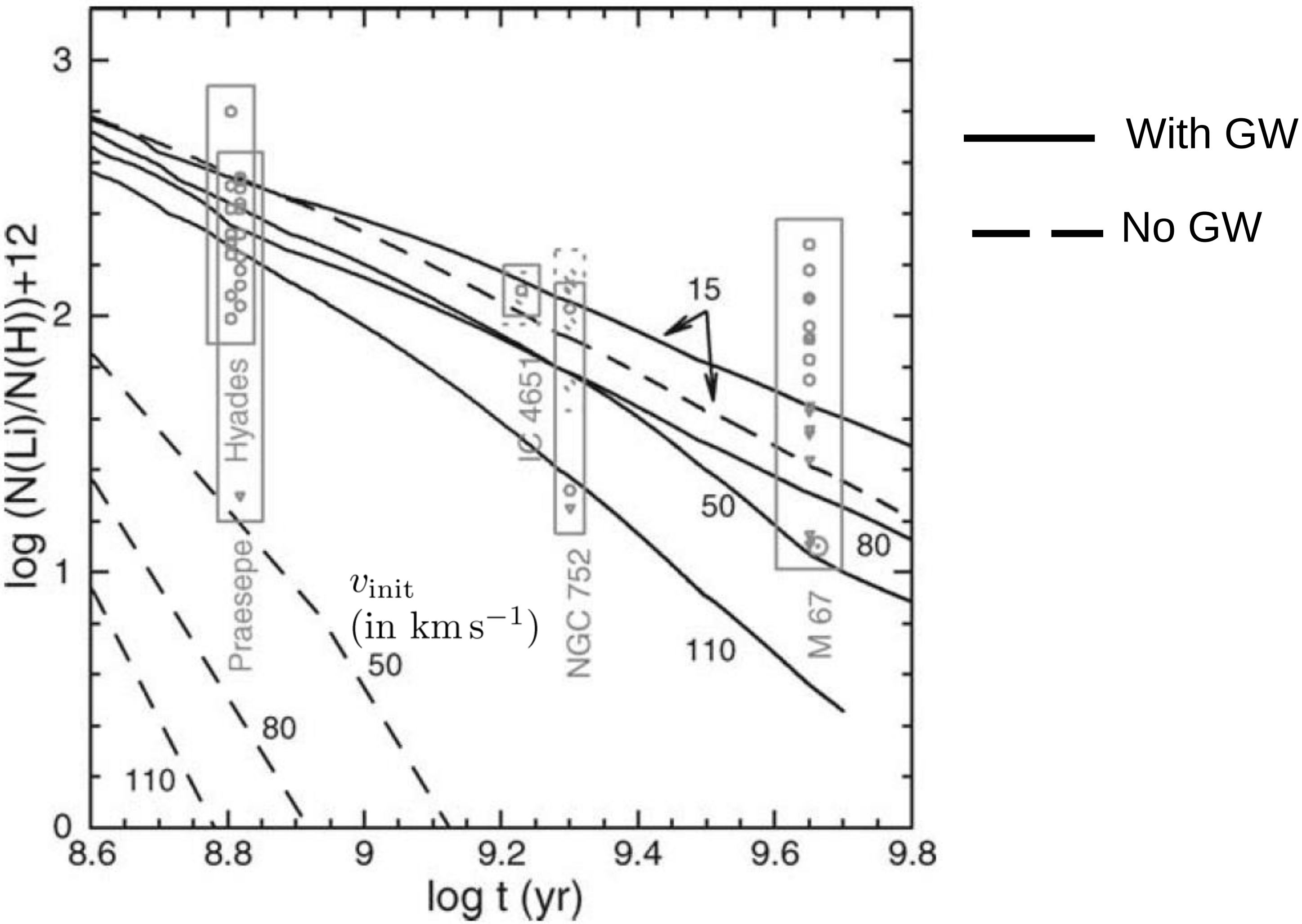
Plumb & McEwan (1978; J. Atmos. Science 35, 1827)
(See http://owww.phys.au.dk/~jcd/stel-struc/plumb_mcewan_Z35b.mov)

Gravity-wave transport?



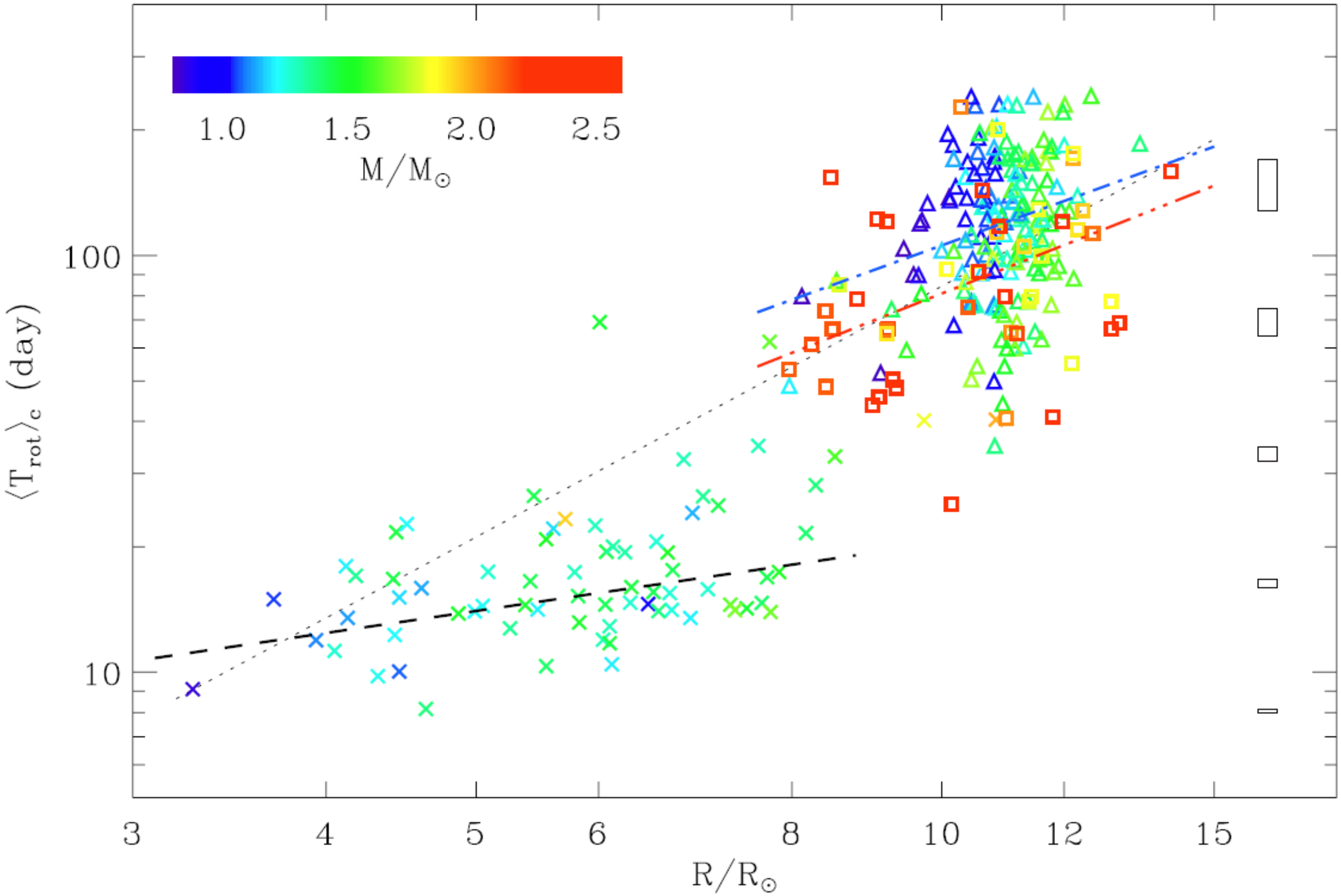
Charbonnel & Talon (2005; Science 309, 2189)

Gravity waves and lithium



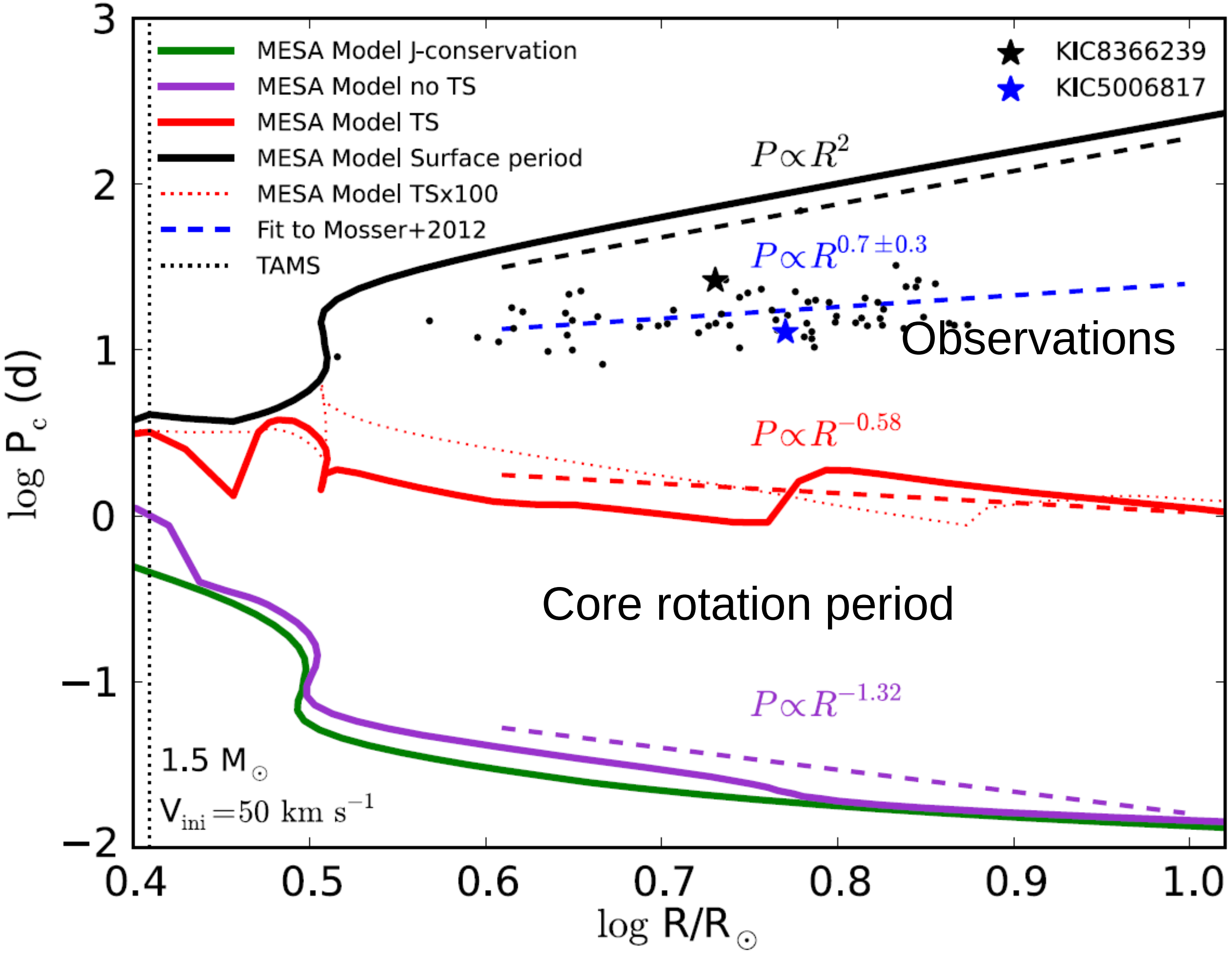
Charbonnel & Talon (2005; Science 309, 2189)

Ensemble rotation



Mosser et al. (2013; A&A 548, A10)

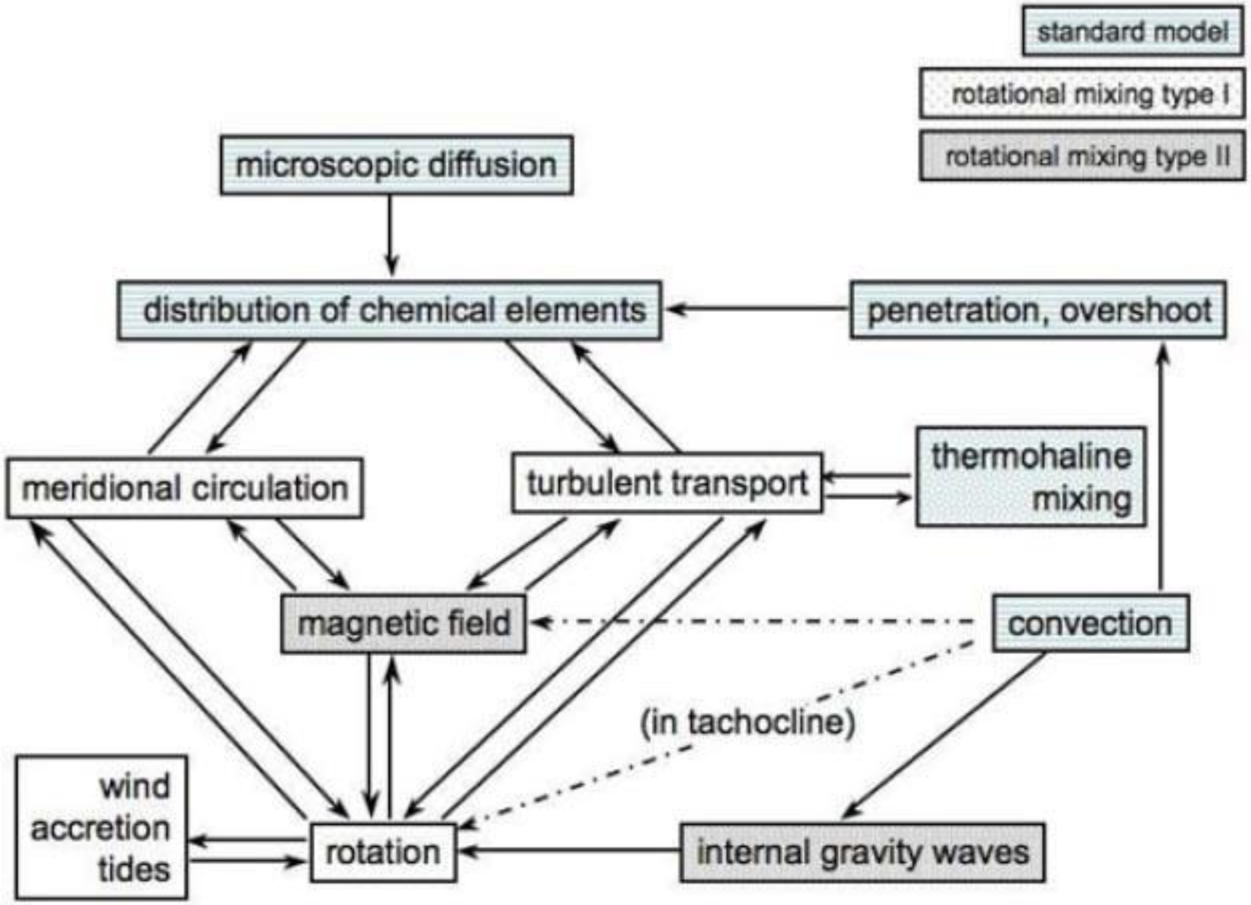
Rotation evolution



Cantiello et al. (2014; ApJ 788, 93)

TS: Tayler-Spruit dynamo

The true (or perhaps somewhat simplified) story of stellar evolution



Mathis & Zahn (2005; A&A 440, 653);