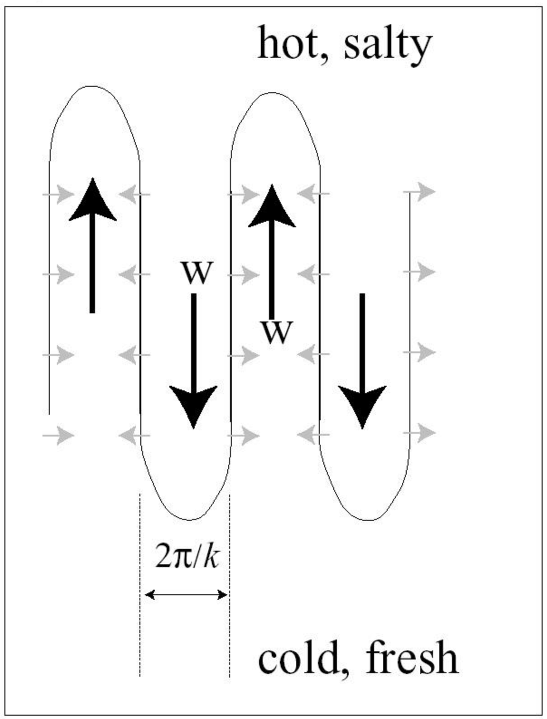
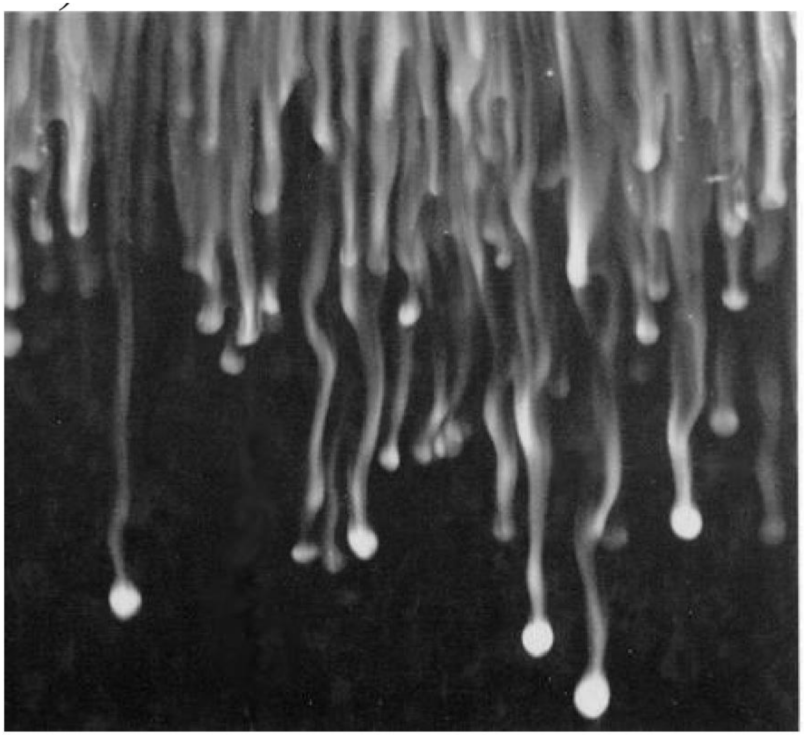
What is thermohaline convection?

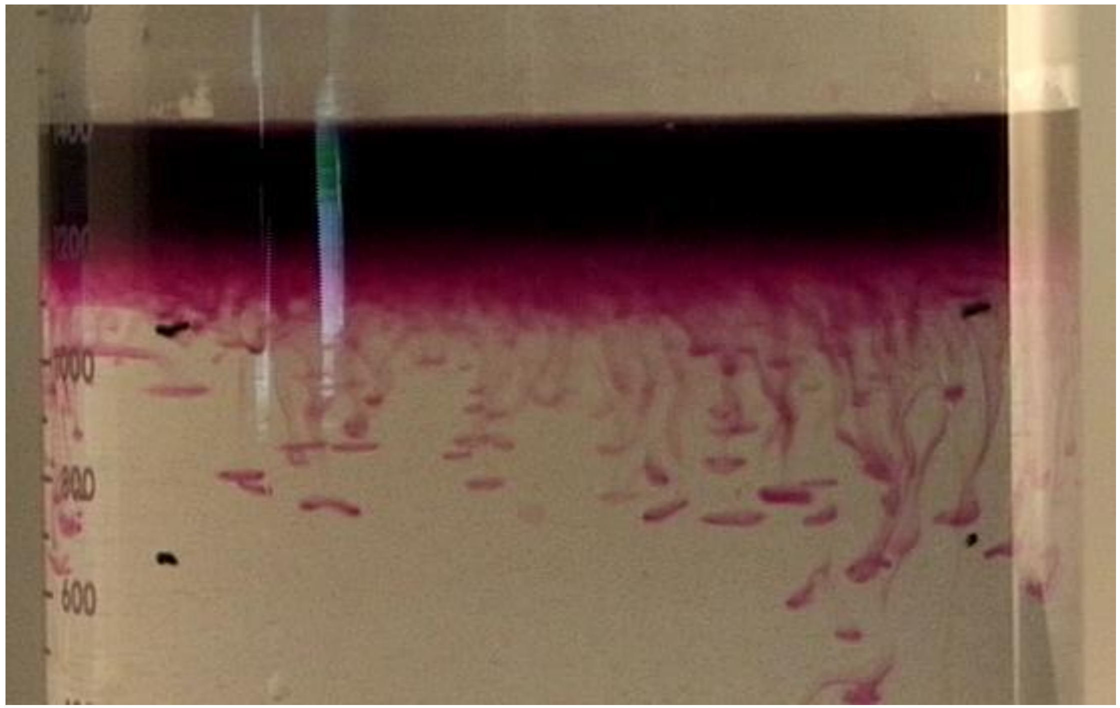




special issue of « progress in Oceanography » , 2003

Sylvie Vauclair

Salt-finger experiment



http://www.ualberta.ca/~bsuther/eifl/teaching/saltfingers/index.html

Convective transport

Mixing-length treatment

Fluxes

$$\nabla_{\text{rad}} = \frac{3}{16\pi acG}\frac{\kappa P}{T^4}\frac{l}{m}$$

$$F_{\text{rad}} = \frac{4acG}{3}\frac{T^4 m}{\kappa P r^2}\nabla = \frac{4ac}{3}\frac{T^4}{\rho\kappa H_P}\nabla$$

$$F = F_{\text{rad}} + F_{\text{con}} = \frac{4ac}{3}\frac{T^4}{\rho\kappa H_P}\nabla_{\text{rad}}$$

$$\nabla_{\text{rad}} = \nabla + \frac{3\kappa\rho H_P}{4acT^4}F_{\text{con}}$$

Fluxes

$$\nabla_{\mathrm{rad}} = \nabla + \frac{3\kappa\rho H_P}{4acT^4} F_{\mathrm{con}}$$

$$F_{\mathrm{con}} = \rho c_P T \sqrt{g\delta} \frac{\ell_{\mathrm{m}}^2}{4\sqrt{2}} H_P^{-3/2} (\nabla - \nabla_{\mathrm{e}})^{3/2}$$

$$\begin{aligned}
\nabla_{\mathrm{rad}} - \nabla &= \frac{3\kappa\rho^2 c_P}{4acT^3} \sqrt{\frac{g\delta}{H_P}} \frac{\ell_{\mathrm{m}}^2}{4\sqrt{2}} (\nabla - \nabla_{\mathrm{e}})^{3/2} \\
&= \frac{3\kappa\rho^2 c_P}{8acT^3} \sqrt{\frac{g\delta}{8H_P}} \ell_{\mathrm{m}}^2 (\nabla - \nabla_{\mathrm{e}})^{3/2} = \frac{9}{8} U^{-1} (\nabla - \nabla_{\mathrm{e}})^{3/2}
\end{aligned}$$

$$U = \frac{3acT^3}{c_P \rho^2 \kappa \ell_{\mathrm{m}}^2} \sqrt{\frac{8H_P}{g\delta}}$$

Heat loss

$$\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{e}} = \left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{ad}} - \frac{\lambda}{\rho c_P V v}$$

$$\nabla_{\mathrm{e}} - \nabla_{\mathrm{ad}} = \frac{\lambda H_P}{\rho c_p T V v}$$

$$\lambda = \frac{8acT^3}{3\kappa\rho}\frac{DT}{\ell_{\mathrm{m}}}S = \frac{8acT^3}{3\kappa\rho}\frac{1}{2}(\nabla - \nabla_{\mathrm{e}})\frac{T}{H_p}S$$

$$\nabla_{\mathrm{e}} - \nabla_{\mathrm{ad}} = \frac{4acT^3}{\kappa\rho^2 c_P v}(\nabla - \nabla_{\mathrm{e}})\frac{S}{V}$$

$$\frac{\nabla_{\mathrm{e}} - \nabla_{\mathrm{ad}}}{\nabla - \nabla_{\mathrm{e}}} = \frac{6acT^3}{\kappa\rho^2 c_P \ell_{\mathrm{m}} v}$$

Heat loss

$$\frac{\nabla_{\mathrm{e}} - \nabla_{\mathrm{ad}}}{\nabla - \nabla_{\mathrm{e}}} = \frac{6acT^3}{\kappa\rho^2 c_P \ell_{\mathrm{m}} v}$$

$$v^2 = g\delta(\nabla - \nabla_{\mathrm{e}})\frac{\ell_{\mathrm{m}}^2}{8H_P}$$

$$\frac{\nabla_{\mathrm{e}} - \nabla_{\mathrm{ad}}}{\nabla - \nabla_{\mathrm{e}}} = \frac{6acT^3}{\kappa\rho^2 c_P}\sqrt{\frac{8H_P}{g\delta}}\frac{1}{\ell_{\mathrm{m}}^2}(\nabla - \nabla_{\mathrm{e}})^{-1/2} = 2U(\nabla - \nabla_{\mathrm{e}})^{-1/2}$$

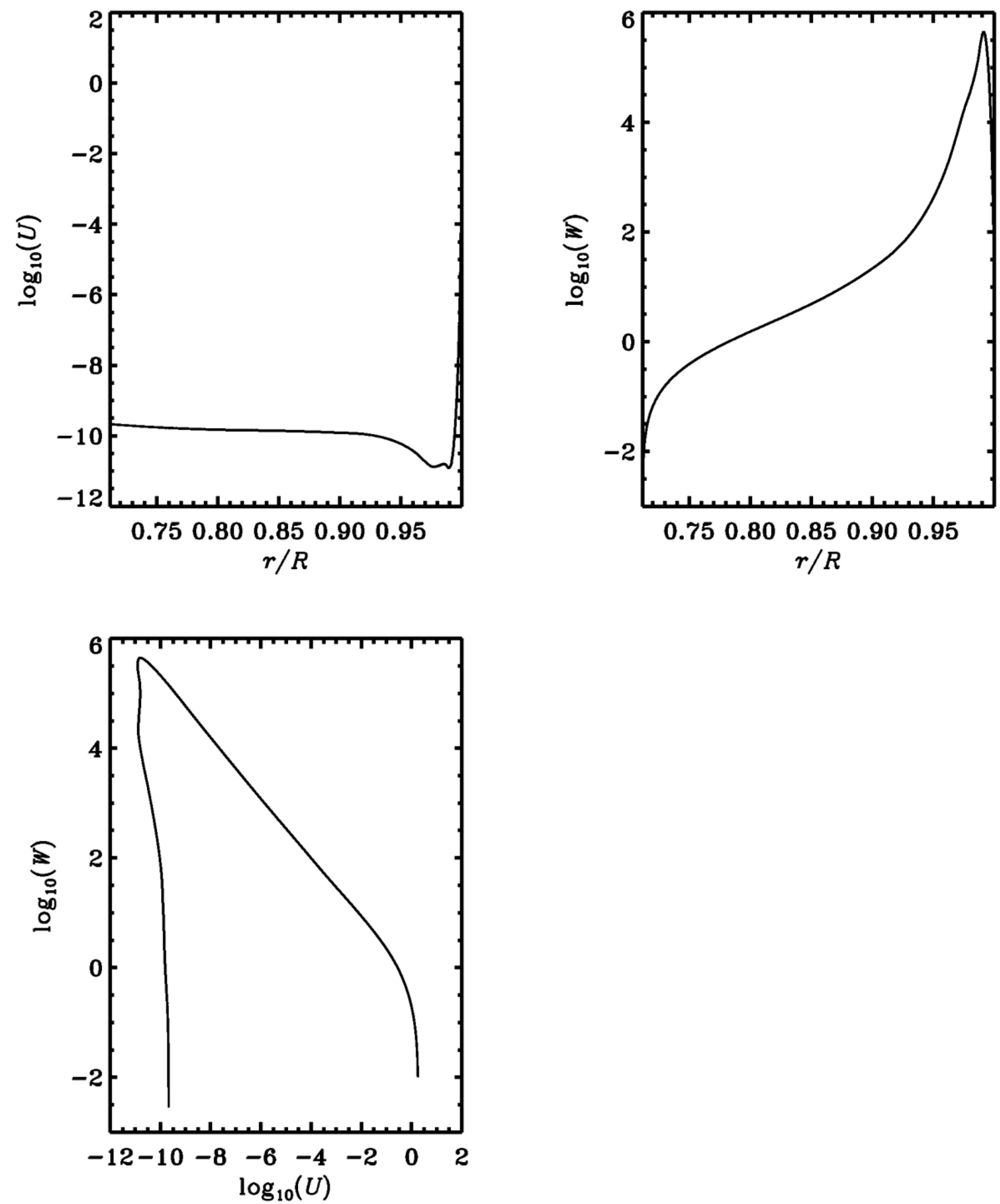
Combined equations

$$U = \frac{3acT^3}{c_P \rho^2 \kappa \ell_{\mathrm{m}}^2} \sqrt{\frac{8H_P}{g\delta}} \,, \qquad W = \nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}$$

$$U \sim \frac{\tau_{\mathrm{dyn}}}{\tau_{\mathrm{adj}}}$$

$$\begin{aligned} \nabla_{\mathrm{e}} - \nabla_{\mathrm{ad}} &= 2U\sqrt{\nabla - \nabla_{\mathrm{e}}} \\ (\nabla - \nabla_{\mathrm{e}})^{3/2} &= \frac{8}{9} U (\nabla_{\mathrm{rad}} - \nabla) \end{aligned}$$

Solar model



Combined equations

$$
\begin{aligned}
\nabla_{\mathrm{e}} - \nabla_{\mathrm{ad}} &= 2U\sqrt{\nabla - \nabla_{\mathrm{e}}} \\
(\nabla - \nabla_{\mathrm{e}})^{3/2} &= \frac{8}{9}U(\nabla_{\mathrm{rad}} - \nabla)
\end{aligned}
$$

$$
(\nabla - \nabla_{\mathrm{e}}) - (\nabla - \nabla_{\mathrm{ad}}) = -2U\sqrt{\nabla - \nabla_{\mathrm{e}}}
$$

Combined equations

$$\begin{aligned} \nabla_{\mathrm{e}} - \nabla_{\mathrm{ad}} &= 2U\sqrt{\nabla - \nabla_{\mathrm{e}}} \\ (\nabla - \nabla_{\mathrm{e}})^{3/2} &= \frac{8}{9}U(\nabla_{\mathrm{rad}} - \nabla) \end{aligned}$$

$$(\nabla - \nabla_{\mathrm{e}}) - (\nabla - \nabla_{\mathrm{ad}}) = -2U\sqrt{\nabla - \nabla_{\mathrm{e}}}$$

$$\sqrt{\nabla - \nabla_{\mathrm{e}}} = -U + \xi\,, \qquad \xi^2 = \nabla - \nabla_{\mathrm{ad}} + U^2$$

$$(\xi - U)^3 + \frac{8}{9}U(\xi^2 - U^2 - W) = 0$$

Combined equations

$$\begin{aligned} \nabla_{\rm e} - \nabla_{\rm ad} &= 2U\sqrt{\nabla - \nabla_{\rm e}} \\ (\nabla - \nabla_{\rm e})^{3/2} &= \frac{8}{9}U(\nabla_{\rm rad} - \nabla) \end{aligned}$$

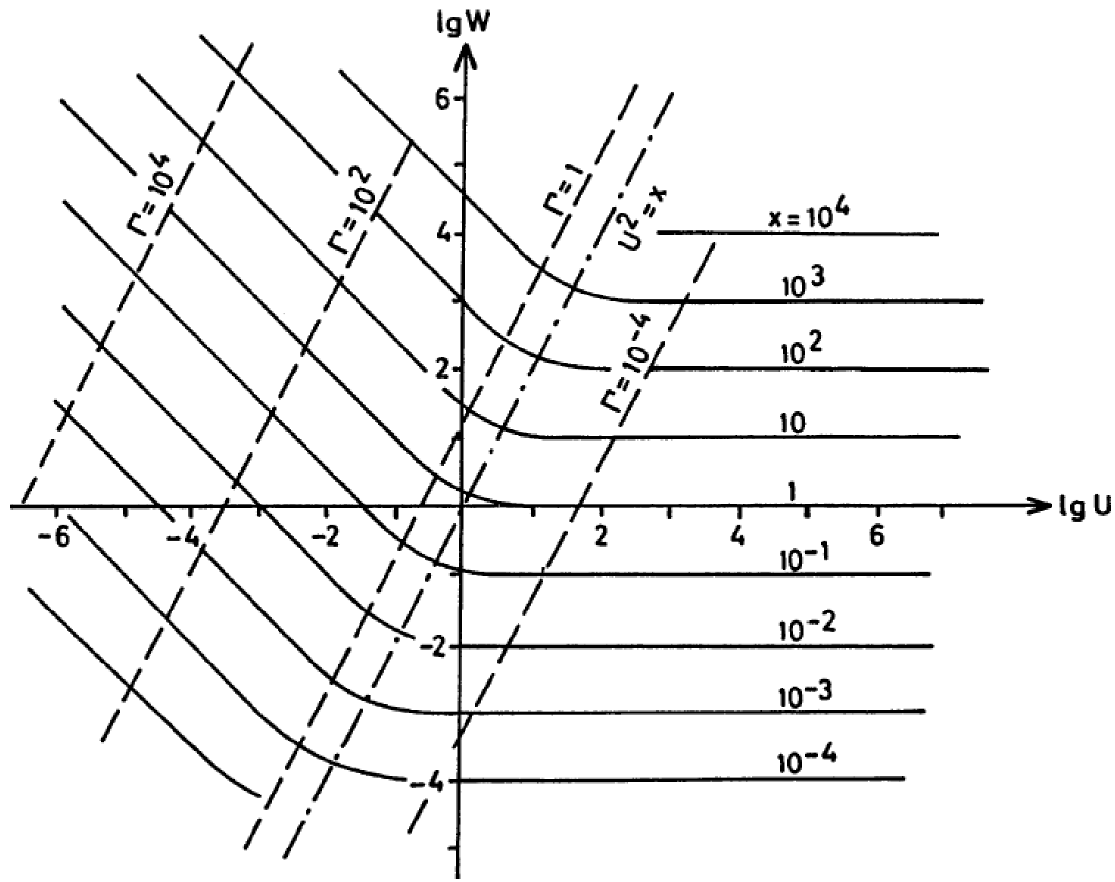
$$\sqrt{\nabla - \nabla_{\rm e}} = -U + \xi\ , \qquad \xi^2 = \nabla - \nabla_{\rm ad} + U^2$$

$$(\xi - U)^3 + \frac{8}{9}U(\xi^2 - U^2 - W) = 0$$

$$x = \nabla - \nabla_{\rm ad}\ , \qquad \left[\sqrt{x + U^2} - U\right]^3 + \frac{8}{9}U(x - W) = 0$$

Limiting cases

$$\Gamma = \frac{(\nabla - \nabla_e)^{1/2}}{2U} = \frac{\nabla - \nabla_e}{\nabla_e - \nabla_{ad}} \sim \frac{\text{energy transported}}{\text{energy lost}}$$

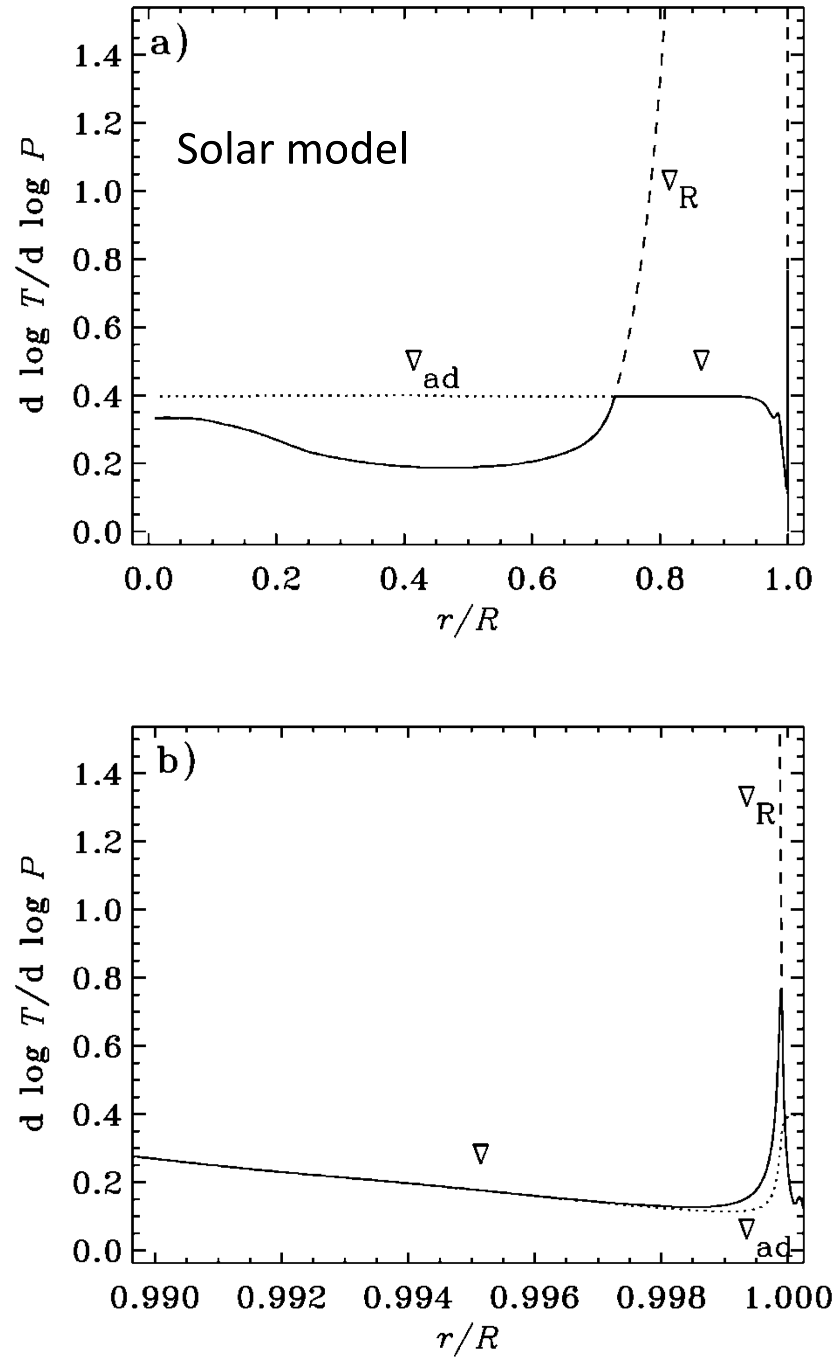


Energy transport by convection

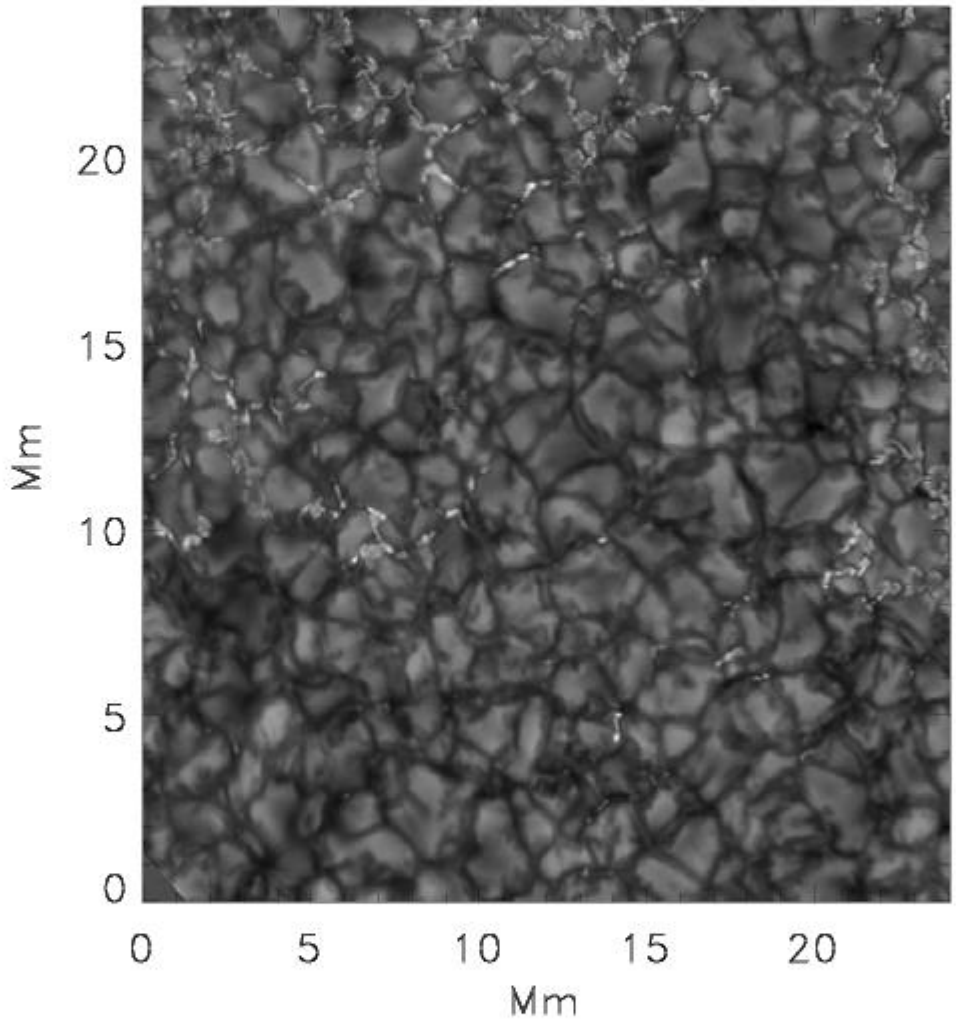
$$v \propto (\nabla - \nabla_{\mathrm{ad}})^{1/2}$$

$$F_{\mathrm{con}} \propto \rho D T c_p v \propto \rho c_p T (\nabla - \nabla_{\mathrm{ad}})^{3/2}$$

In most of the star ρ is so large that even a minute $\nabla - \nabla_{\mathrm{ad}}$ is enough to transport the energy.



Solar granulation



Nordlund et al. (2009; LRSP 6, 2)

Convection simulations

Conservation of mass:

$$\frac{\partial \ln\rho}{\partial t} = -\bar{\mathbf{v}} \cdot \nabla \ln\rho - \nabla \cdot \bar{\mathbf{v}} \quad (1)$$

Conservation of momentum:

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} = -\bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}} + \bar{\mathbf{g}} - \frac{P}{\rho} \nabla \ln P + \frac{1}{\rho} \nabla \cdot \sigma \quad (2)$$

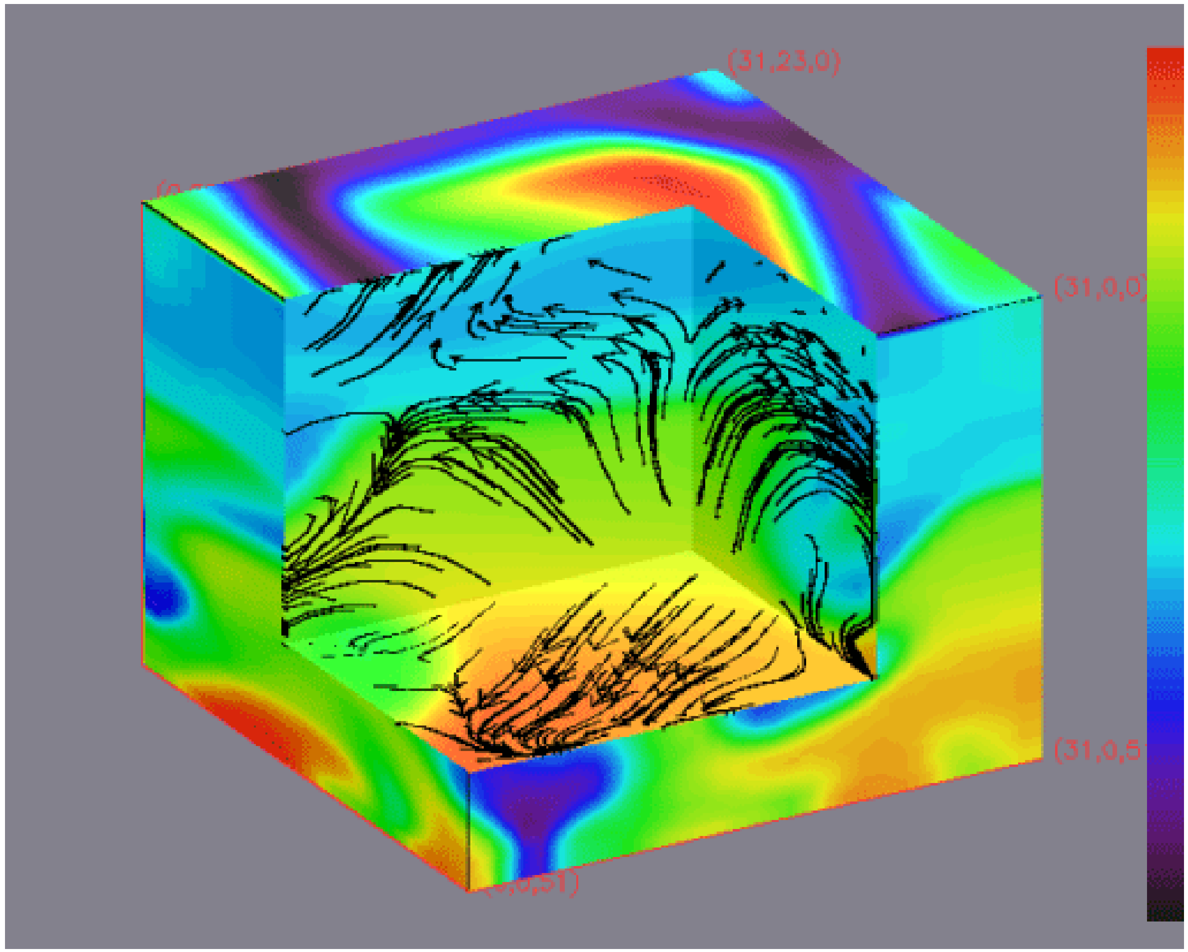
Conservation of energy:

$$\frac{\partial e}{\partial t} = -\bar{\mathbf{v}} \cdot \nabla e - \frac{P}{\rho} \nabla \cdot \bar{\mathbf{v}} + Q_{\mathrm{rad}} + Q_{\mathrm{visc}} \quad (3)$$

Q_{rad} = radiative heating/cooling rate

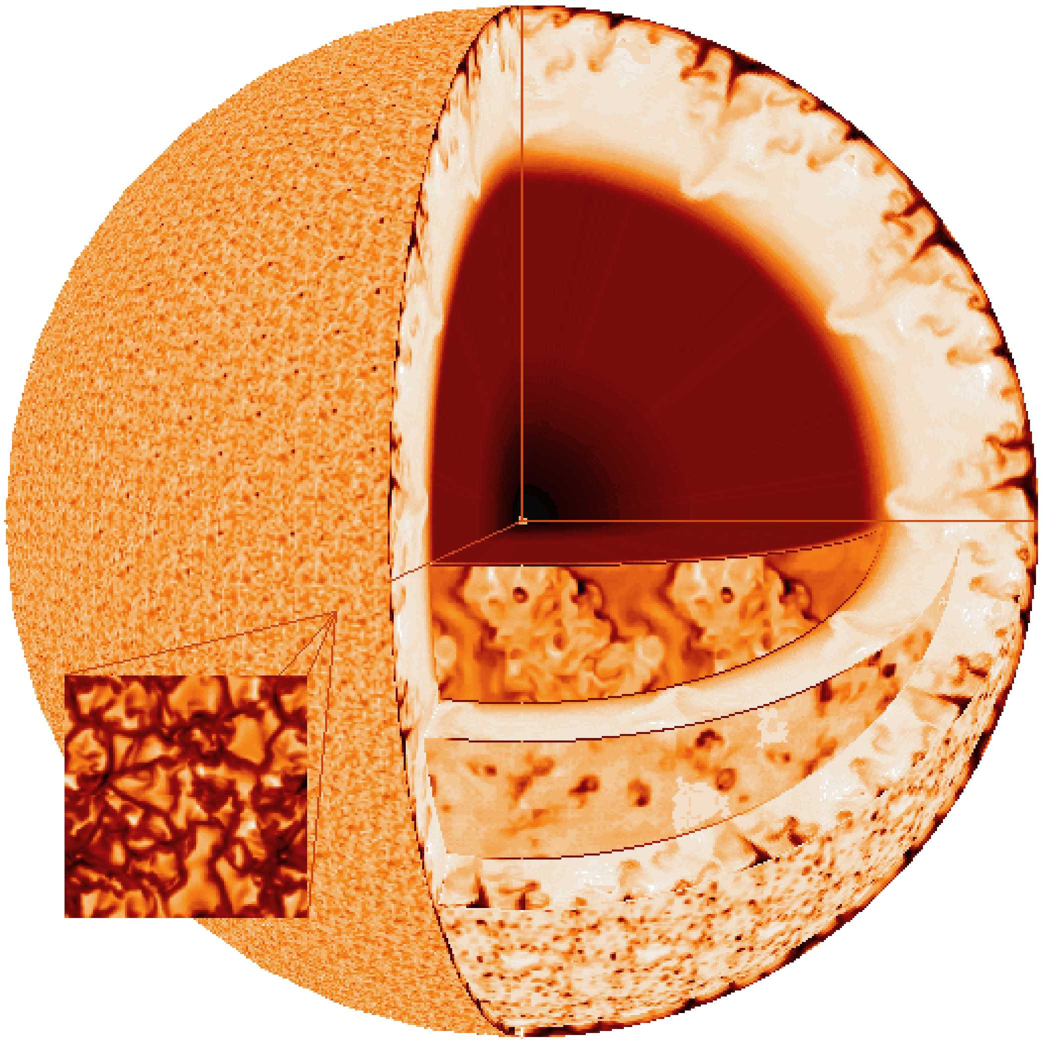
Q_{rad} obtained from the equation of radiative transfer

Granulation simulation

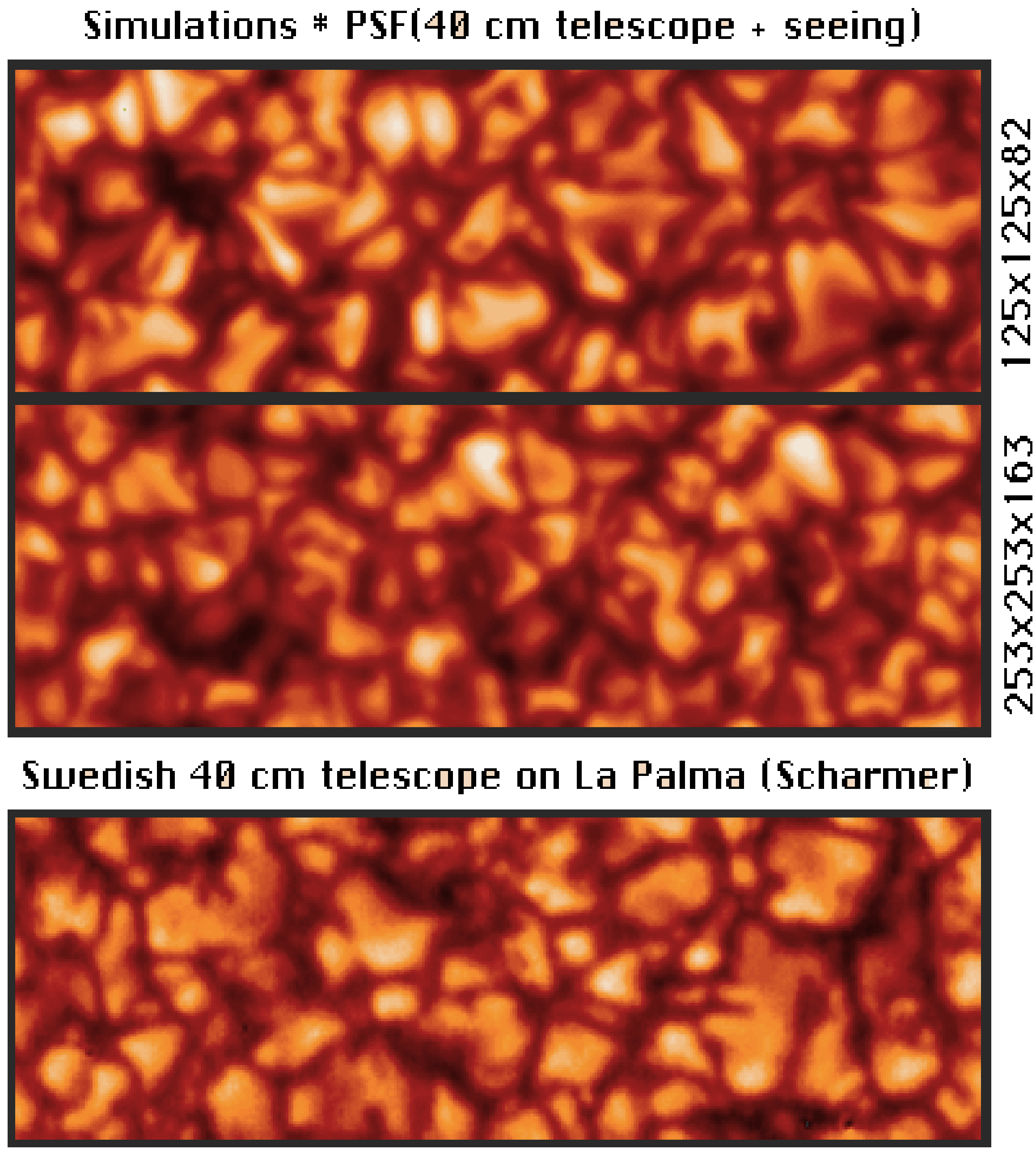


Nordlund et al. (2009; LRSP 6, 2)

Convection cartoon



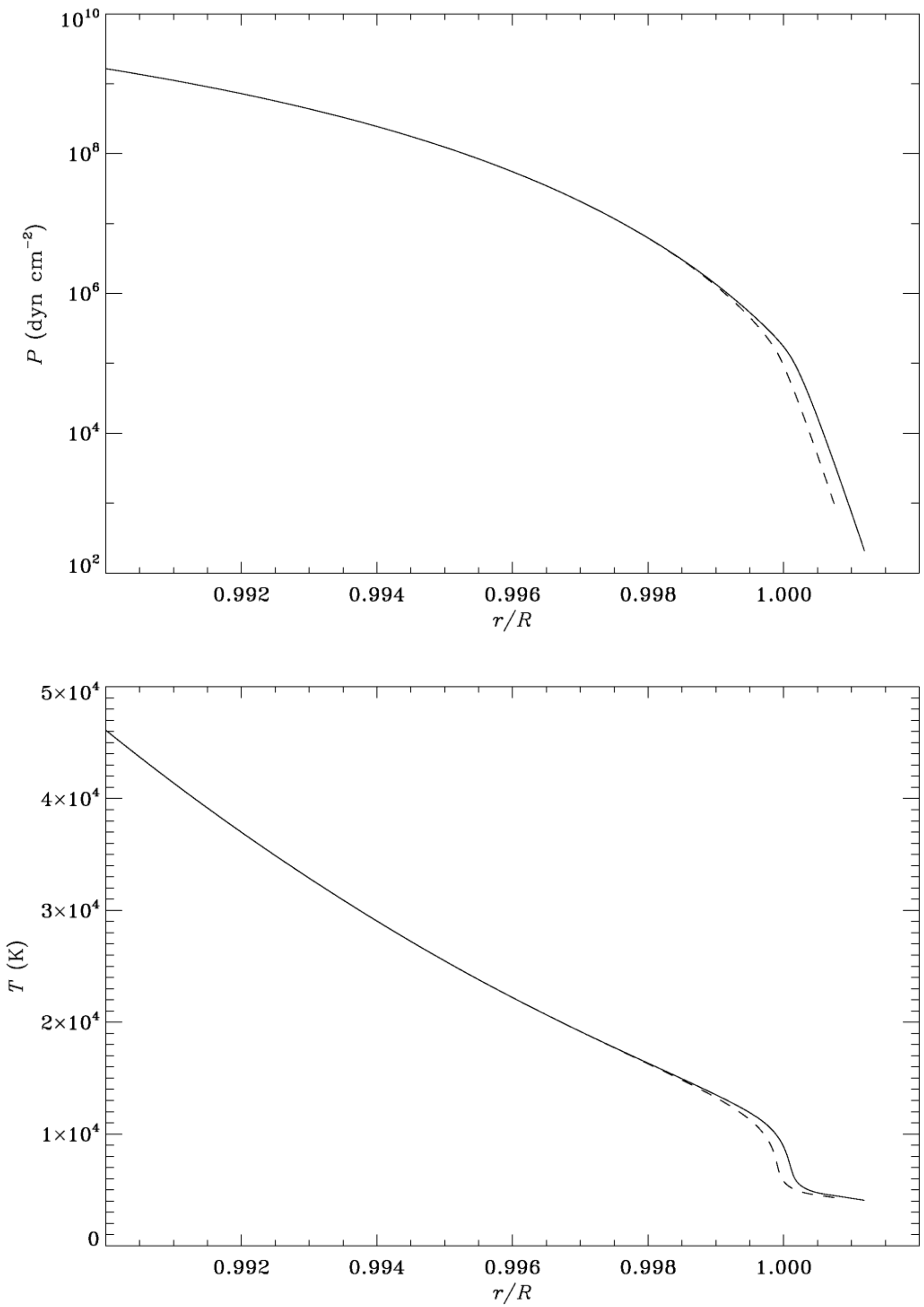
Comparing with reality



ML vs simulations

3D simulation

ML treatment



Solar model
Trampedach et al.
(2013; ApJ 769, 18)

ML vs simulations

3D simulation

ML treatment

Solar model
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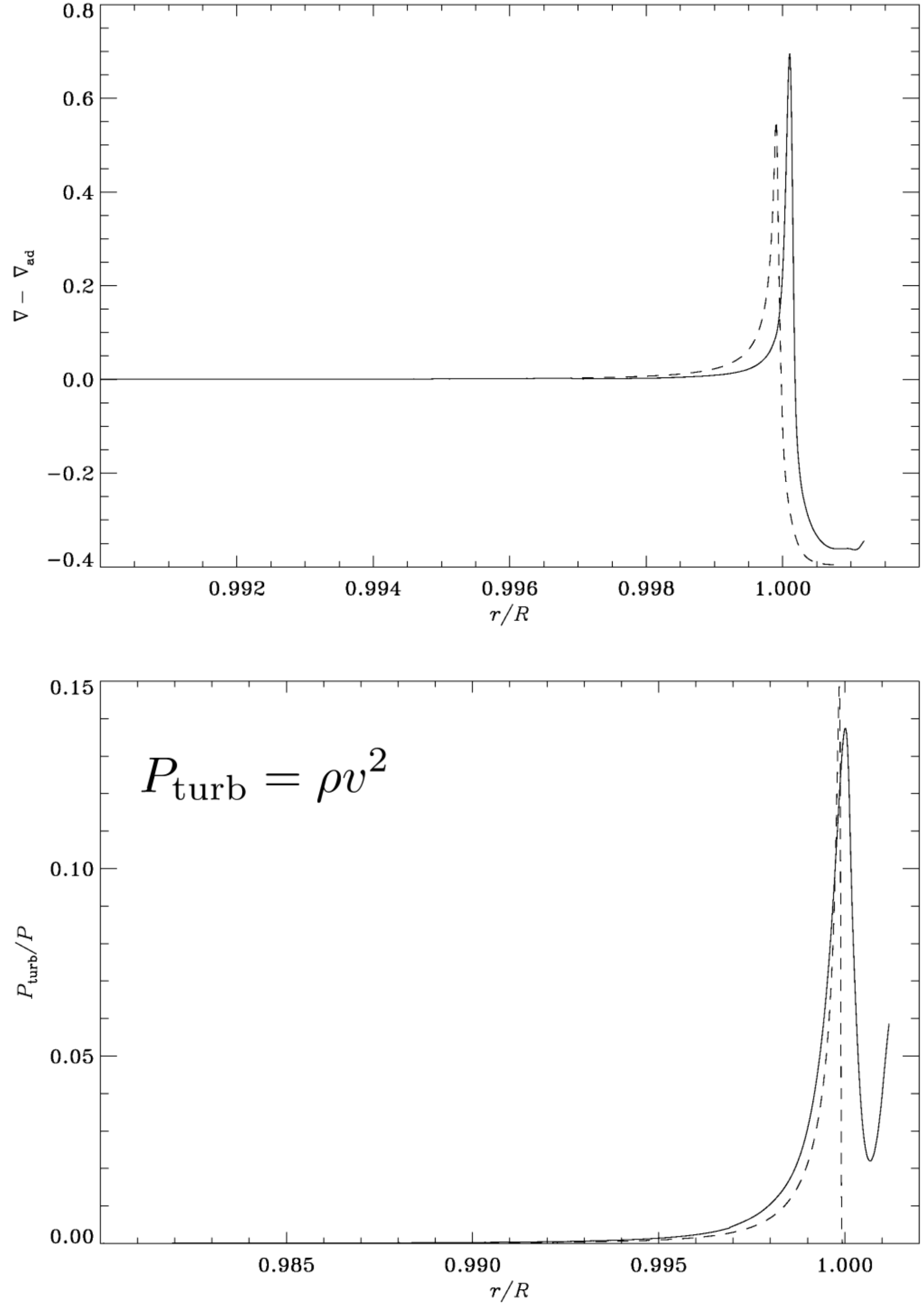
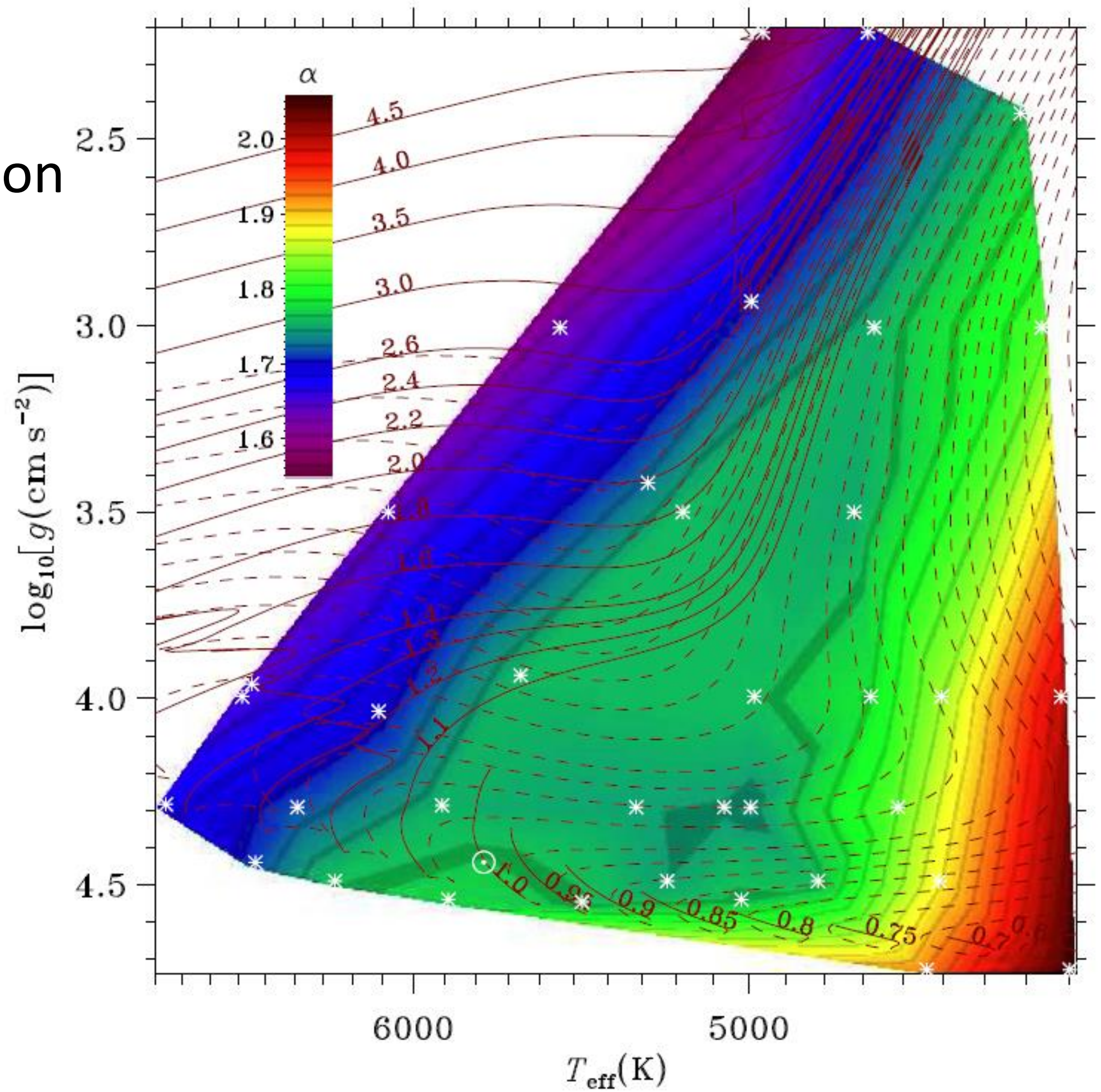


How do we determine mixing length?

$$\ell_{\mathrm{m}} = \alpha_{\mathrm{ML}} H_P$$

- α_{ML} from solar calibration
- From 3D simulations



Tranpedach et al. (2014; MNRAS 445, 4366)