Stellar surface convection simulations

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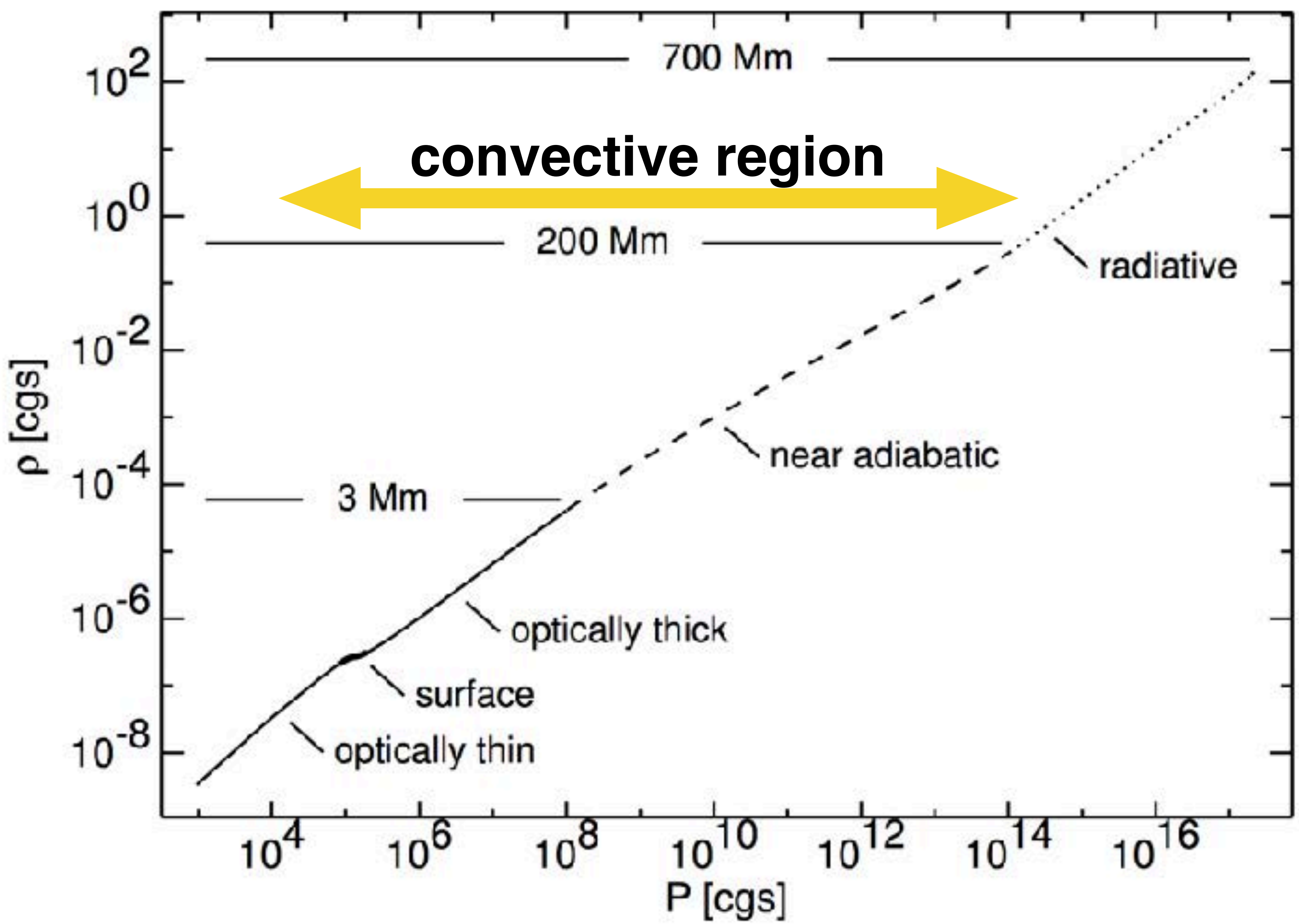
Convection

- One of the main **energy transport** mechanisms in stellar interiors and envelopes together with radiation and conduction

- Occurs in regions where the **temperature gradient** required for energy transport by means of radiation and/or conduction alone would be **too steep**, leading to **dynamical instability**

Solar structure

Average mass density - pressure stratification in the Sun



Classical analysis (1)

- Start with 1D hydrostatic stationary radiative stratification, then

- Radiative flux, diffusion approximation (1D):

$$F_{\rm rad} = -\frac{4\,a\,c\,G}{3}\,\frac{T^4\,m}{\kappa\,P\,r^2}\,\nabla$$

- When does convection occur?

Classical analysis (2)

- Acceleration of buoyant element:

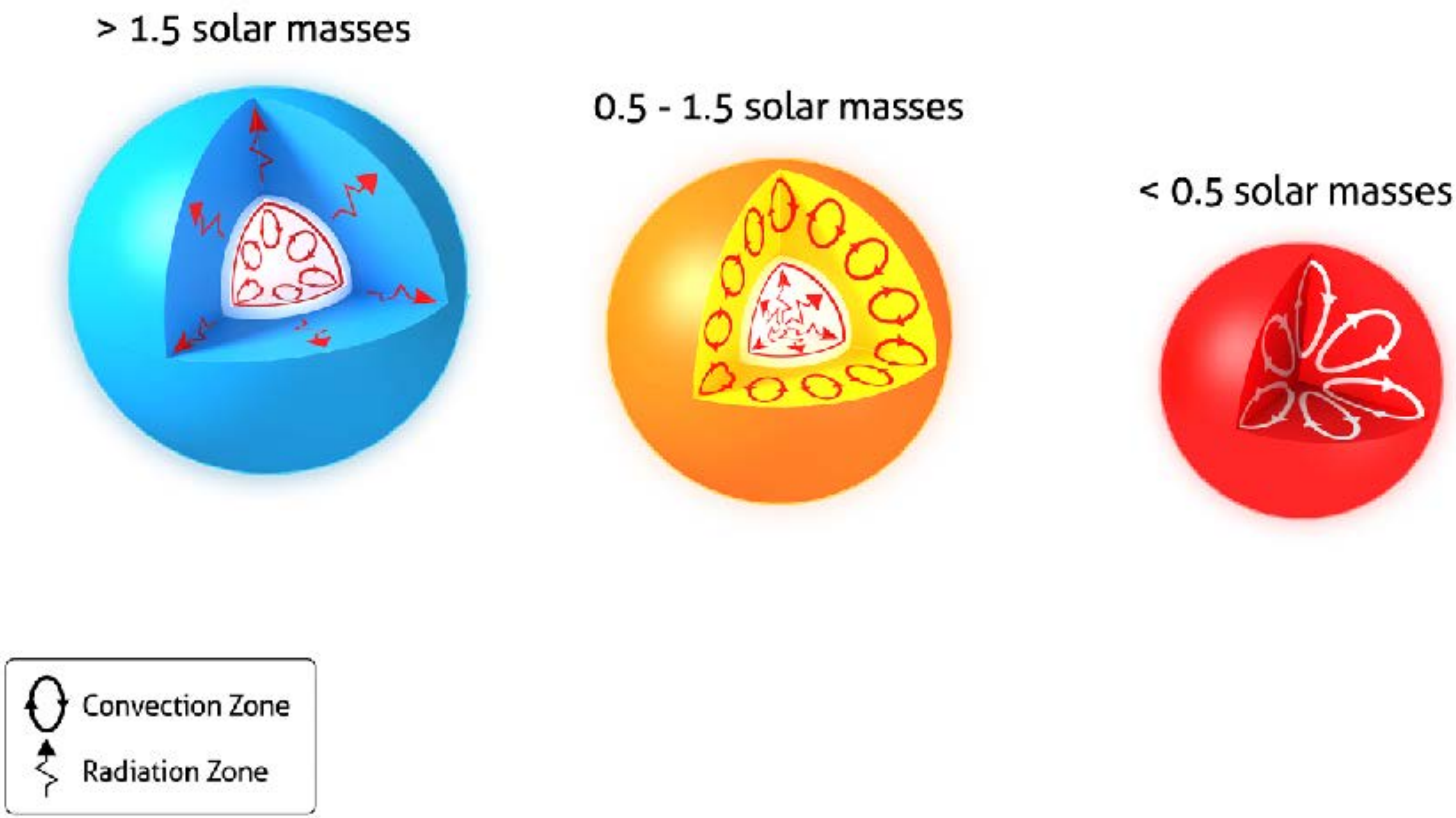
$$\frac{\partial^2 \Delta r}{\partial t^2} = -\frac{g\,\delta}{H_P}\left(\nabla_e - \nabla + \frac{\phi}{\delta}\nabla_\mu\right)$$

- Convective instability (Schwarzschild-Ledoux criterion):

$$\left(\nabla_e - \nabla + \frac{\phi}{\delta}\nabla_\mu\right) < 0$$

- It can be shown that instability according to Schwarzschild criterion implies a **positive entropy gradient with depth**

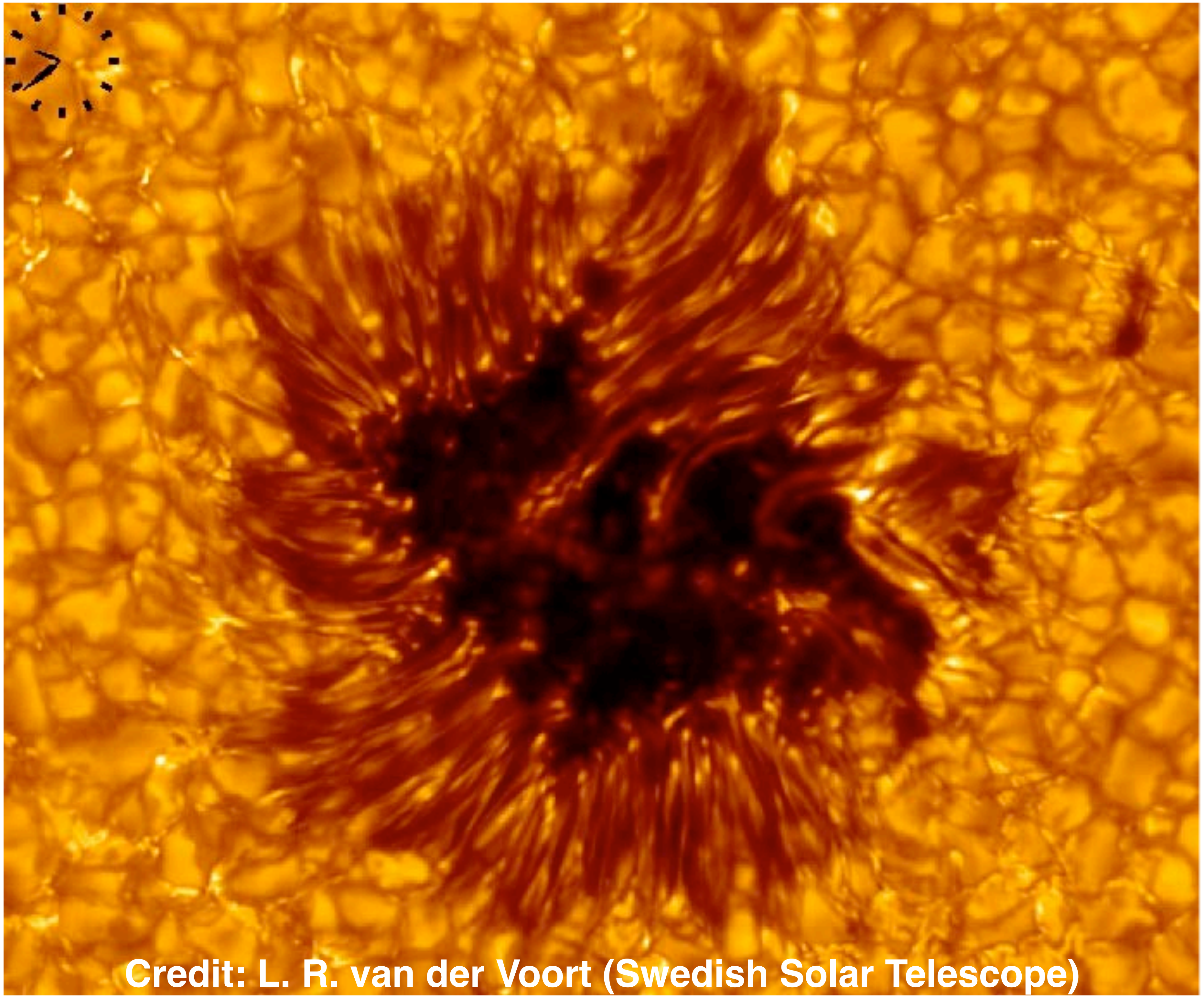
Convective regions in stars



1D stellar models

- 1D Stratification
- Stationary
- Hydrostatic
- Convection: simplified treatment via Mixing-Length Theory or similar (e.g. Full Turbulence Spectrum)
- Free parameters, e.g. mixing length: $\ell = \alpha_{\rm MLT}\, H_P$
- Magnetic fields: generally neglected

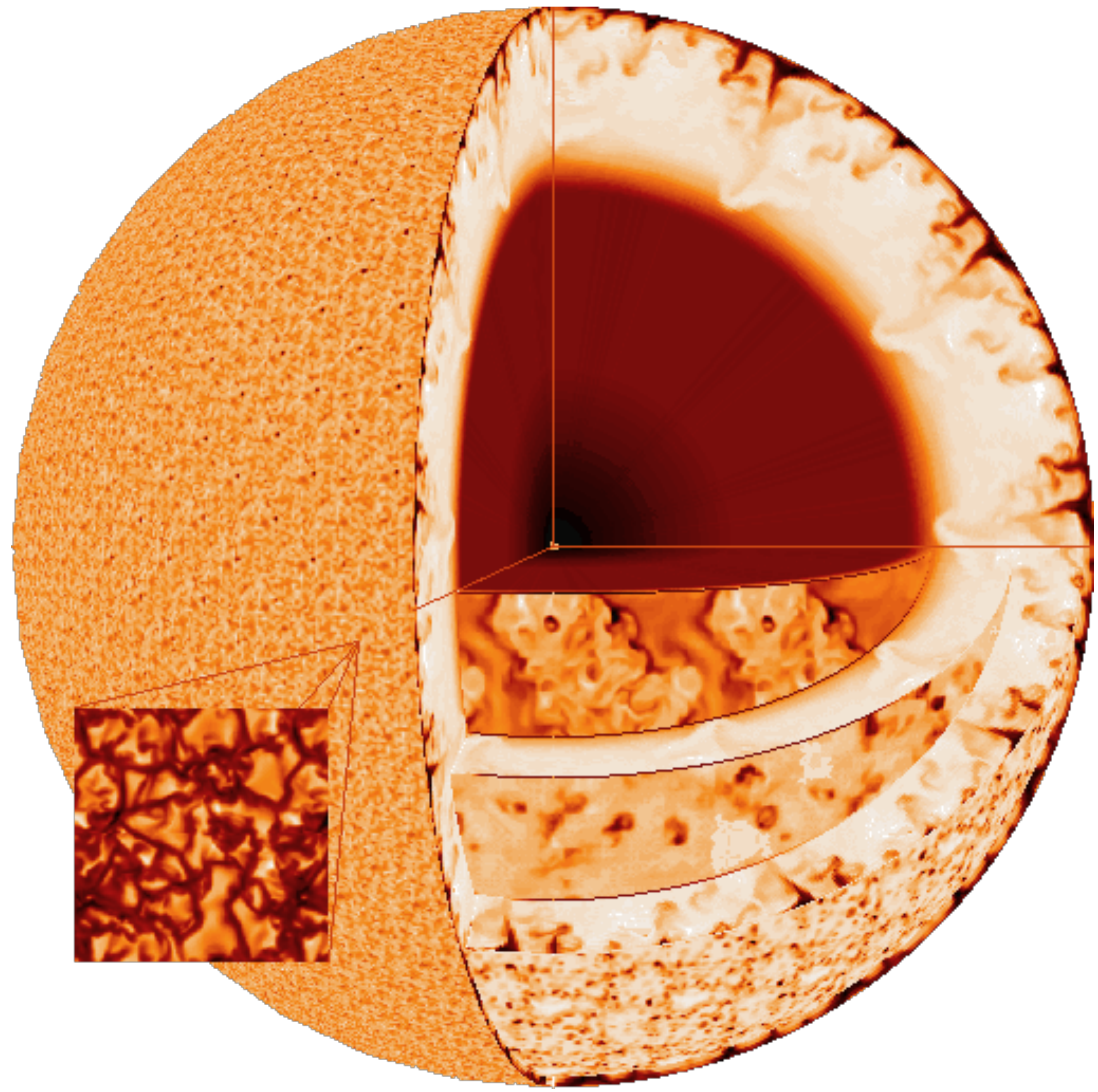
Solar surface convection



3D simulations: motivation

- Convection affects stellar structure and evolution, stellar spectra and oscillations

- Convection is inherently a multi-dimensional, time-dependent, non-local, non-linear phenomenon



Courtesy: Å. Nordlund

3D simulations

- Solution of the mass, momentum, and energy conservation equations
- 3D Geometry
- Time-dependent
- 3D non-grey radiative transfer
- (Magnetic induction equation)
- Convection: no need for free parameters!

Hydrodynamics equations

- Mass, momentum, and energy conservation equations:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{u}) \\
\frac{\partial (\rho \mathbf{u})}{\partial t} &= -\nabla \cdot (\rho \mathbf{u}\mathbf{u}) - \nabla P - \rho \nabla \Phi - \nabla \cdot \tau_{\mathrm{visc}} \\
\frac{\partial E}{\partial t} &= -\nabla \cdot (E \mathbf{u}) - P(\nabla \cdot \mathbf{u}) + \underline{Q_{\mathrm{rad}}} + Q_{\mathrm{visc}}
\end{aligned}$$

- Viscous dissipation and shear stress tensor:

$$Q_{\mathrm{visc}} = \sum_{i,j} \tau_{ij} \frac{\partial u_i}{\tau_j} \qquad \tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \mu' \nabla \cdot \mathbf{u}\, \delta_{ij}$$

Radiative heating

- Radiation plays a crucial role in the overall energy transport. The local radiative heating rate is given by:

$$Q_{\mathrm{rad}} = \nabla \cdot \mathbf{F}_{\mathrm{rad}}$$

- This is equivalent to integrating the difference between the intensity of the radiation field and the source function over frequency and solid angle:

$$Q_{\mathrm{rad}} = \int_{\nu} \int_{\Omega} \rho \kappa_{\nu} (I_{\nu} - S_{\nu}) \, d\Omega \, d\nu$$

Radiative transfer (1)

- Intensities can be computed by solving the (time-independent) radiative transfer equation:

$$\frac{\partial I_\nu}{\partial \tau_\nu} = S_\nu - I_\nu$$

- Optical depth along a given ray:

$$d\tau_\nu = \rho \kappa_\nu \, ds$$

- Note: radiative transfer is **intrinsically non-local!**

Radiative transfer (2)

- Source function with (coherent) radiative scattering:

$$S_\nu = \epsilon_\nu B_\nu + (1 - \epsilon_\nu) J_\nu$$

- Mean intensity:

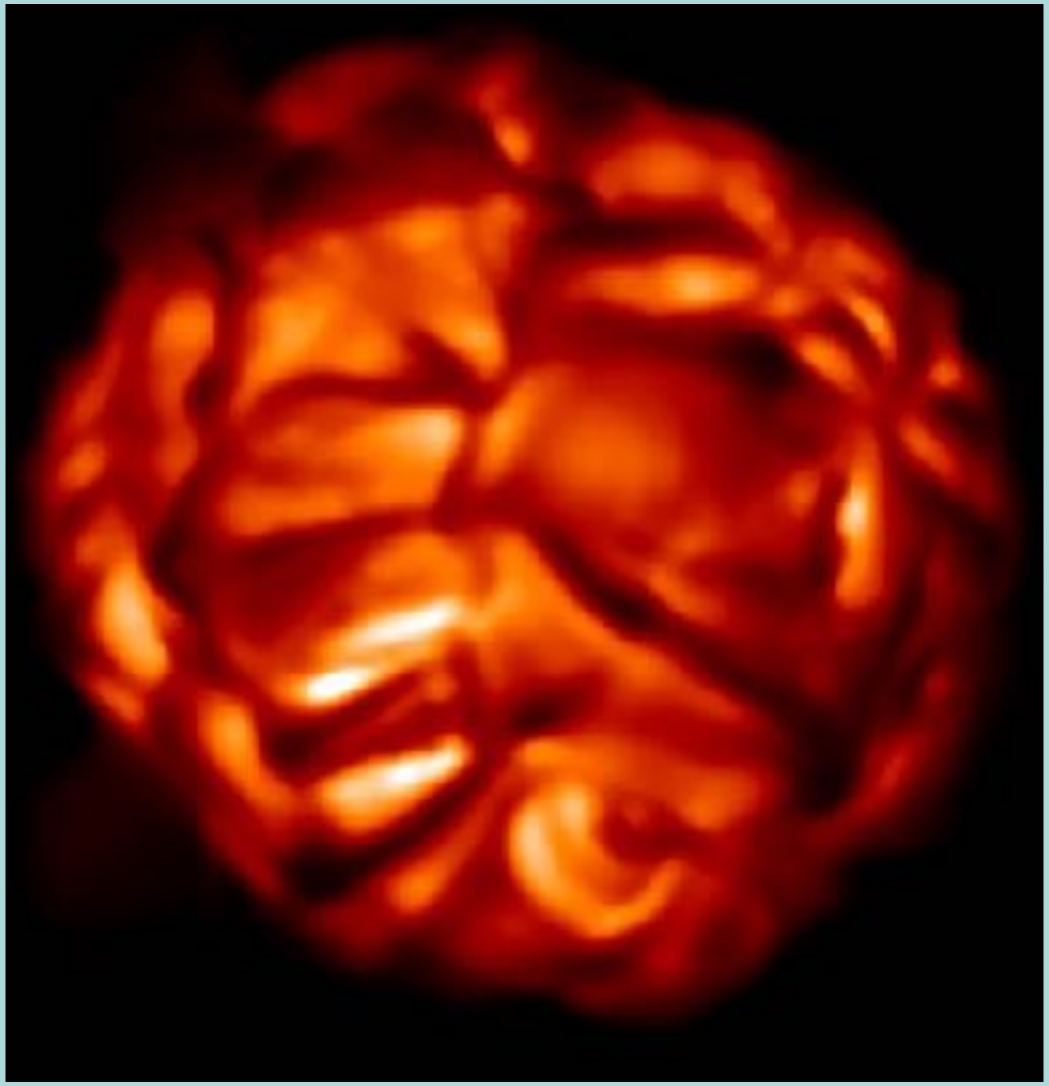
$$J_\nu = \frac{1}{4\pi} \int_\Omega I_\nu \, d\Omega$$

- Simplification: local thermodynamic equilibrium (LTE) and Planck source function:

$$S_\nu \approx B_\nu$$

Simulation setups: “Star in the box”

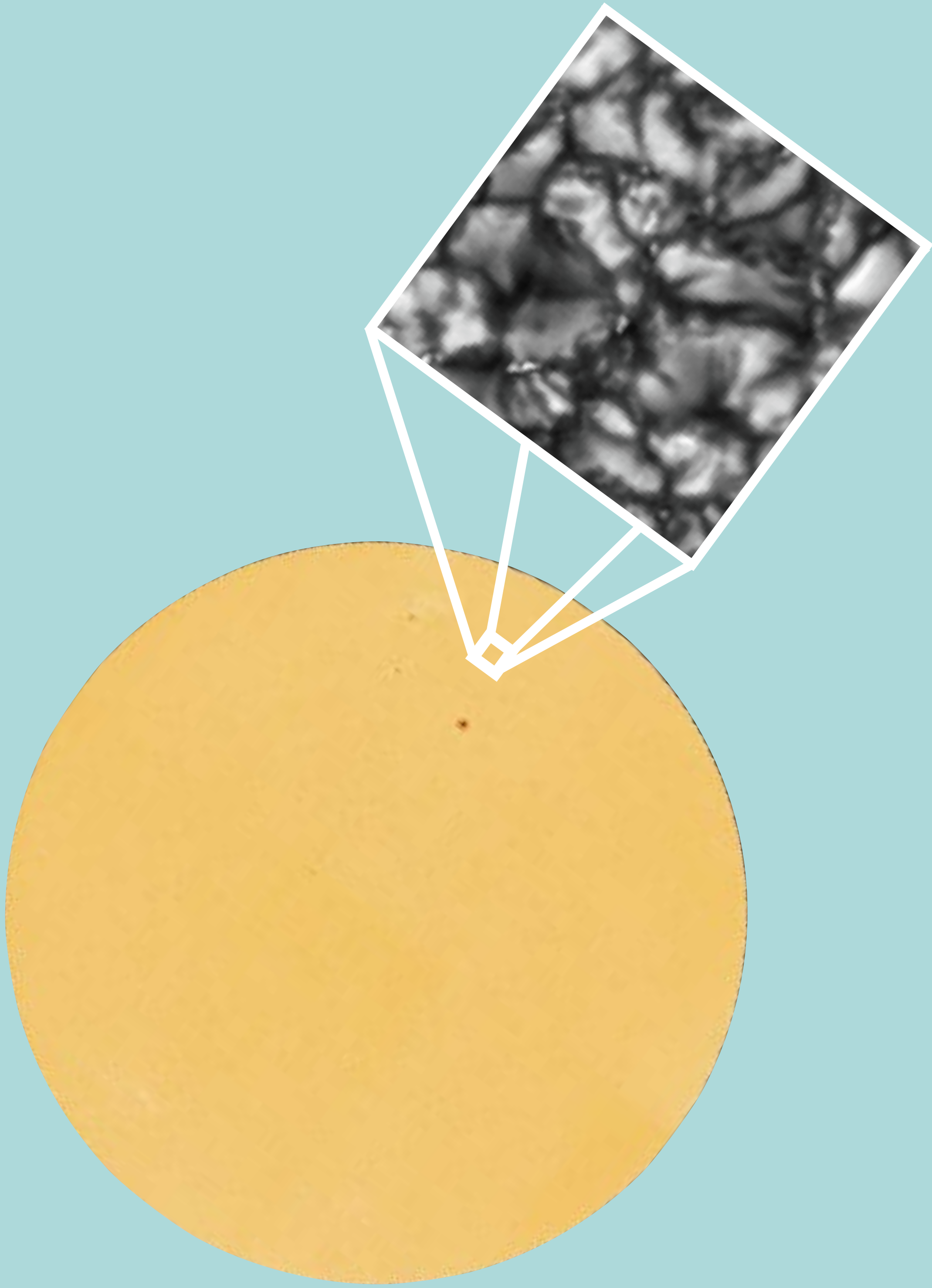
- Star completely enclosed in simulation domain
- Red supergiant simulations: Freytag et al. (2002); Dorch (2004); Chiavassa et al. (2010)
- Boundary conditions: central luminosity source, open external boundaries
- Global simulations, but low spatial resolution



Simulation: Betelgeuse
Freytag et al. (2002)

"Box in the star"

- Periodic boundary conditions horizontally
- Open/closed boundaries vertical
- Local simulations, but high resolution
- e.g. Stagger-Code (Nordlund et al.)

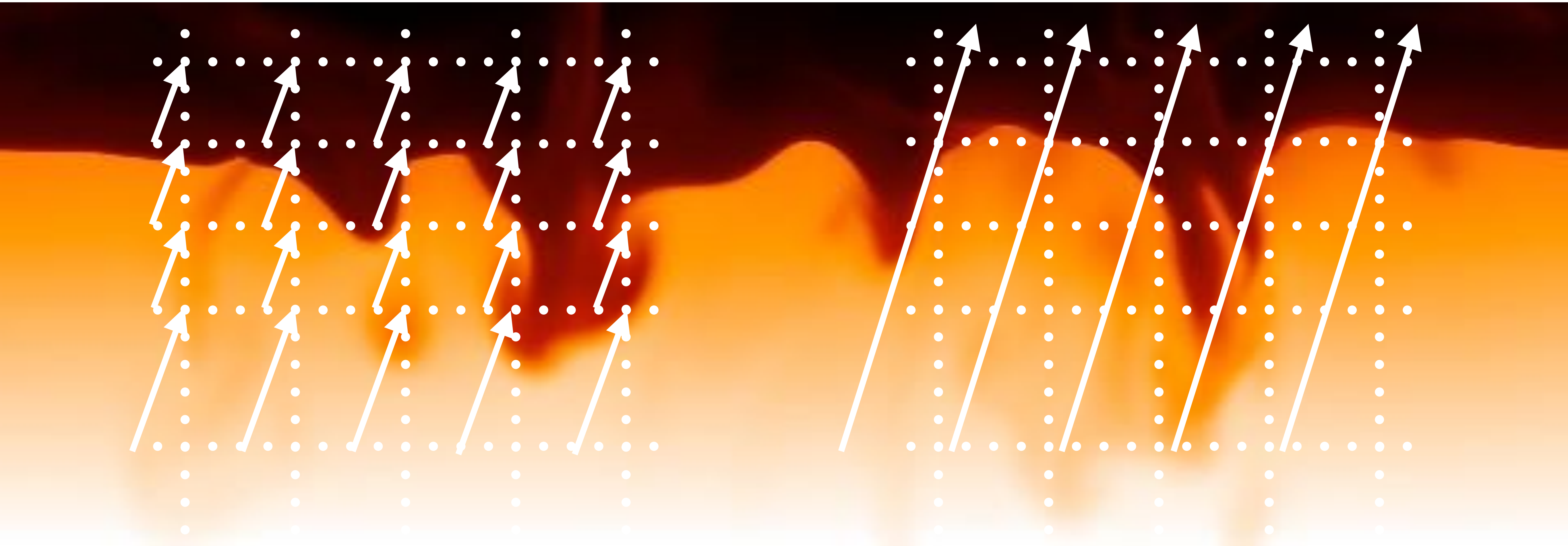


Numerical discretisation

- Example: Stagger-Code
- Domain discretised using a structured 3D Eulerian grid with Cartesian topology
- Derivatives approximated using high-order finite-difference scheme
- Explicit time integration using a Runge-Kutta method

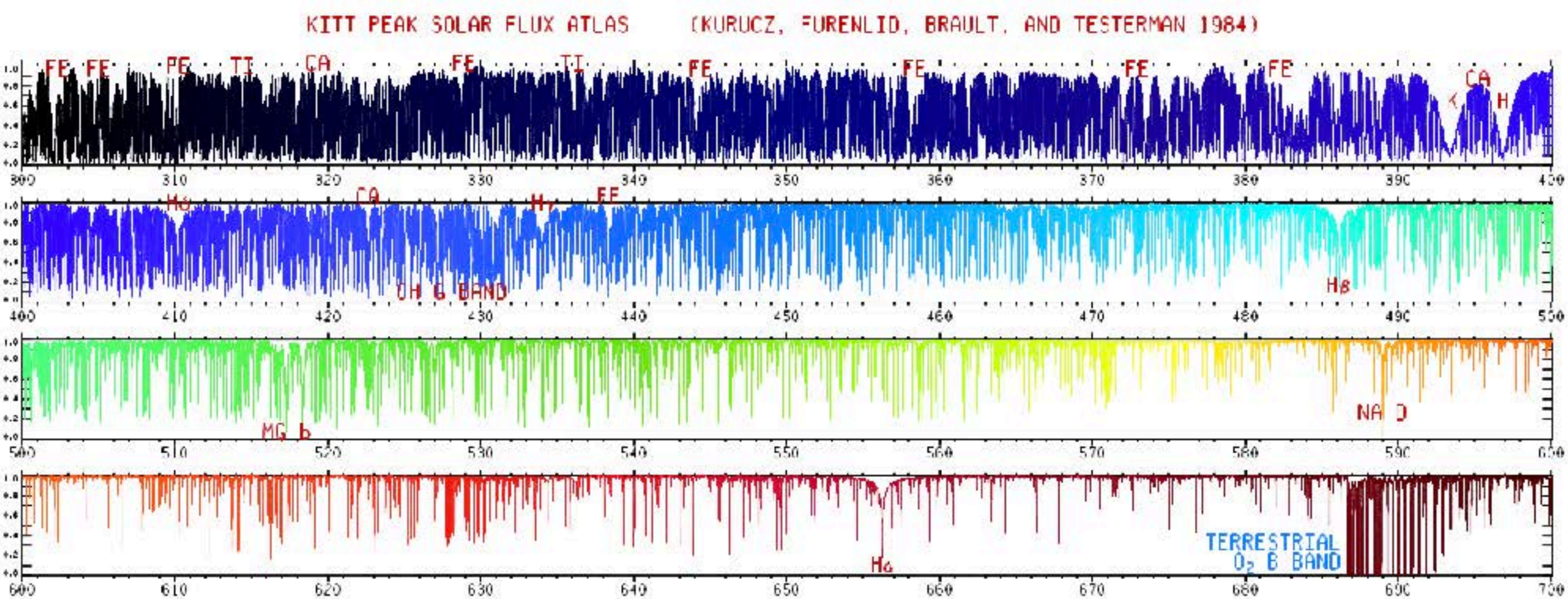
Radiative transfer

- Compute solution along rays crossing all cells in the simulation domain with a range of inclinations
- Numerical discretisation: short- vs. long-characteristics methods



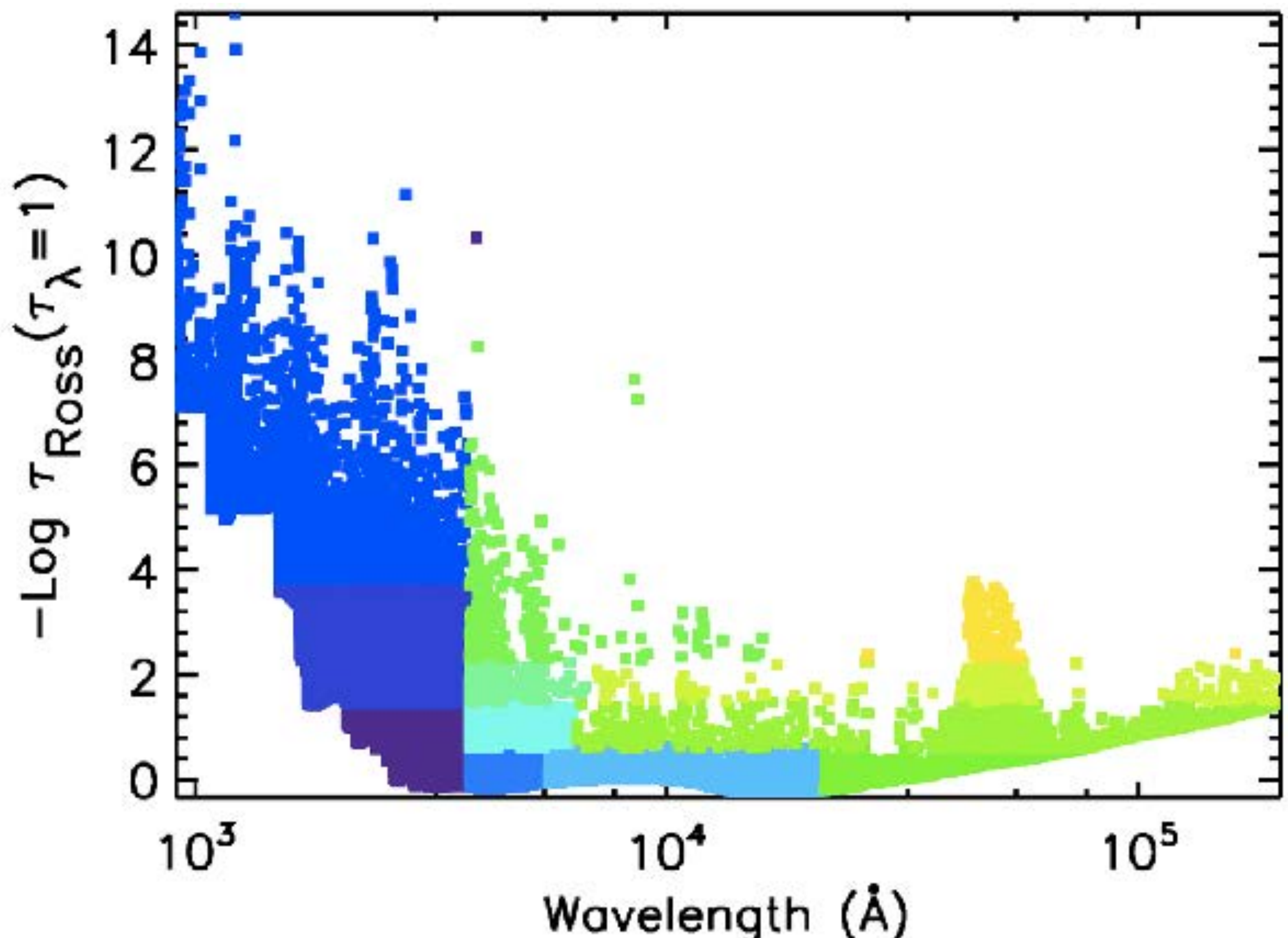
Opacity

- Dependence on temperature, density, and frequency

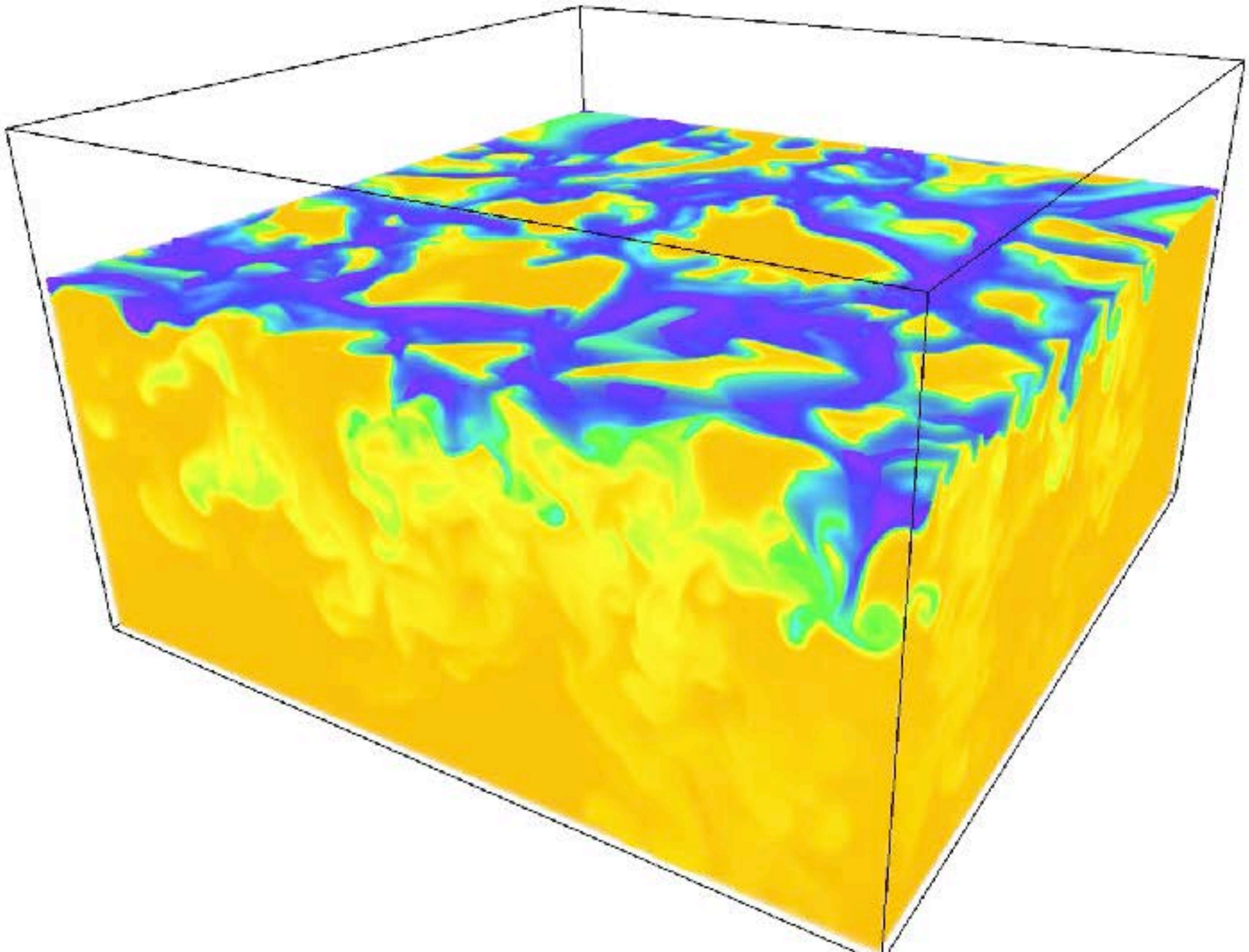


Opacity binning

- Full monochromatic radiative transfer solution in 3D is expensive
- Sort monochromatic wavelengths into groups (opacity bins)
- Solve for average opacities and integrated source functions



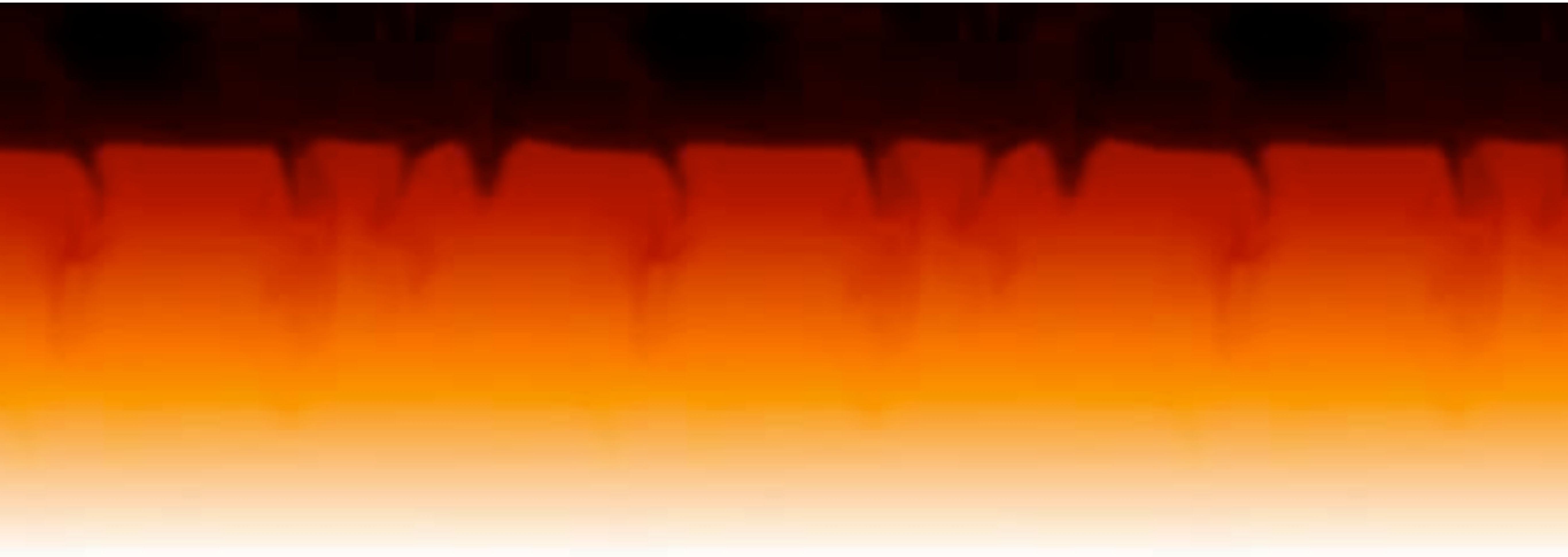
Simulations: surface entropy



Stagger-Code solar surface convection simulation, 6 x 6 x 4 Mm^3 (R. Collet)

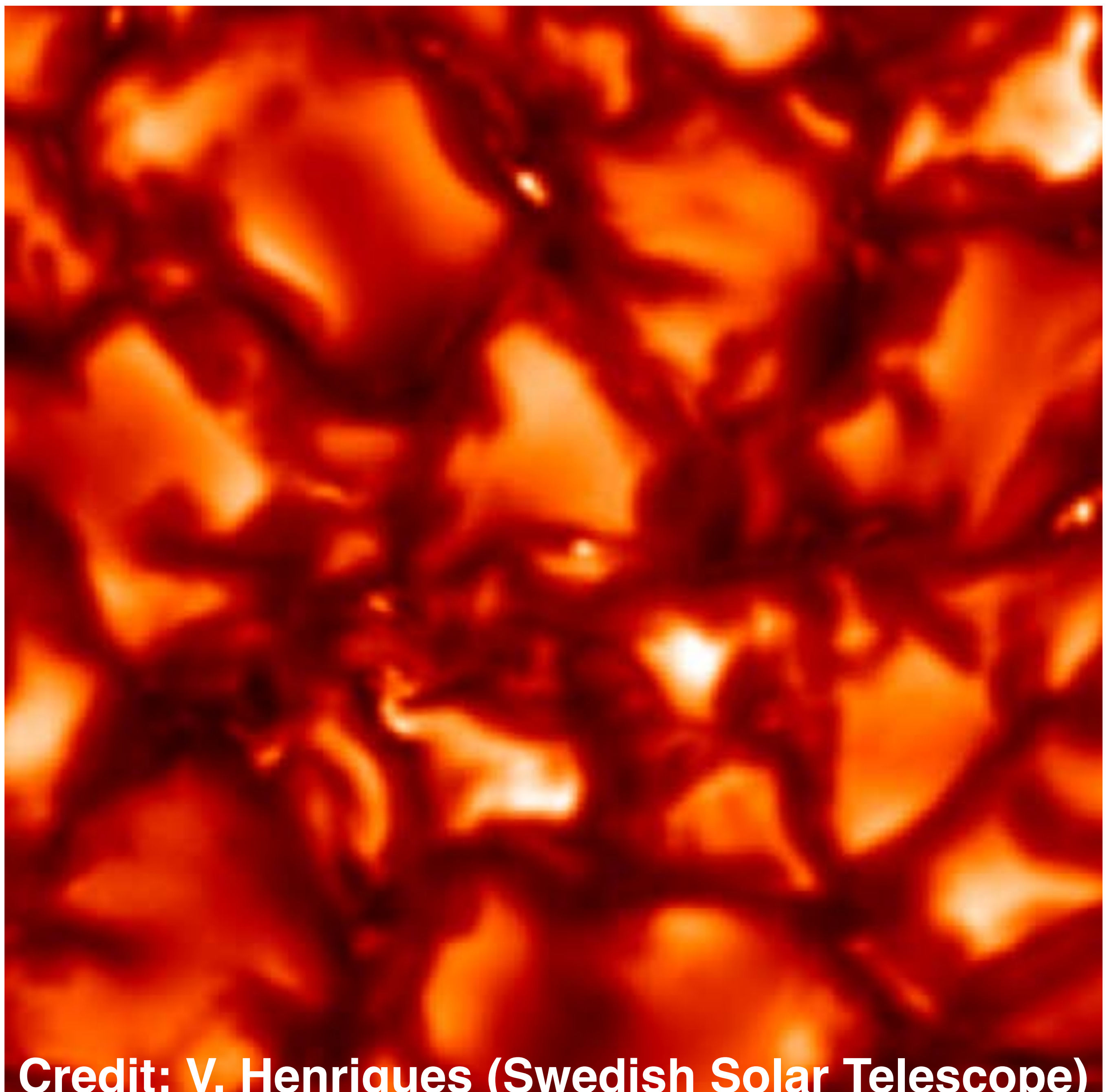
Temperature, vertical slice

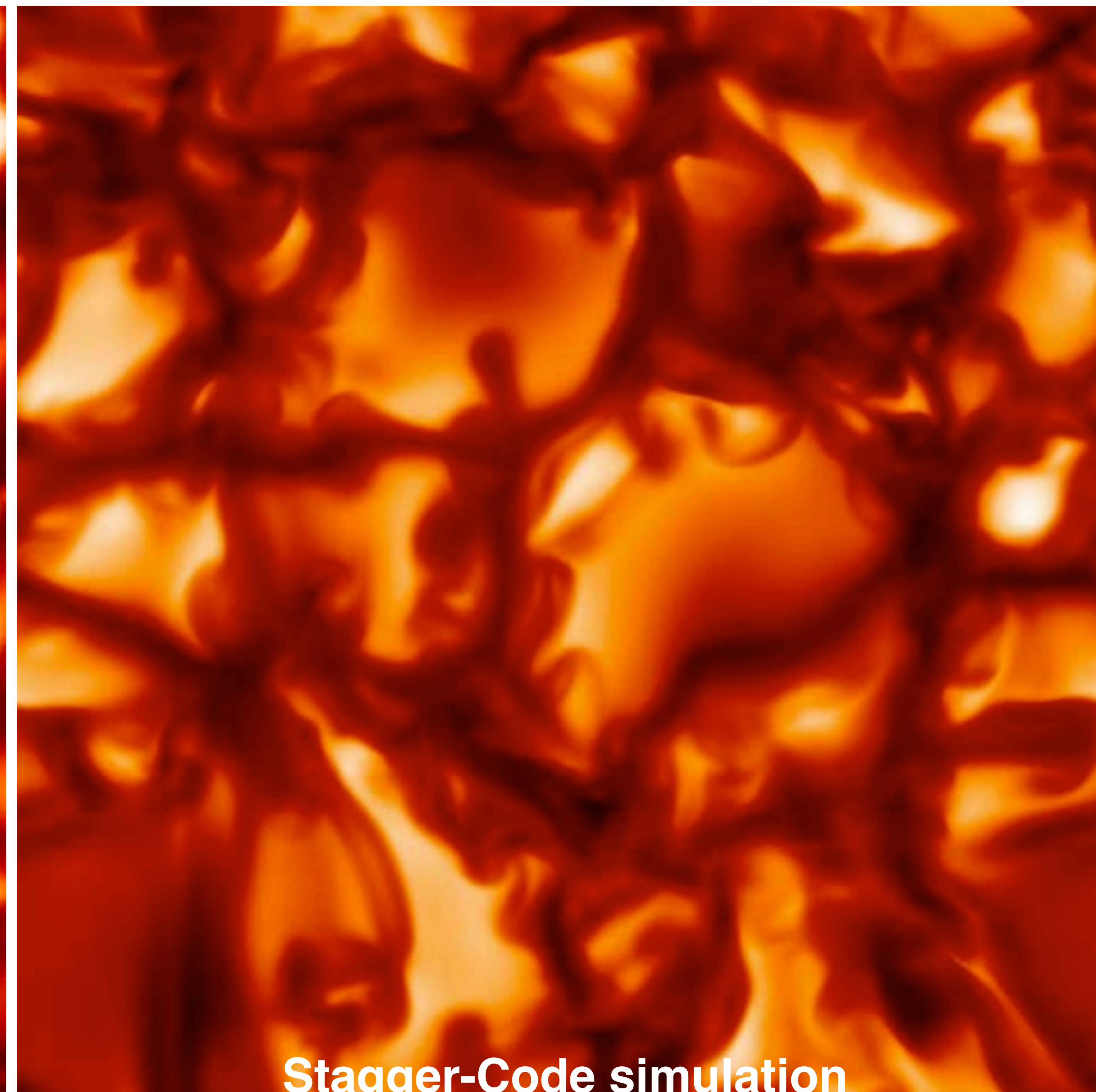
Solar simulation (width:12 Mm, depth: 4 Mm)



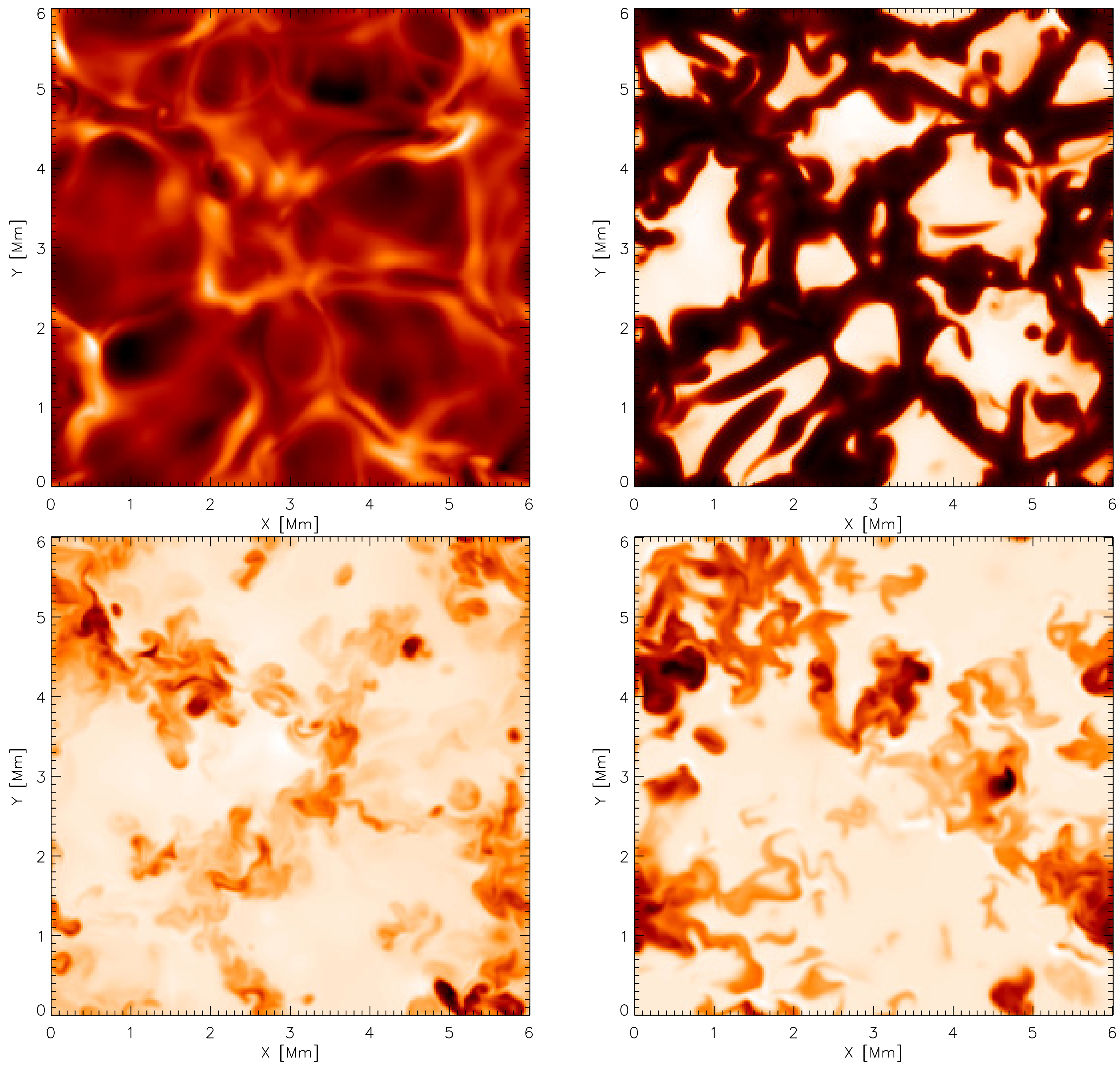
Granulation (simulated)

Solar surface intensity





Temperature patterns at four different depths



-0.4 Mm

0 Mm

0.8 Mm

2.7 Mm

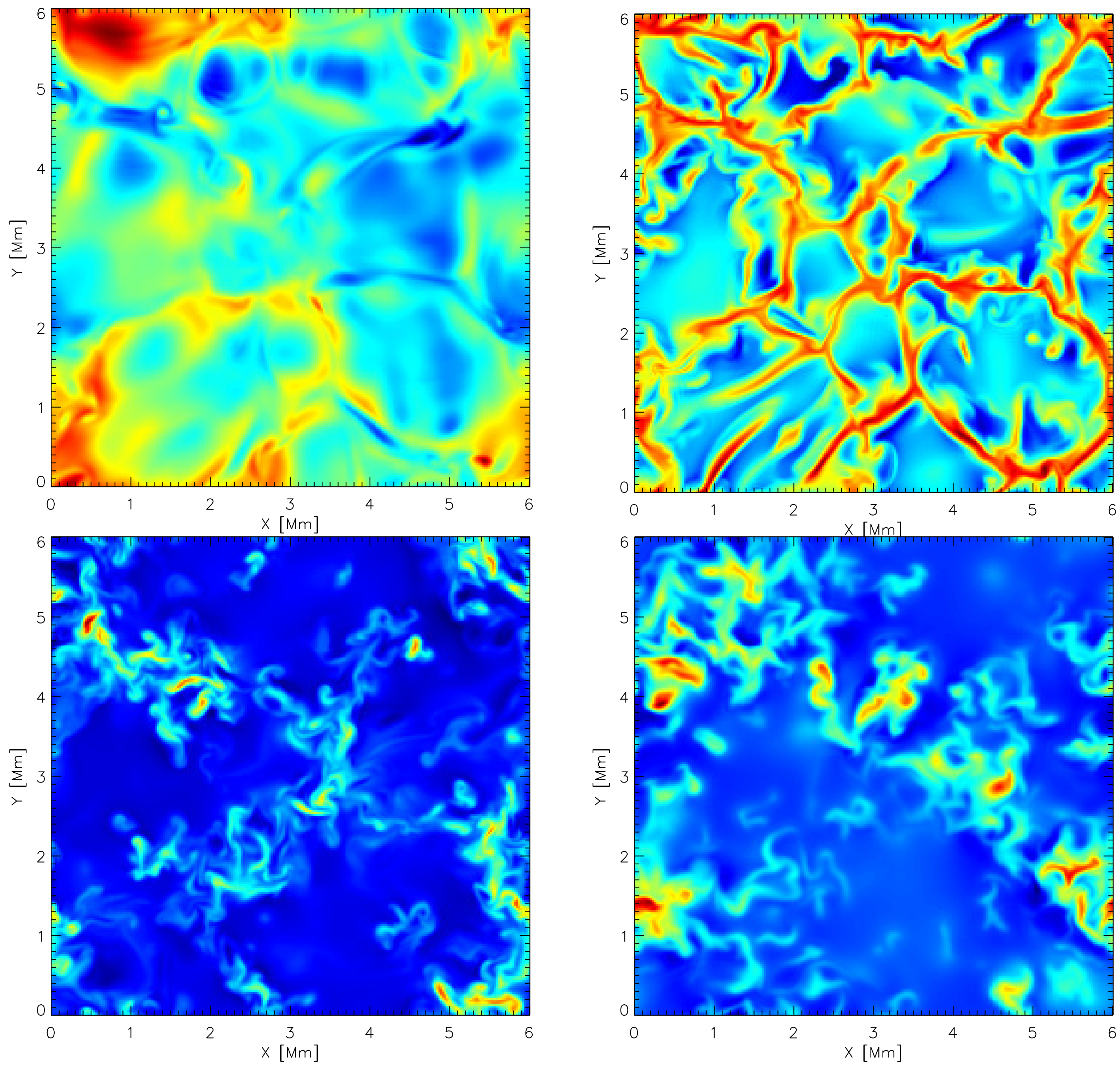
Vertical velocity patterns at four different depths

-0.4 Mm

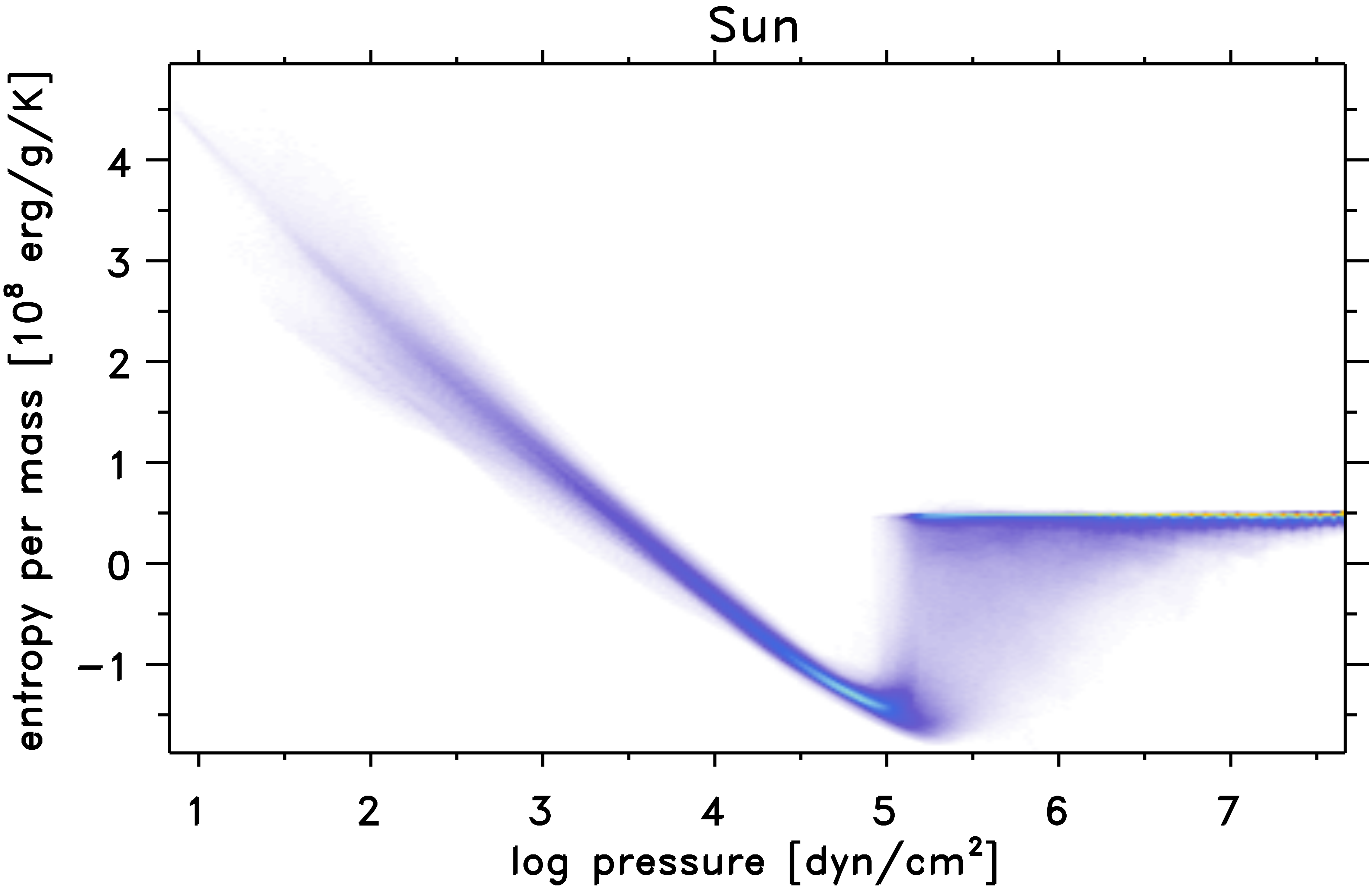
0 Mm

0.8 Mm

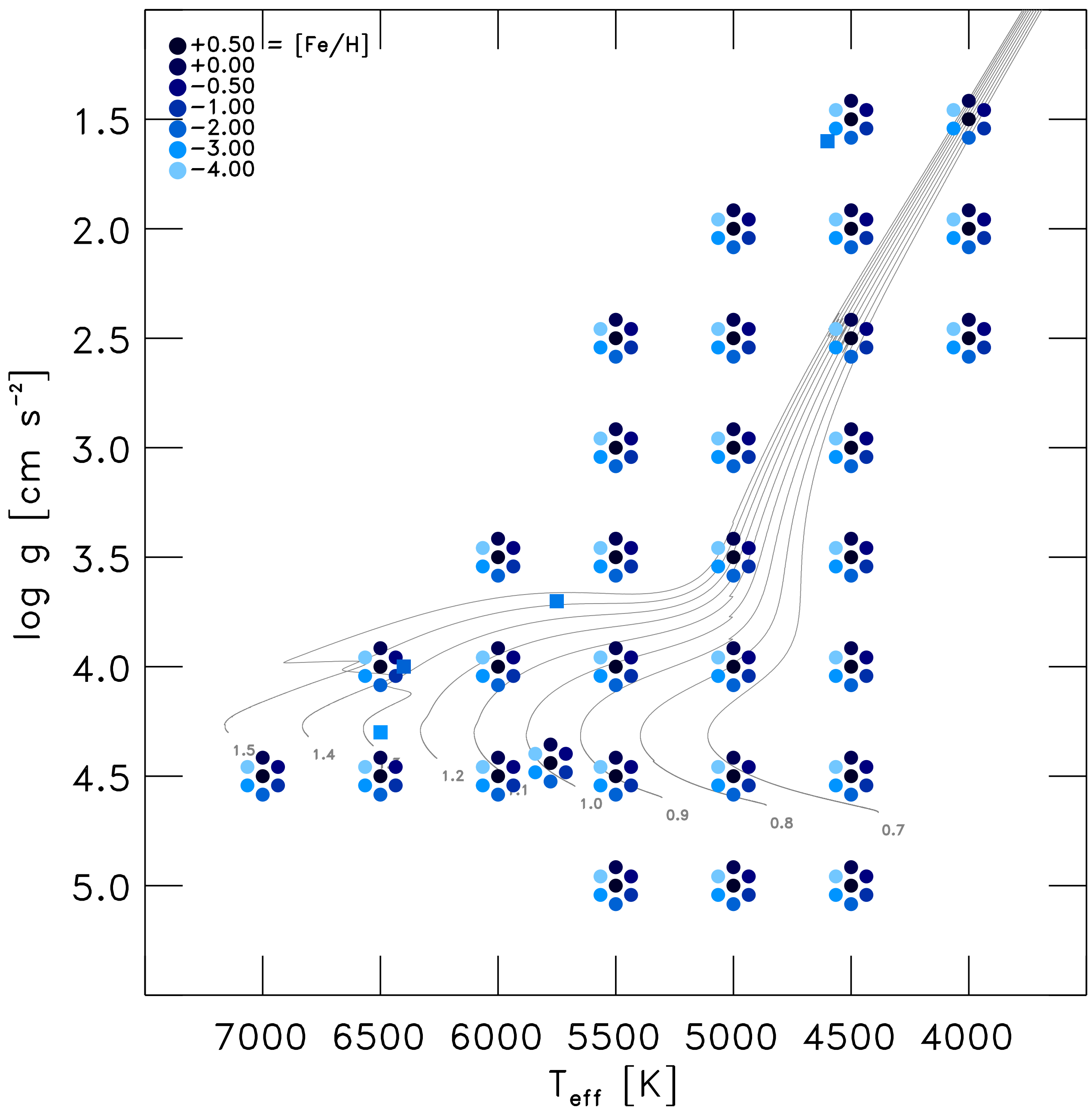
2.7 Mm



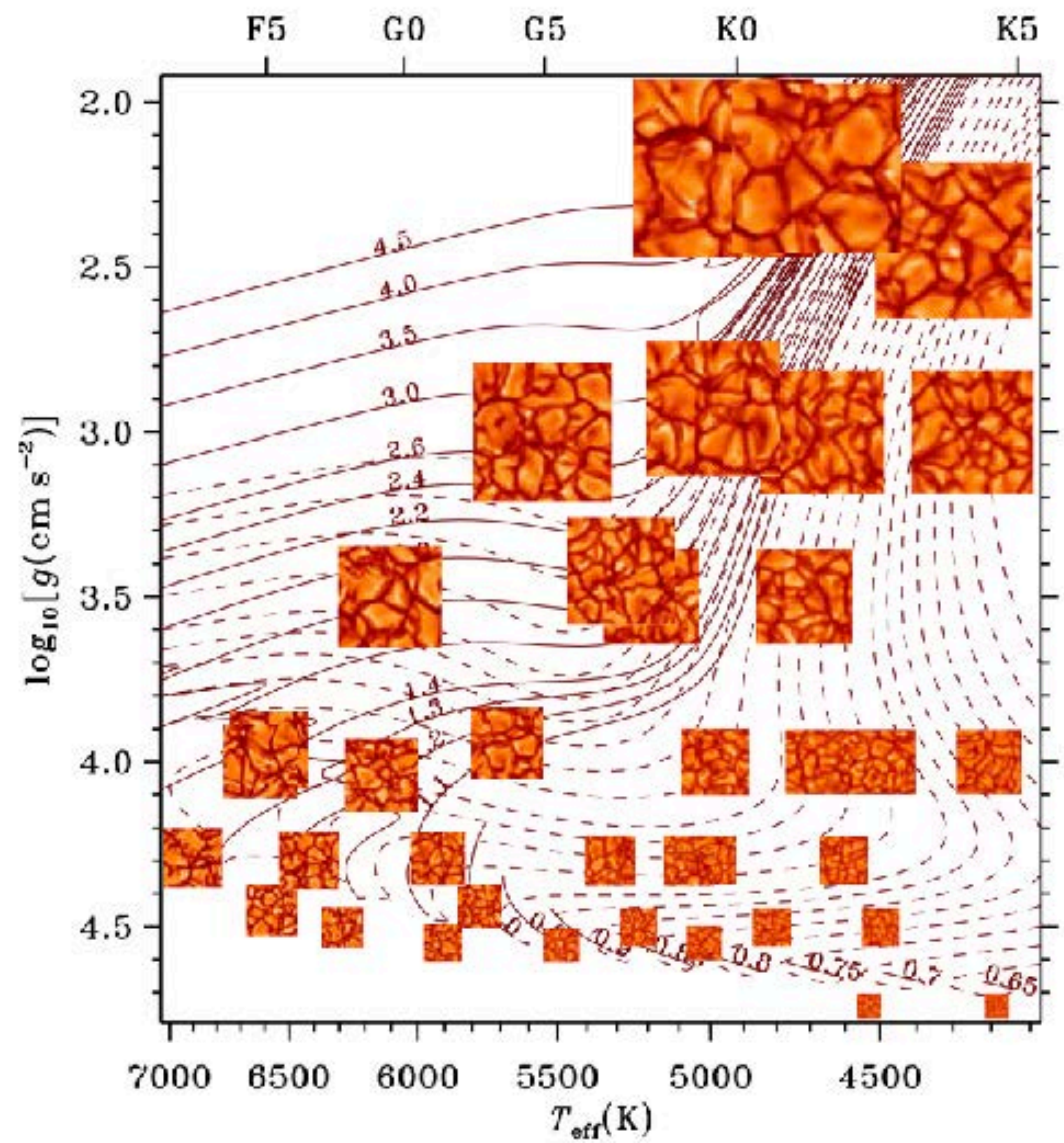
Entropy stratification



3D Simulations across the H-R diagram



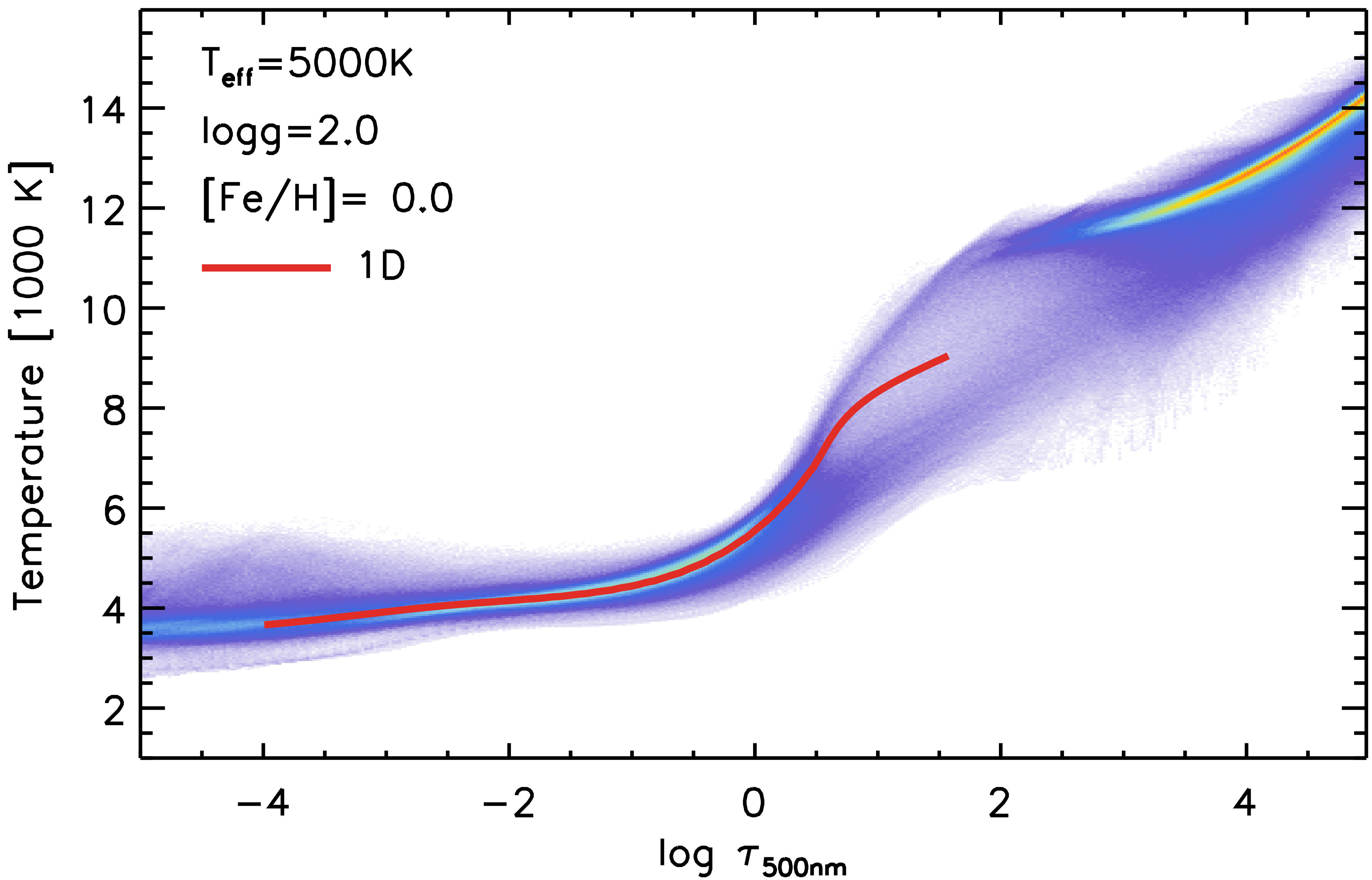
Magic et al. (2013)



Trampedach et al. (2014)

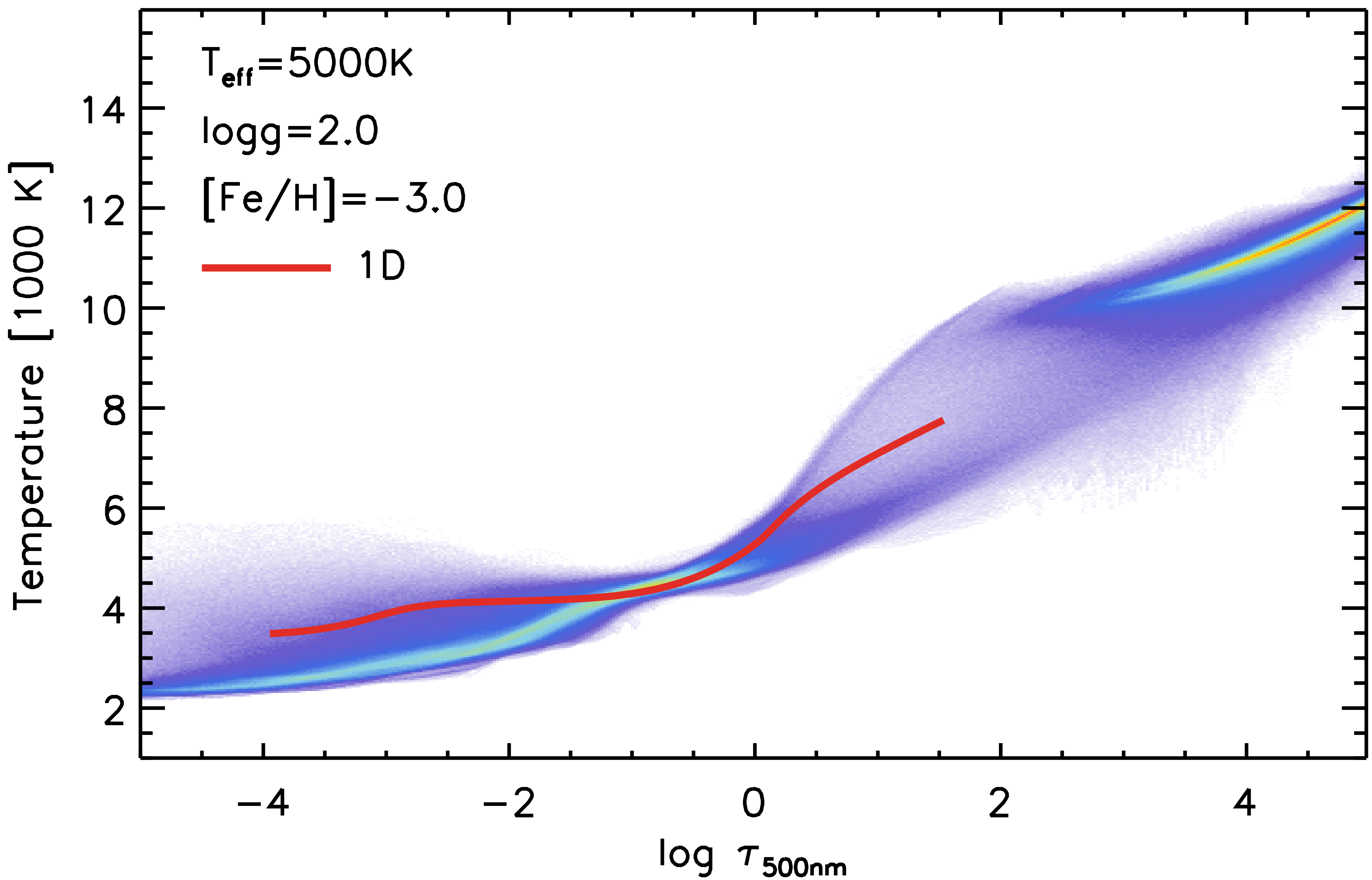
Temperature stratification

Solar metallicity



Temperature stratification

Low metallicity



Energy balance

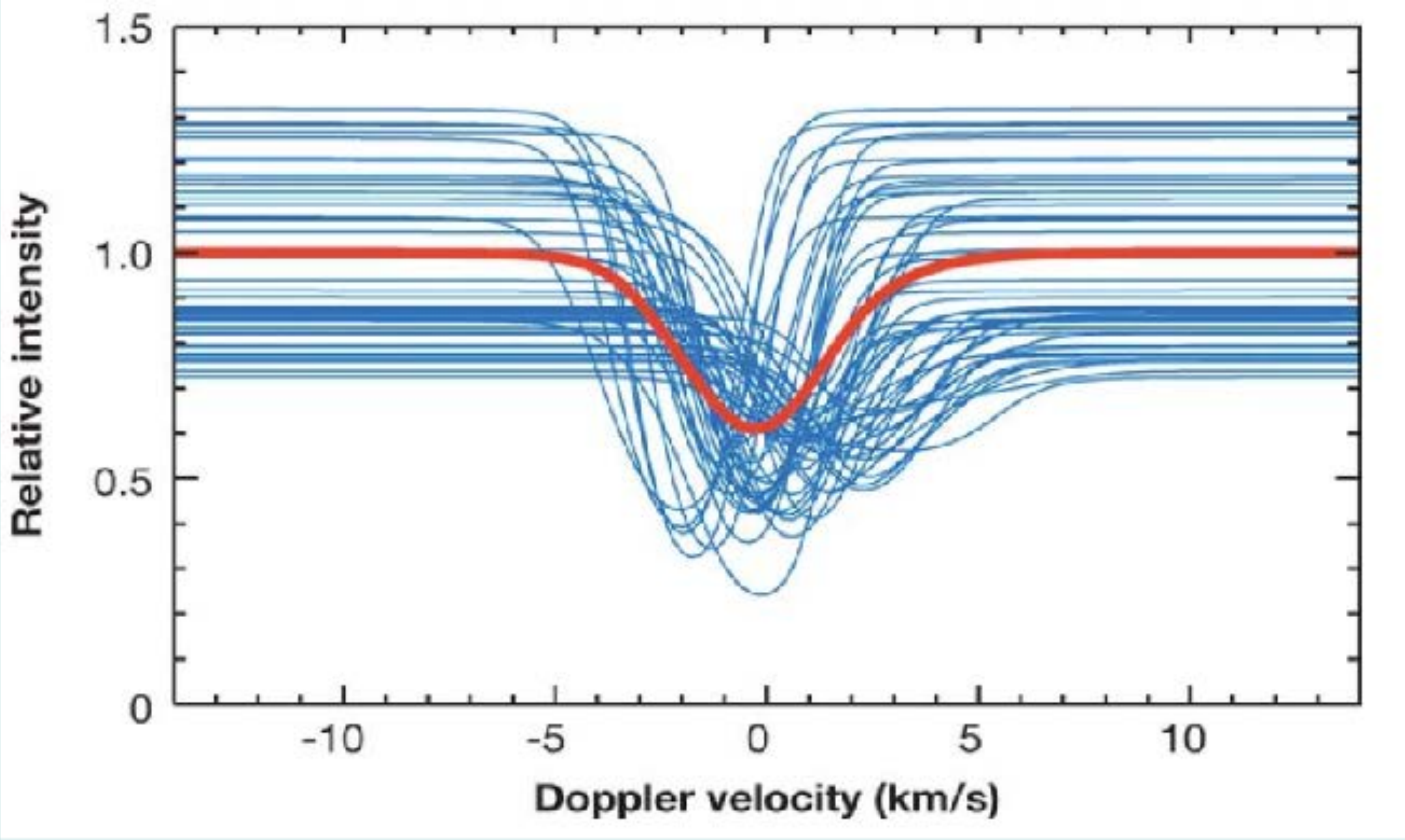
- Competition between **adiabatic cooling** and **radiative heating**:

$$\partial E / \partial t = -\nabla \cdot E\mathbf{u} \; -P\, \nabla \cdot \mathbf{u} \; + Q_{rad} \; + Q_{visc}$$

- Lower metallicity —> lower line opacity —> less radiative heating

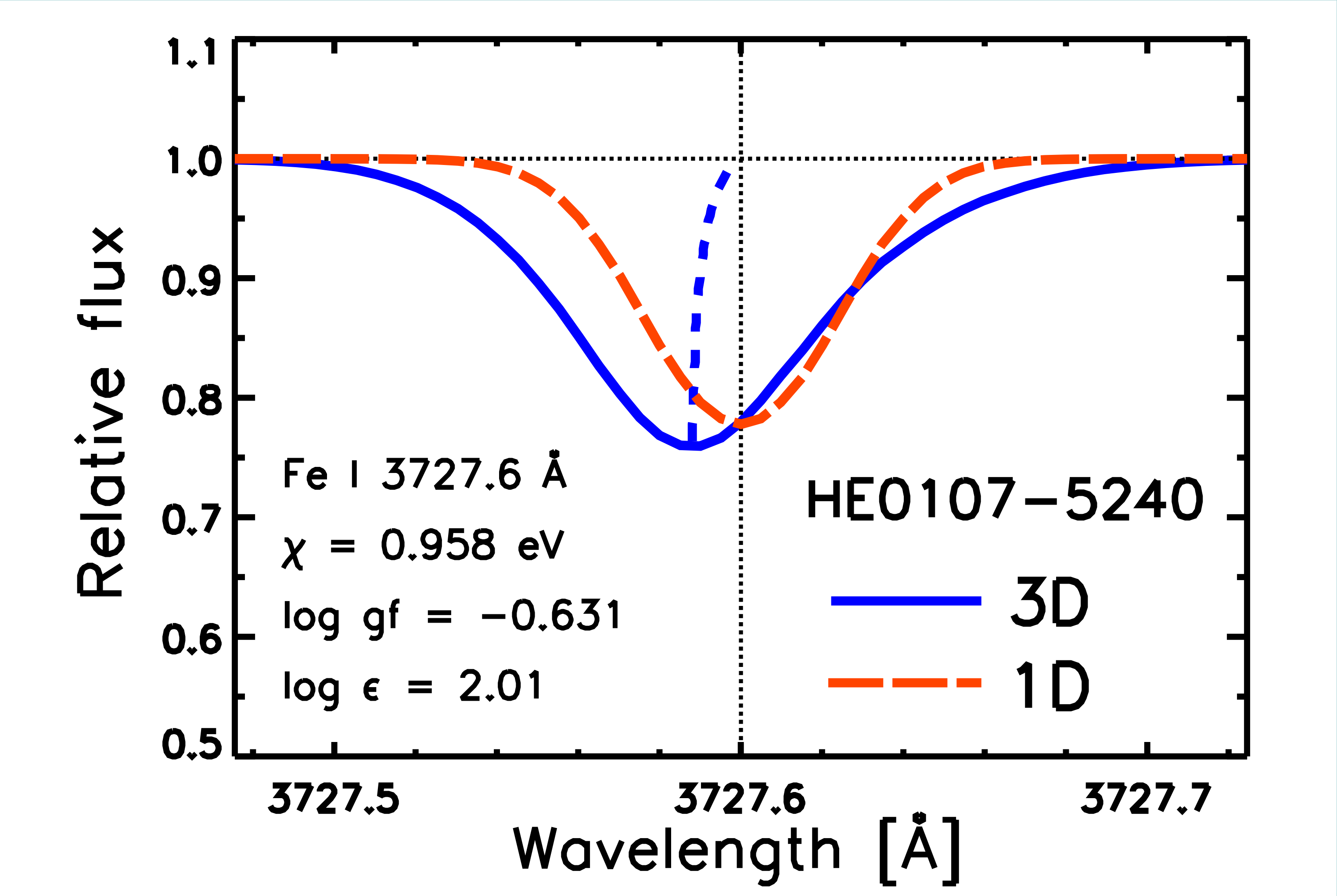
3D Spectral line formation

- Use physical structure from 3D simulations in post-processing calculations to synthesise spectral lines
- Temperature and density inhomogeneities, Doppler shifts
- No need for additional broadening (“turbulence”) parameters as in 1D!



Spectral lines: 3D vs 1D

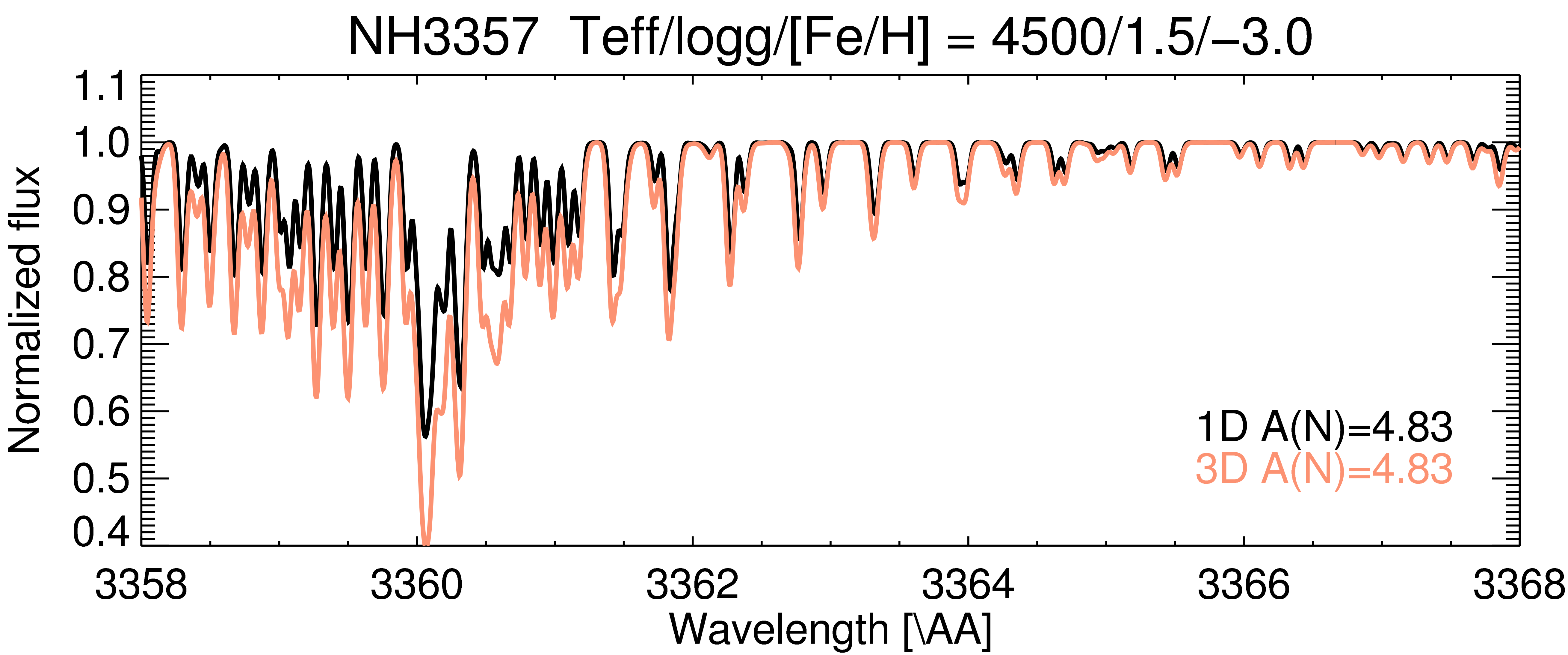
- 3D: wavelength shifts and line asymmetries
- 3D vs. 1D: different line strengths, hence different derived elemental abundances



Applications of 3D stellar surface convection simulations

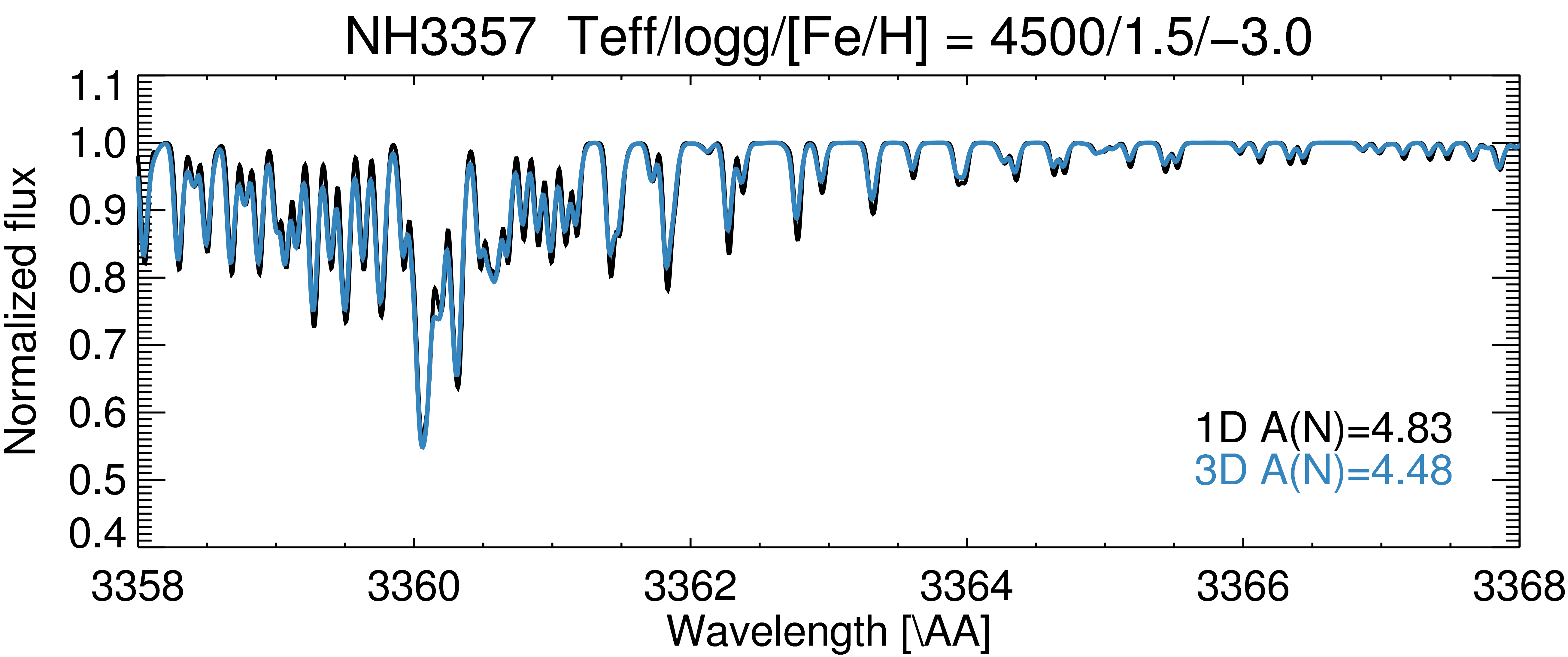
Stellar spectroscopic abundance analyses

Example: NH molecular lines in 3D and 1D, same nitrogen abundance



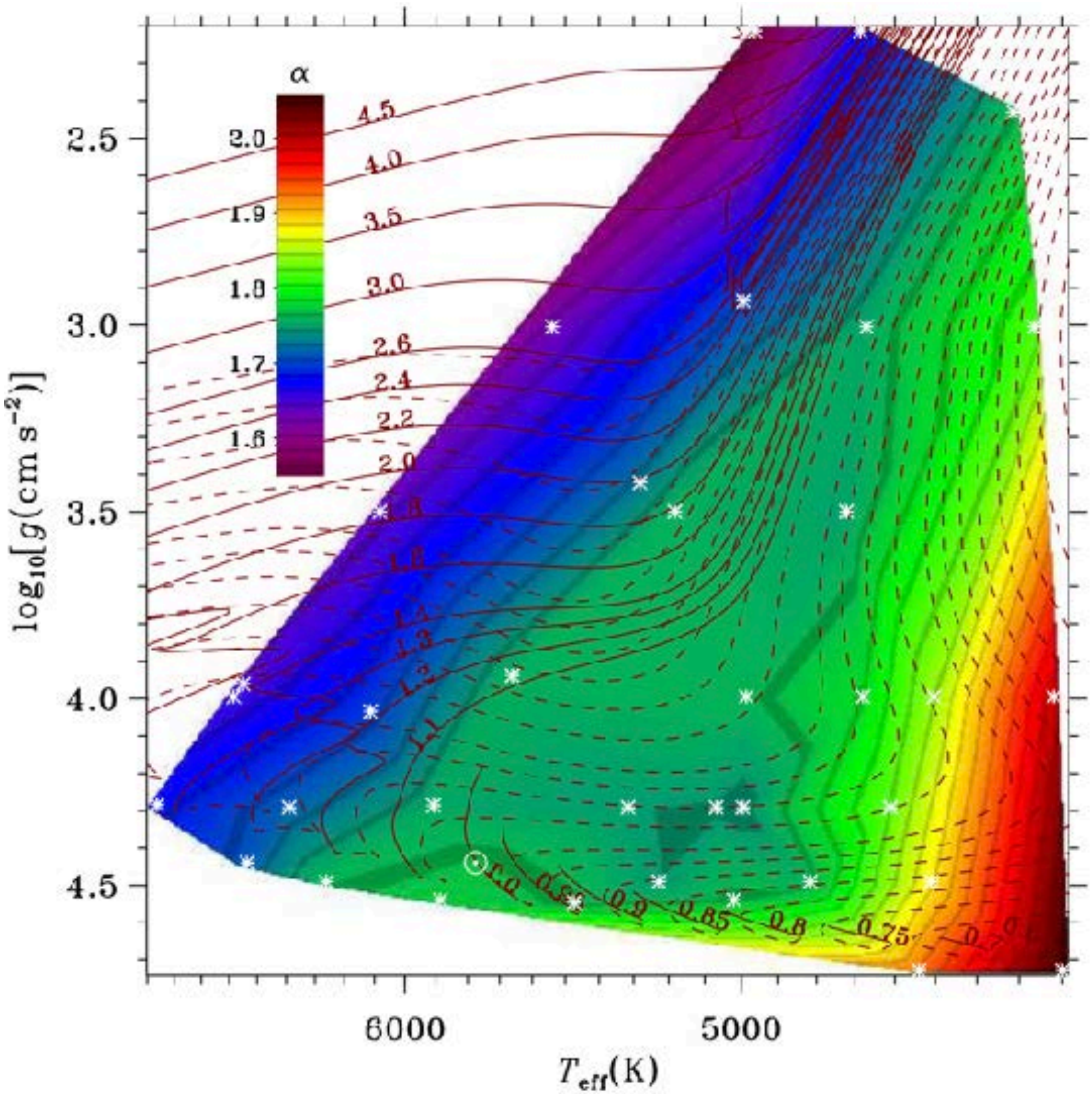
Stellar spectroscopic abundance analyses

Example: NH molecular lines in 3D and 1D, lower nitrogen abundance in 3D



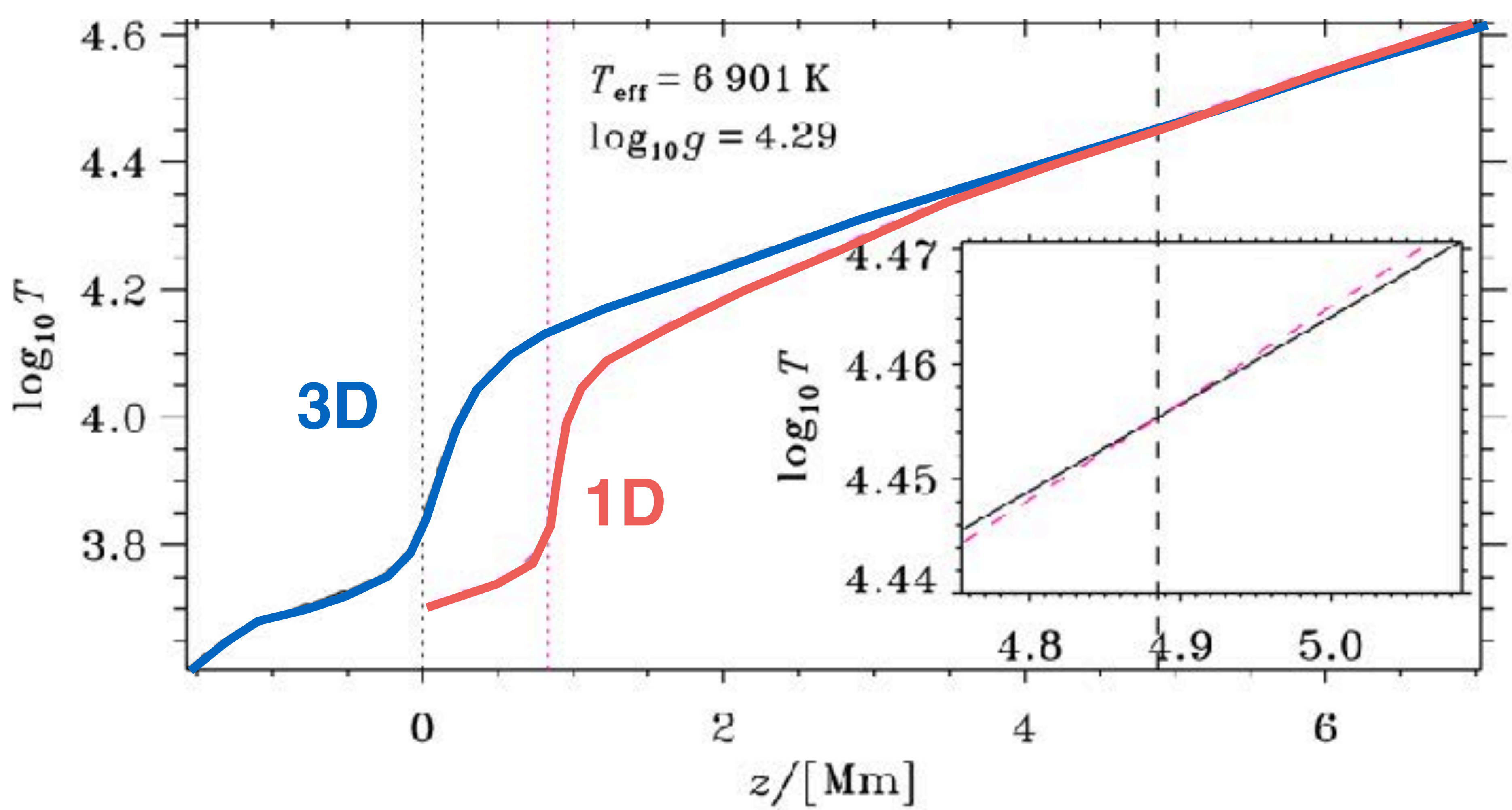
Calibration of MLT

- 3D simulations can be used to calibrate the α_{MLT} parameter used in 1D stellar structure and evolution models (Trampedach et al. 2014; Magic et al. 2015)



Surface effects

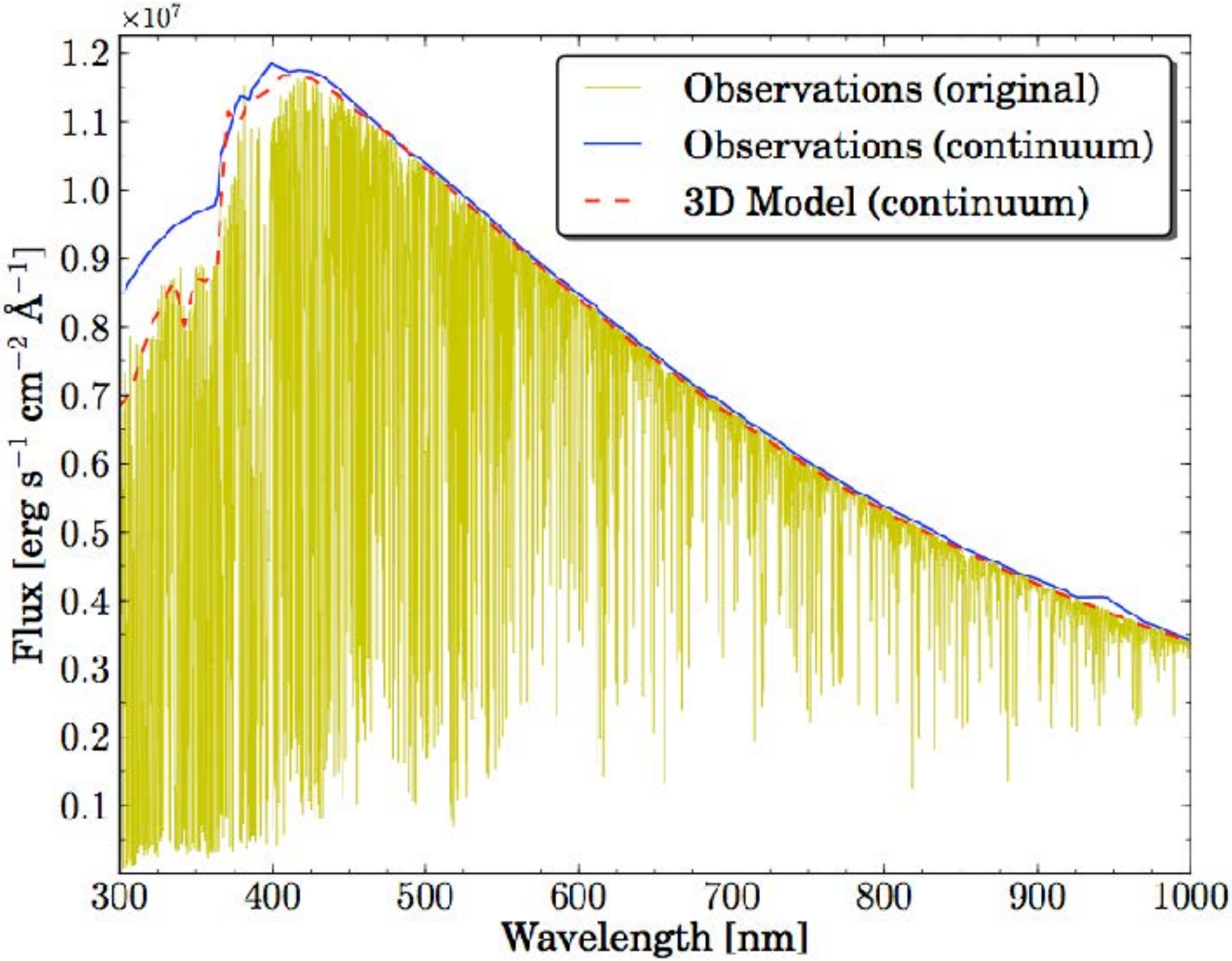
- 3D simulations predict convective expansion of stellar envelopes
- Affects predicted frequencies of stellar oscillations; closer agreement with observed ones than in 1D



Trampedach et al. (2016)

Validation of
3D Simulations

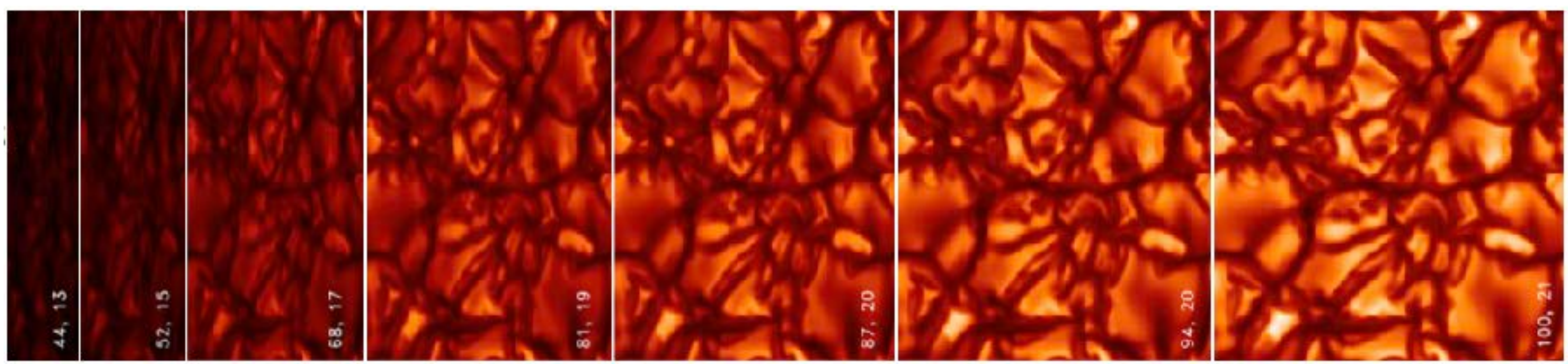
Sun: Spectral Energy Distribution



Pereira, Asplund, Collet et al. (2013)

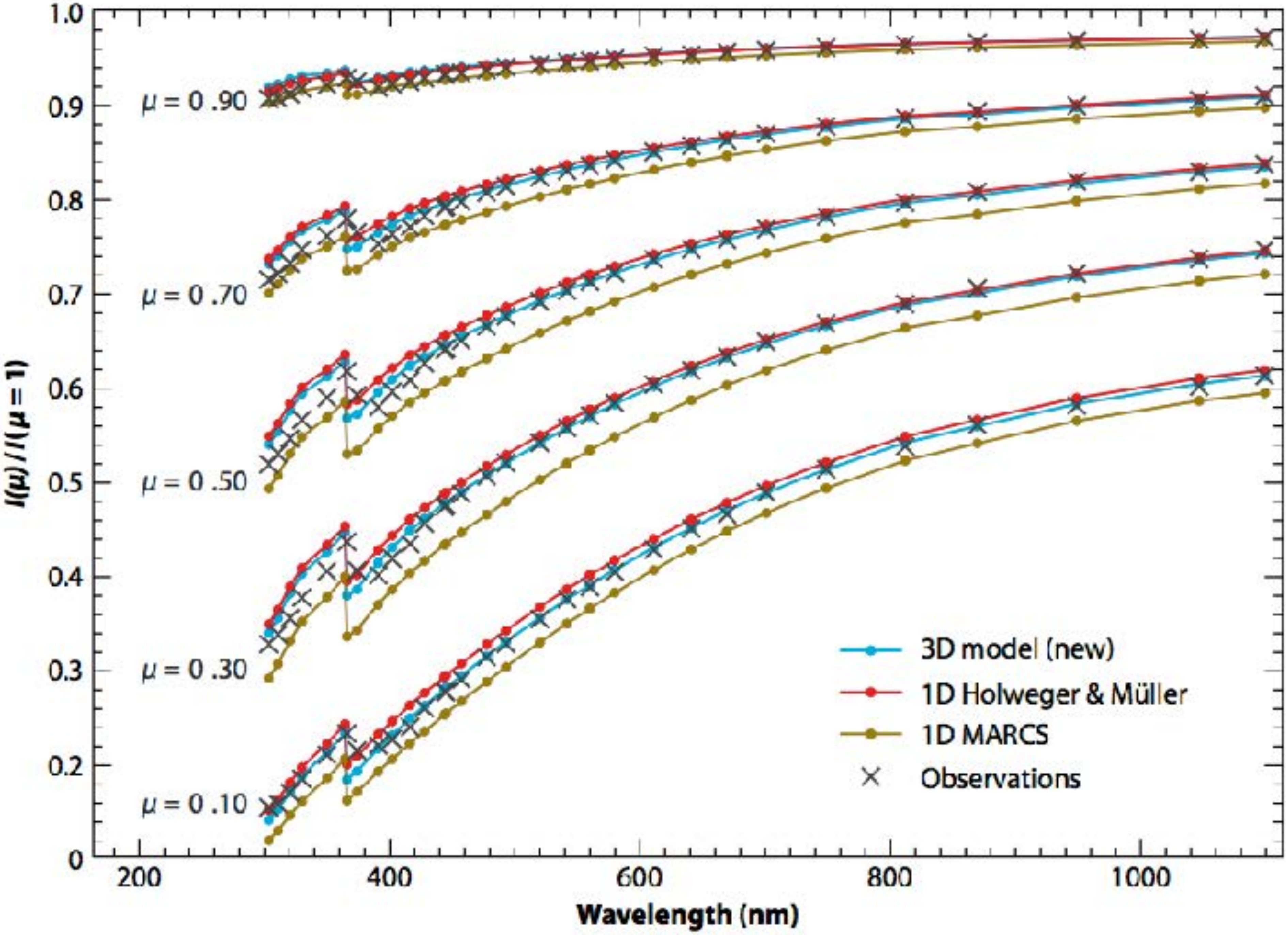
Sun: limb darkening

- Intensity of emitted radiation is lower toward the limb of the stellar disc



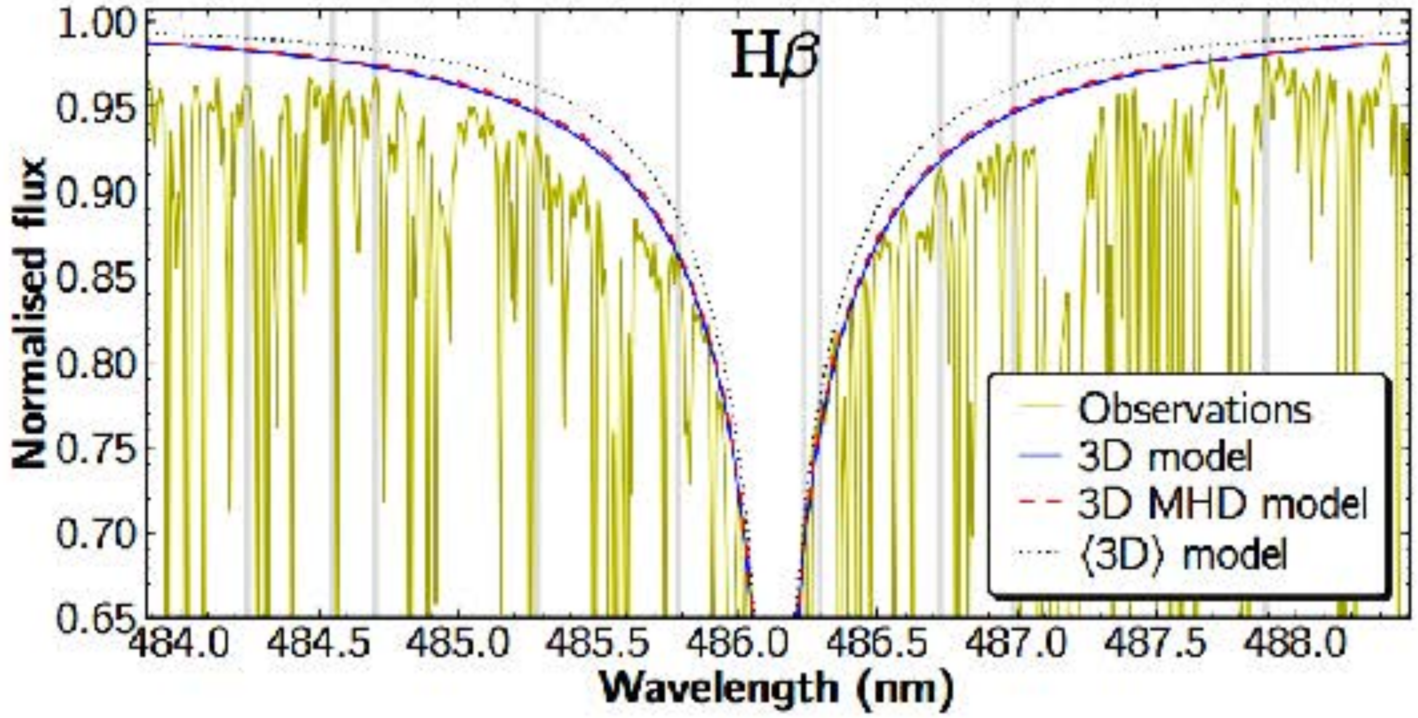
limb ← → **disc centre**

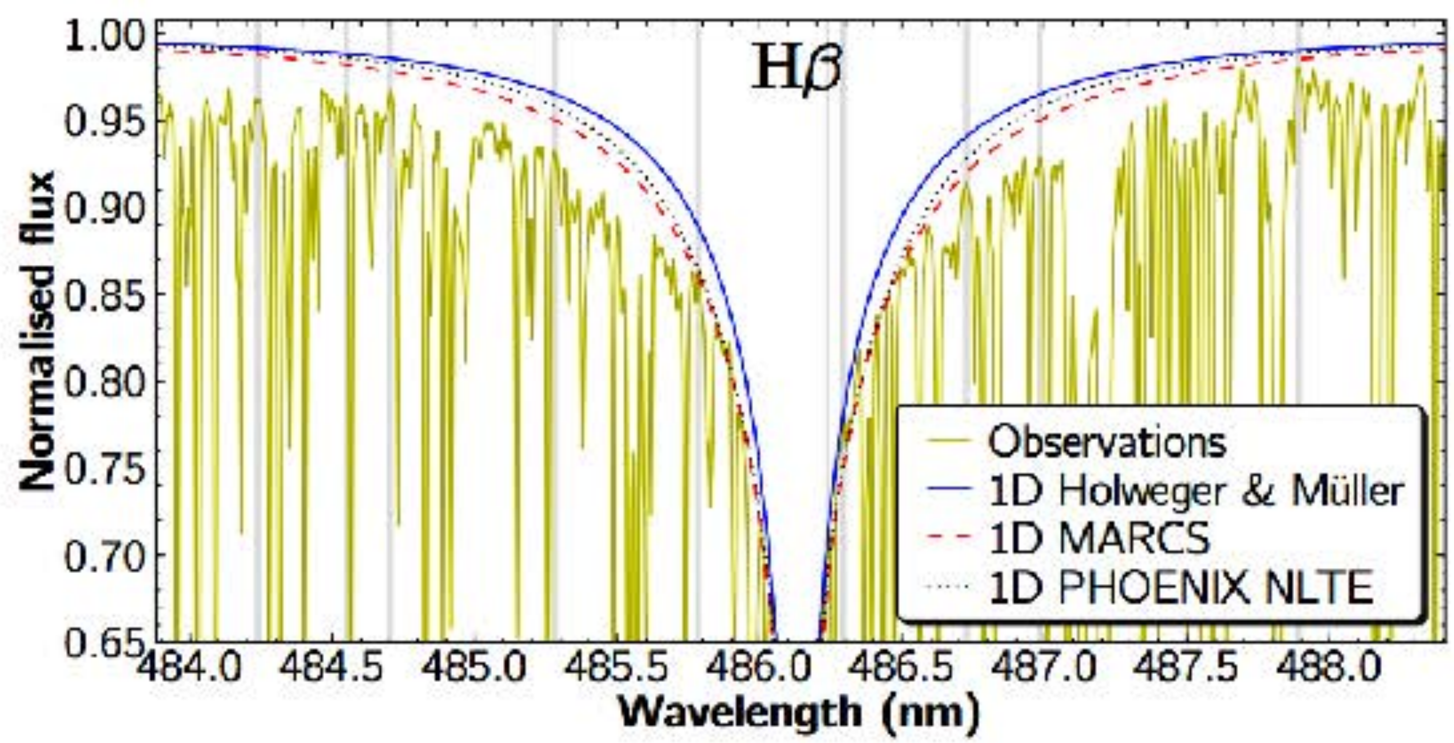
Sun: Limb Darkening



Pereira, Asplund, Collet et al. (2013)

Sun: Hydrogen Lines

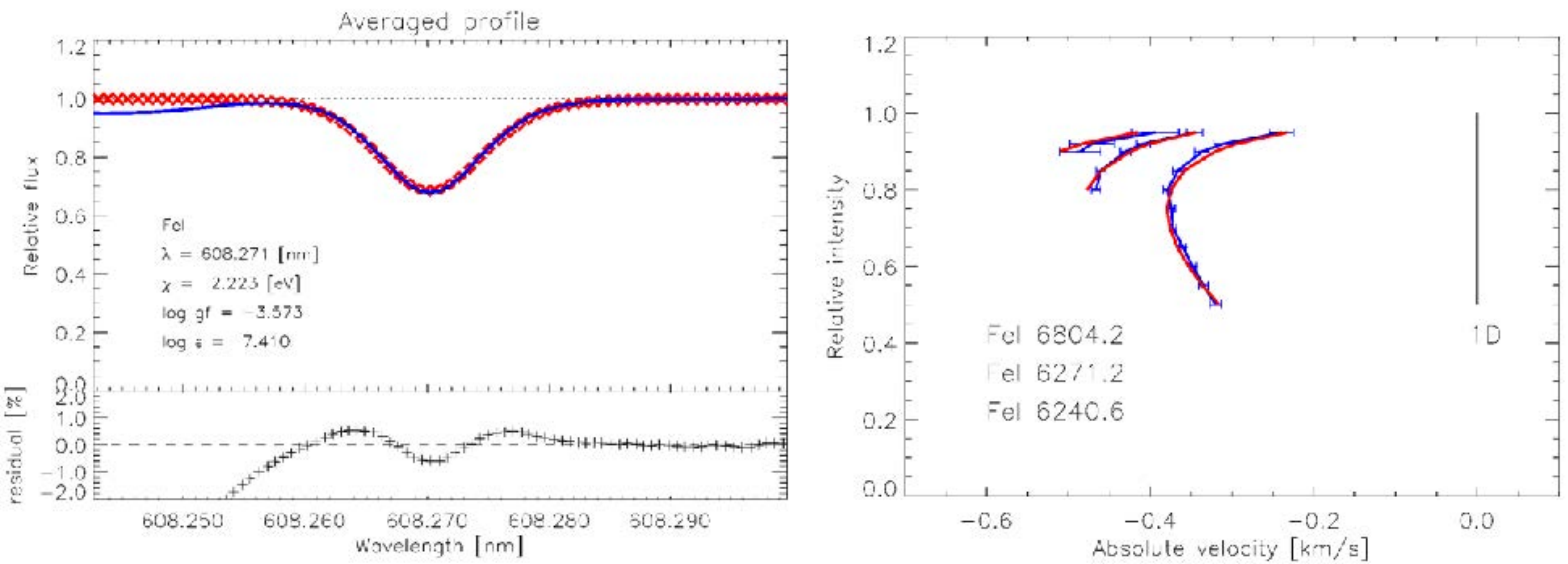




Pereira, Asplund, Collet et al. (2013)

Spectral lines

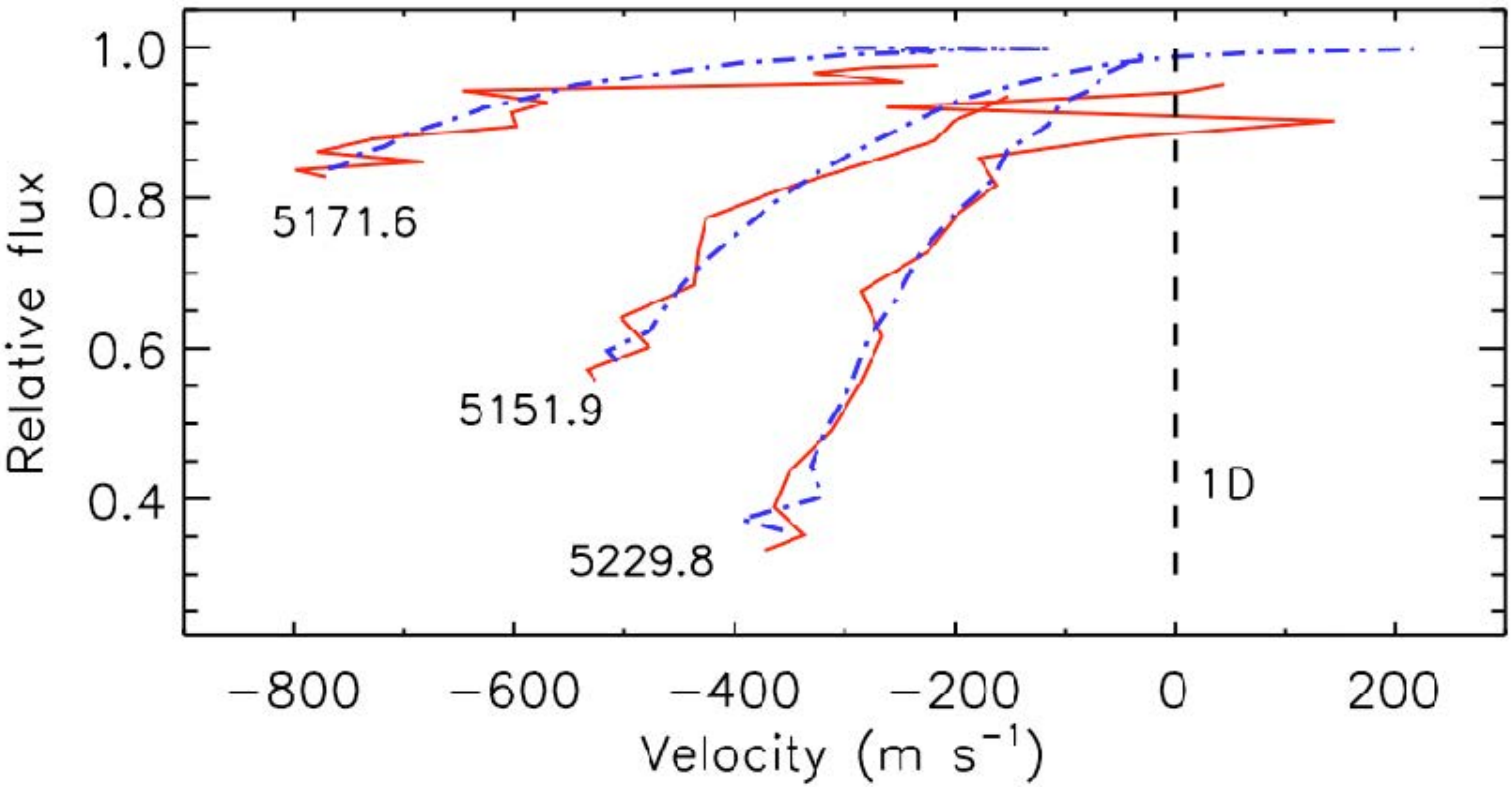
- Spectral synthesis with 3D solar surface convection simulations reproduces observed line shapes and asymmetries
- Relevant spatial scales and velocities are resolved



(Asplund et al. 2000)

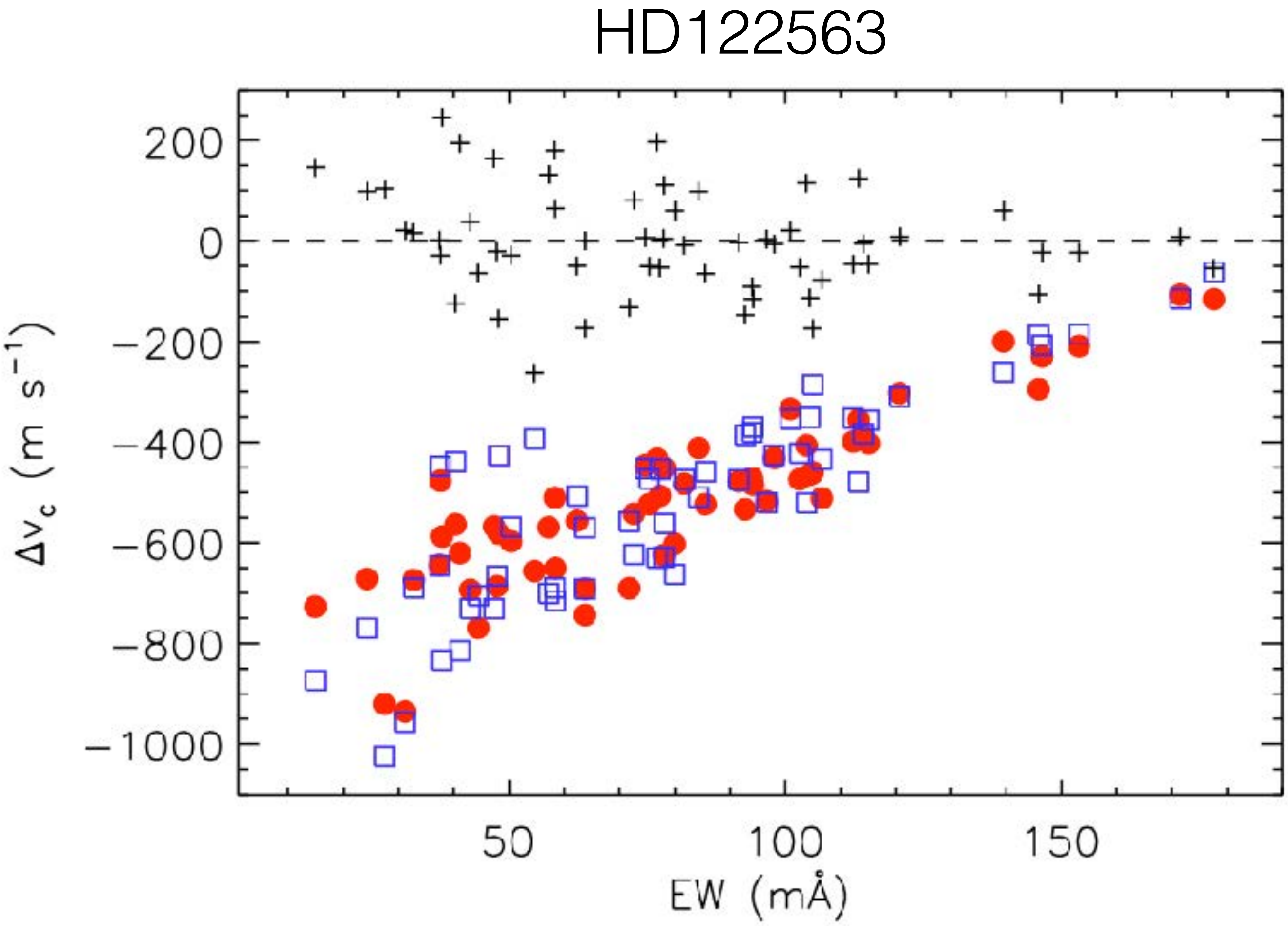
Line asymmetries in other stars

Very metal-poor giant HD122563, McDonald Observatory



Ramírez, Collet, Lambert et al. 2010

Wavelength shifts



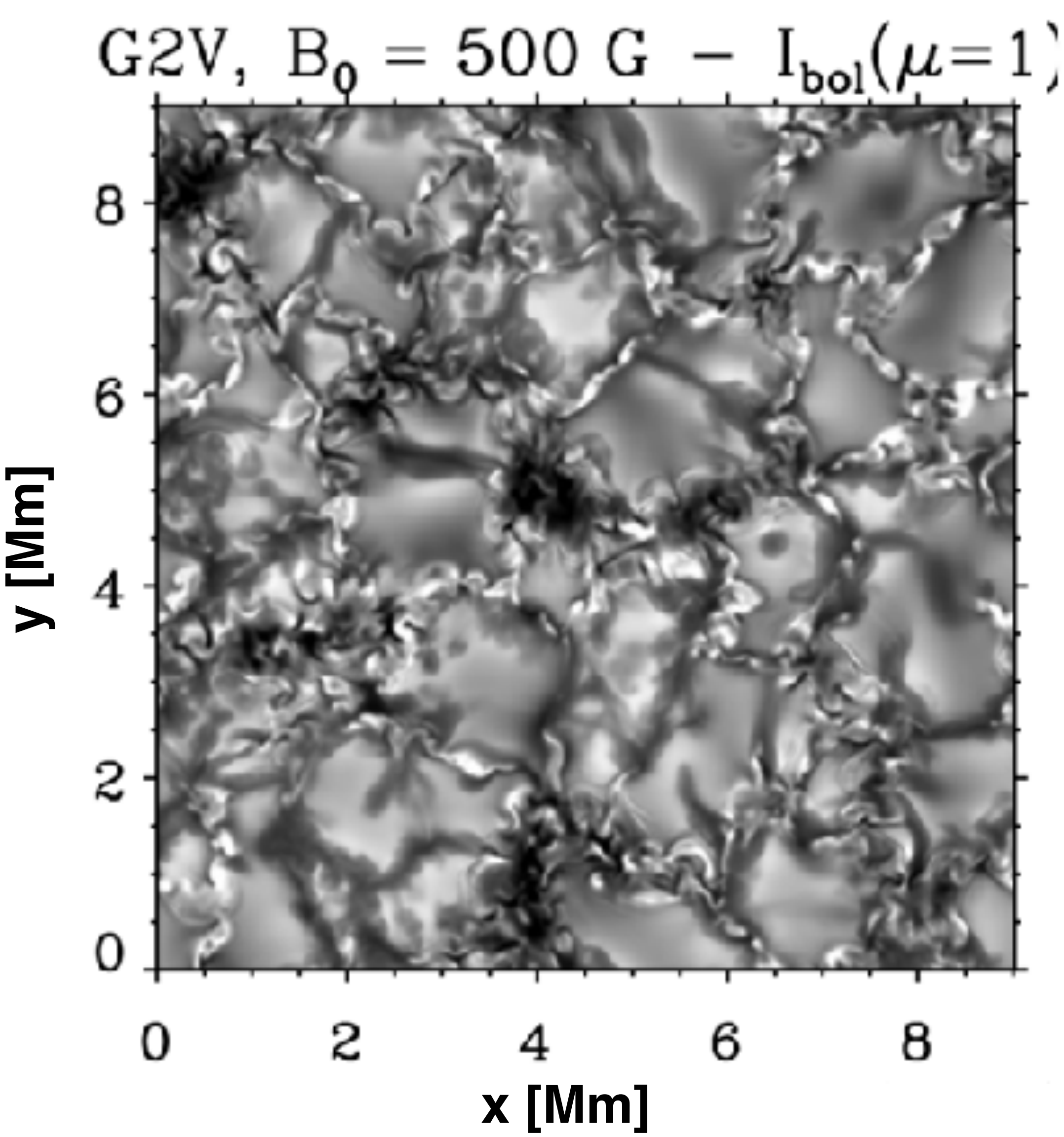
Ramírez, Collet, Lambert et al. 2010

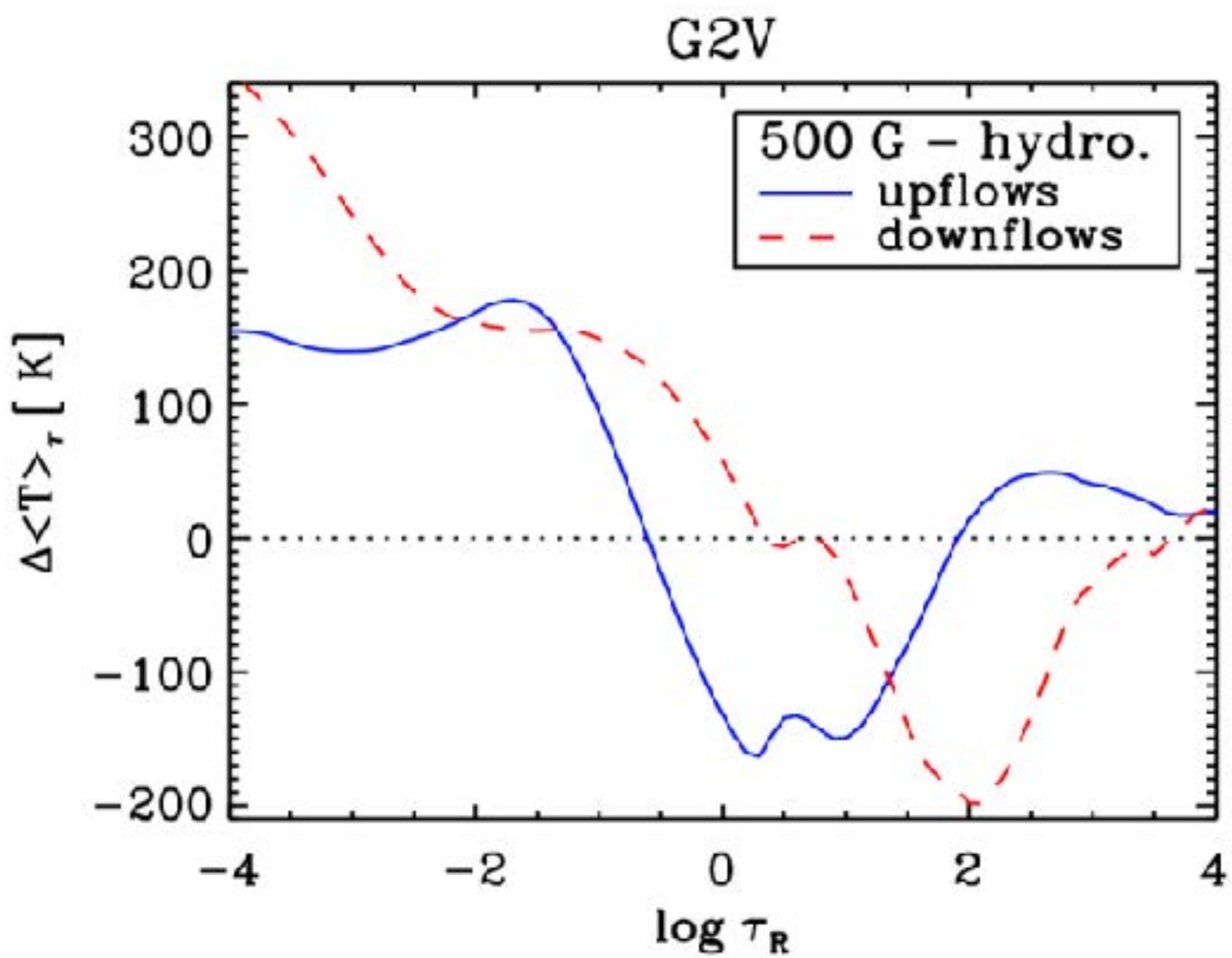
Magneto-hydrodynamic (MHD) simulations

- Magnetic fields: additional degrees of freedom and increased complexity
- Differences in magnetic field configurations
- Dependence on boundary conditions

Magnetic fields

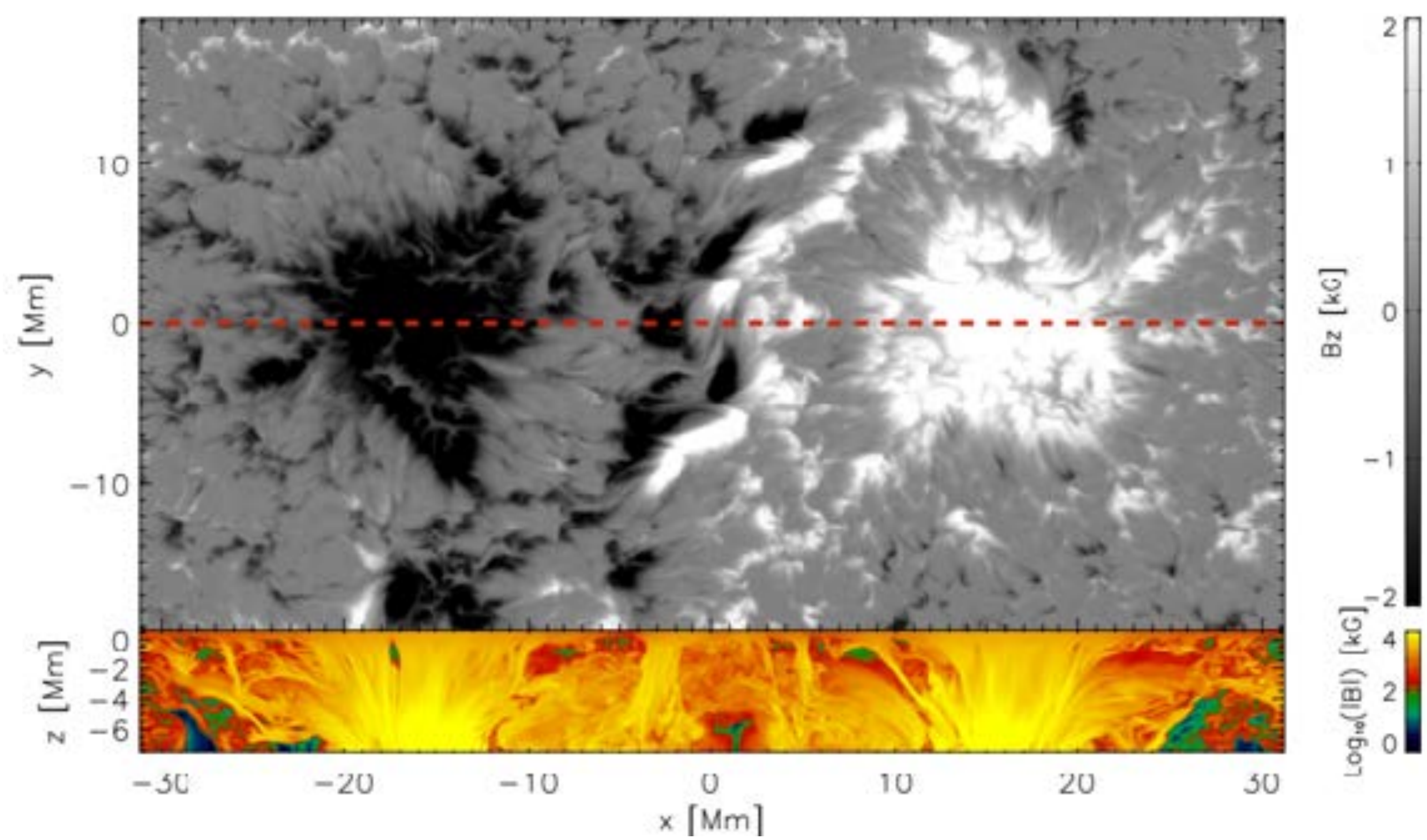
- Effect of vertical magnetic field on physical stratification in main-sequence stars (e.g. Beeck et al. 2015)





Flux emergence and active regions

- Cheung et al. (2007, 2010): twisted horizontal flux tube advected upwards through bottom boundary
- Stein et al. (2006, 2010, 2011): horizontal magnetic field sheet
- Stein & Nordlund (2012): no need for initial coherent flux tube



Cheung et al. 2010

Sunspots and penumbrae

- Rempel et al. (2009), Heinemann et al. (2007)
- Difficult to maintain stability, need deep boundary