Neutron stars
NS models

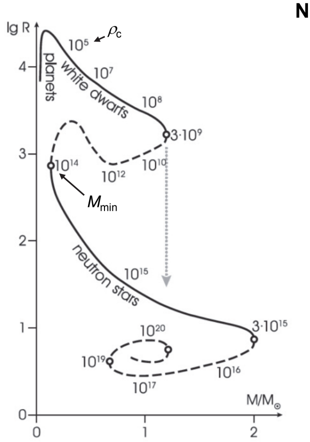

Fig. 38.3. Schematic mass-radius relation (R in km) for configurations of cold catalyzed matter, from the planetary regime to ultradense neutron stars. Some values of ϱ_c (in g cm^{-3}) are indicated along the curve. At the extrema of M (*open circles*) the stability problem has a zero eigenvalue. Solid branches are stable, dashed branches are unstable.

The grey, vertical arrow indicates the collapse of a white dwarf exceeding the maximum stable mass to a neutron star

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Neutron stars
History

- Baade & Zwicky (Dec. 1933, published 1934): “ With all reserve we advance the view that supernovae represent the transitions from ordinary stars to **neutron stars**, which in their final stages consist of extremely closely packed neutrons”.

- But: Lev Landau completed a paper in (1931) in which he writes for stars heavier than 1.5 M⊙ “the density of matter becomes so great that atomic nuclei come in close contact, forming one gigantic nucleus.”

Note: the neutron was discovered later by J. Chadwick in January 1932 (publ. Feb. 1932).

- 1939: Oppenheimer & Volkoff compute first neutron star model, assuming a free (ideal) neutron gas (EOS) and hydrostatic equilibrium in General Relativity (GR) $\longrightarrow M_{max} = 0.7M_\odot$.

- 1967: First Pulsar discovered by J. Bell & A. Hewish, identified as a NS, independently by Pacini (1967) and T. Gold (1968).

- Today: more than 1700 radio pulsars discovered. 10^9 NS in our Galaxy expected!

Neutron stars
Introduction

- NS are remnants of supernova (II) explosions of massive stars, with initial masses between ~(8-10) M⊙ and ~(25-30) M⊙. Also, if a WD accretes matter from its companion in a close binary system and exceeds M_{ch}, then it may collapse & explode as a supernova type Ia, or collapse to become a NS.

- The main source of the pressure needed to counter balance the gravitational force is the degenerate neutrons, since e^- degeneracy cannot support objects with $M > M_{ch}$.

remember: M_{ch} is obtained for UR, degenerate (ideal) e^- gas, and is independent of rest mass m_ec^2! Since neutrons are also fermions (i.e. obeying Pauli’s exclusion principle) M_{ch} is also valid as a limiting mass for and **ideal**, degenerate, UR neutron gas with μ_e=1, but this **limit is too high**!

Neutron stars
Introduction

- NSs are born hot (10^{10}-10^{11} K) in the collapse of a highly evolved star, but after a few days T~10^9…10^{10} K, due to efficient neutrino losses; it may even drop to 10^8 K after 100 years.

- 10^8 K (kT≈10 keV) is low (cold) compared to the characteristic excitation energies in the nuclei of ~1 MeV (10^{10}K) and to the nearly relativistic & degenerate neutrons (E_F ≈ 1000 MeV).

- During NS formation (i.e. relatively high T) nuclear reactions are very rapid and thermodynamic equilibrium (TD) is reached very quickly. Such matter is called **catalyzed matter** (T and ρ are the catalysts), also called **nuclear equilibrium**. The free energy of a macroscopic volume V, containin N_b baryons (nucleons, hyperons, deconfined quarks,…), which is electrically neutral, is

$$\text{Free Energy} = (E - TS)_{V,N_b} \rightarrow \text{ minimum.}$$

- if extrapolated to degenerate (cold) state, the TS term can be neglected and, because EOS (and therefore E) does not depend on T $\longrightarrow$ **cold catalyzed matter**, i.e. NS is in its lowest energy state, the ***ground state of matter***.

remember: ground state (@ zero pressure) consists of
(i) n & p are packaged in to ^{56}Fe nuclei, the most tightly bound nucleus,
(ii) Nuclei are arranged in a lattice to minimize configuration energy (free energy, see above)
(iii) e^- are in ferromagnetic state
$\longrightarrow$ expect NS surface layers to be a ferromagnetic iron lattice.

Neutron stars

Introduction

NS characteristics:

From observations: $M \sim 1 - 2\,M_\odot$, $R \sim 10 - 20$ km → $\bar{\rho} \simeq \frac{M_\odot}{(10\,\mathrm{km})^3} \simeq 10^{15}\,\mathrm{g\,cm^{-3}}$.

another observation: most rapidly rotating pulsar (ν = 716 Hz; ω = 4499 s^{-1}):

$$\underbrace{m\omega^2_{\max}R}_{\text{centrifugal force}} < \underbrace{\frac{GMm}{R^2}}_{\text{gravitational force}} \to \frac{M}{R^3} > \frac{\omega^2_{\max}}{G} \to \bar{\rho} \geq 7.2\times10^{13}\,\mathrm{g\,cm^{-3}}\,.$$

(central density is much larger).

Note: nuclear density: $\rho_{\mathrm{nuc}} := \rho_0 \simeq m_{\mathrm{u}}/(4\pi/3)r_0^3 \simeq 2.8\times10^{14}\,\mathrm{g\,cm^{-3}}$; $r_0 \simeq 10^{-15}\mathrm{m}$

nuclei start to touch each other @ $\rho \simeq 1.5\times\rho_0 \simeq 4\times10^{14}\,\mathrm{g\,cm^{-3}}$

→ NS are like giant atomic nuclei: but NS is hold together by gravity, nuclei by strong forces.

Average inter-particle distance: < 1fm (10^{-15} m) → strong interactions → crucial for EOS

Temperatures: surface ~ 10^6 K, interior up to 10^{11} K

Magnetic fields: 10^{12} – 10^{15} G !

Schwarzschild radius (event horizon), no signal can escape from within sphere with radius r_g

Space-time is strongly curved: $\frac{R}{r_g} \simeq 2 - 4$ where $r_g := \frac{2GM}{c^2} = 2.95\,M/M_\odot\,\mathrm{km}\,.$

Neutron stars

Theoretical NS compositions (structures)

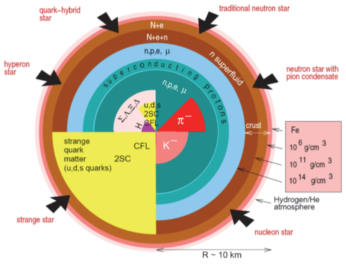

Fig. 1. Neutron star compositions predicted by theory.

-depending on NS mass and rotational frequ., NS may be compressed to densities ~10xρ_{nuc}.

↓

numerous subatomic particle processes are likely to compete with each other (Figure):

e.g., generation of hyperons and baryons resonances ($\Sigma,\Lambda,\Xi,\Delta$), quark (u,d,s) deconfinement, formation of boson condensates (π, K$^-$, H - matter).

Mesons are bosons (integer spin)

Baryons are fermions (1/2 integer spin)

H… H dybarion (a six-quark composite)
CFL … colour-flavoured locked quark pairing (QCD): all 3 quark flavours participate symmetrically
2SC … two-flavour superconducting phase

Neutron stars

particle overview

$m_\mu \approx 207\, m_e$
$t_\mu \approx 2.2\ \mu$s

$m_\tau \approx 207$x$16.8\, m_e$
$t_\tau \approx 2.9$x10^{-13} s

V • T • E	Particles in physics			
Elementary	Fermions	Quarks		u • ū • d • d̄ • c • c̄ • s • s̄ • t • t̄ • b • b̄
		Leptons		e^- • e^+ • μ^- • μ^+ • τ^- • τ^+ • ν_e • $\bar\nu_e$ • ν_μ • $\bar\nu_\mu$ • ν_τ • $\bar\nu_\tau$
	Bosons	Gauge		γ • g • $W^\pm$ • Z
		Scalar		H^0
	Others	Ghosts		
	Hypothetical	Superpartners	Gauginos	Gluino • Gravitino
			Others	Axino • Chargino • Higgsino • Neutralino • Sfermion
		Others		A^0 • Dilaton • G • J • Majorana fermion • m • Tachyon • X • Y • W' • Z' • Sterile neutrino
Composite	Hadrons	Baryons / Hyperons		N (p • n) • Δ • Λ • Σ • Ξ • Ω
		Mesons / Quarkonia		π • ρ • η • η′ • φ • ω • J/ψ • ϒ • θ • K • B • D • T
	Others	Atomic nuclei • Atoms • Diquarks • Exotic atoms (Positronium • Muonium • Tauonium • Onia) • Superatoms • Molecules		
	Hypothetical	Exotic hadrons	Exotic baryons	Dibaryon • Pentaquark • Skyrmion
			Exotic mesons	Glueball • Tetraquark
		Others		Mesonic molecule • Pomeron

Baryons (composed of 3 quarks) Neutron stars

$J^P = \frac{1}{2}^+$ baryons

Particle name	Symbol	Quark content	Rest mass (MeV/c^2)	I	J^P	Q (e)	S	C	B'	Mean lifetime (s)
nucleon/proton[7]	p / p^+ / N^+	uud	938.272013 ±0.000023	½	½$^+$	+1	0	0	0	Stable
nucleon/neutron[8]	n / n^0 / N^0	udd	939.565346 ±0.000023	½	½$^+$	0	0	0	0	$8.857 \pm 0.008 \times 10^2$
Lambda[9]	Λ^0	uds	1,115.683 ± 0.006	0	½$^+$	0	−1	0	0	$2.631 \pm 0.020 \times 10^{-10}$
charmed Lambda[10]	Λ_c^+	udc	2,286.46 ± 0.14	0	½$^+$	+1	0	+1	0	$2.00 \pm 0.06 \times 10^{-13}$
bottom Lambda[11]	Λ_b^0	udb	5,620.2 ± 1.6	0	½$^+$	0	0	0	−1	$1.391\,^{+0.038}_{-0.037} \times 10^{-12}$
Sigma[12]	Σ^+	uus	1,189.37 ± 0.07	1	½$^+$	+1	−1	0	0	$8.018 \pm 0.026 \times 10^{-11}$
Sigma[13]	Σ^0	uds	1,192.642 ± 0.024	1	½$^+$	0	−1	0	0	$7.4 \pm 0.7 \times 10^{-20}$
Sigma[14]	Σ^-	dds	1,197.449 ± 0.030	1	½$^+$	−1	−1	0	0	$1.479 \pm 0.011 \times 10^{-10}$

$J^P = \frac{3}{2}^+$ baryons

Particle name	Symbol	Quark content	Rest mass (MeV/c^2)	I	J^P	Q (e)	S	C	B'	Mean lifetime (s)
Delta[27]	Δ^{++}(1232)	uuu	1,232 ±1	3⁄2	3⁄2$^+$	+2	0	0	0	$5.58 \pm 0.09 \times 10^{-24}$
Delta[27]	Δ^+(1232)	uud	1,232 ±1	3⁄2	3⁄2$^+$	+1	0	0	0	$5.58 \pm 0.09 \times 10^{-24}$
Delta[27]	Δ^0(1232)	udd	1,232 ±1	3⁄2	3⁄2$^+$	0	0	0	0	$5.58 \pm 0.09 \times 10^{-24}$
Delta[27]	Δ^-(1232)	ddd	1,232 ±1	3⁄2	3⁄2$^+$	−1	0	0	0	$5.58 \pm 0.09 \times 10^{-24}$

Neutron stars

Hyperons (baryon including at least 1 strange quark (s))

Hyperons

Particle	Symbol	Makeup	Rest mass MeV/c^2	Isospin I	Spin(Parity) J^P	Q	S	C	B'	Mean lifetime s
Lambda [1]	Λ^0	uds	1 115.683(6)	0	½⁺	0	−1	0	0	2.60×10^{-10} [2]
Sigma [3]	Σ^+	uus	1 189.37(0.7)	1	½⁺	+1	−1	0	0	$8.018 \pm 0.026 \times 10^{-11}$
Sigma [4]	Σ^0	uds	1 192.642(24)	1	½⁺	0	−1	0	0	$7.4 \pm 0.7 \times 10^{-20}$
Sigma [5]	Σ^-	dds	1 197.449(30)	1	½⁺	−1	−1	0	0	$1.479 \pm 0.011 \times 10^{-10}$
Sigma resonance [6]	Σ^{*+}(1385)	uus	1 382.8(4)	1	3⁄2⁺	+1	−1	0	0	
Sigma resonance [6]	Σ^{*0}(1385)	uds	1 383.7±1.0	1	3⁄2⁺	0	−1	0	0	
Sigma resonance [6]	Σ^{*-}(1385)	dds	1 387.2(5)	1	3⁄2⁺	−1	−1	0	0	
Xi [7]	Ξ^0	uss	1 314.83(20)	½	½⁺	0	−2	0	0	$2.90 \pm 0.09 \times 10^{-10}$
Xi [8]	Ξ^-	dss	1 321.31(13)	½	½⁺	−1	−2	0	0	$1.639 \pm 0.015 \times 10^{-10}$
Xi resonance [9]	Ξ^{*0}(1530)	uss	1 531.80(32)	½	3⁄2⁺	0	−2	0	0	
Xi resonance [9]	Ξ^{*-}(1530)	dss	1 535.0(6)	½	3⁄2⁺	−1	−2	0	0	
Omega[10]	Ω^-	sss	1 672.45(29)	0	3⁄2⁺	−1	−3	0	0	$8.21 \pm 0.11 \times 10^{-11}$

Mesons (composed of 1 quark & 1 antiquark) **Neutron stars**

Pseudoscalar mesons

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c^2)	I^G	J^{PC}	S	C	B'	Mean lifetime (s)
Pion[6]	π^+	π^-	$u\bar{d}$	139.570 18 ± 0.000 35	1^-	0^-	0	0	0	$2.6033 \pm 0.0005 \times 10^{-8}$
Pion[7]	π^0	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	134.9766 ± 0.0006	1^-	0^{-+}	0	0	0	$8.4 \pm 0.5 \times 10^{-17}$
Eta meson[8]	η	Self	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$	547.853 ± 0.024	0^+	0^{-+}	0	0	0	$5.0 \pm 0.3 \times 10^{-19}$
Eta prime meson[9]	η'(958)	Self	$\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$	957.78 ± 0.06	0^+	0^{-+}	0	0	0	$3.39 \pm 0.16 \times 10^{-21}$
Charmed eta meson [10]	η_c(1S)	Self	$c\bar{c}$	2,980.3 ± 1.2	0^+	0^{-+}	0	0	0	$2.30 \pm 0.17 \times 10^{-23}$
Bottom eta meson [11]	η_b(1S)	Self	$b\bar{b}$	9,390.9 ± 2.8	0^+	0^{-+}	0	0	0	Unknown
Kaon[12]	K^+	K^-	$u\bar{s}$	493.677 ± 0.016	½	0^-	1	0	0	$1.2380 \pm 0.0021 \times 10^{-8}$
Kaon[13]	K^0	$\bar{K}^0$	$d\bar{s}$	497.614 ± 0.024	½	0^-	1	0	0	

There are also vector Mesons; there are 24 different Mesons.

Neutron stars

Theoretical NS compositions (structures)

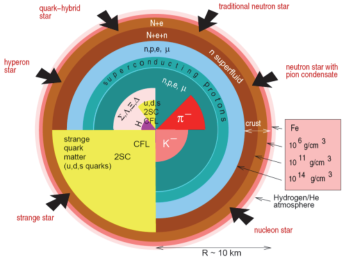

Fig. 1. Neutron star compositions predicted by theory.

- depending on NS mass and rotational frequ., NS may be compressed to densities ~10xρ_{nuc}.

↓

numerous subatomic particle processes are likely to compete with each other (Figure):

Boson condensate:
Mesons obey Bose-Einstein statistics;
fundamental property: at low enough *T* an ideal Bose gas will undergo Bose-Einstein condensation (all particles are in the zero-momentum state).
It is a condensation in momentum space and not in coordinate space. Phenomenon of superfluidity (frictionless, similar to ^{4}He @ low *T*).
The π^- and K⁻ mesons in NS will therefore not
Pion
contribute to the pressure (but to *r* !), and therefore the EOS gets "softer" (i.e. more compressible).
Kaon

H …… H dybarion (a six-quark composite)
CFL … colour-flavoured locked quark pairing (QCD): all 3 quark flavours participate symmetrically
2SC … two-flavour superconducting phase

Neutron stars

Theoretical NS compositions (structures)

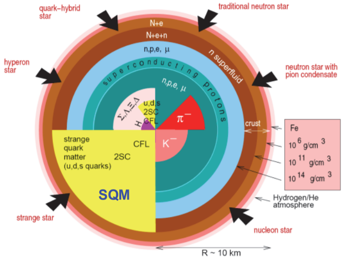

Fig. 1. Neutron star compositions predicted by theory.

- depending on NS mass and rotational frequ., NS may be compressed to densities ~10xρ_{nuc}.

↓

numerous subatomic particle processes are likely to compete with each other (Figure):

SQM: quark (u,d,s) deconfinement:
Note: free quarks do not exist, because the force between them increases with distance.
One can therefore argue that for very short distances (high T) the force between quarks (gluon) can be neglected ⟶ Fermi sea of quarks ⟶ deconfined (quark-gluon matter) with $n_u = n_d = n_s$ and total baryon density $n_b = (n_u + n_d + n_s) / 3$.

H …… H dybarion (a six-quark composite)
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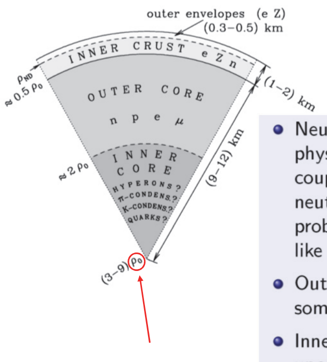
Neutron stars

Crust and outer core



- Neutron star crusts: plasma and nuclear physics (10^5 g cm^{-3}-10^{14} g cm^{-3} (strongly coupled Coulomb plasma, unstable neutron-rich nuclei, nuclear many-body problem, ...). Basic constituents - $n\ p\ e$ - like in "terrestrial physics".
- Outer core: $(1.5-5)\times 10^{14}$ g cm^{-3} npe, some admixture of muons. (if E_F > 105.7 MeV).
- Inner core: $(0.5-3)\times 10^{15}$ g cm^{-3} - uncertainty increasing with ρ

$\rho_0 \simeq 2.8\times 10^{14}\,\mathrm{g\,cm^{-3}}$

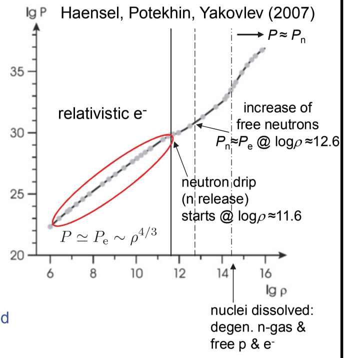
Baryon density = nucleon density $n_0 \simeq 0.16\,\mathrm{fm}^{-3} = 1.6\times 10^{38}\mathrm{cm}^{-3}$

Neutron stars

Nuclear equilibrium @ high densities (crust: below neutron drip)

- the equilibrium composition is that composition, for which the total (Gibbs, T=0) energy G_{cell} (in a Wigner-Seitz cell) is a minimum. Basically one imposes only the baryon number per volume and ask for the corresponding equilibrium composition.

Haensel, Potekhin, Yakovlev (2007)



- with increasing ρ the balance between attractive and repelling forces in isolated nuclei (Z,A) is shifted to heavier and neutron-enriched nuclei, because replacing a proton by a neutron decreases the repulsive Coulomb force inside the nucleus, and resulting β decay is inhibited by Fermi sea (E_F~$\rho^{2/3}$).

Neutron stars

Nuclear equilibrium @ high densities (below neutron drip)

Table 1. Nuclei in the ground state of cold dense matter. Upper part: experimental nuclear masses. Lower part: from mass mass formula of Möller [59]. Last line corresponds to the neutron drip point. After Haensel and Pichon [39].

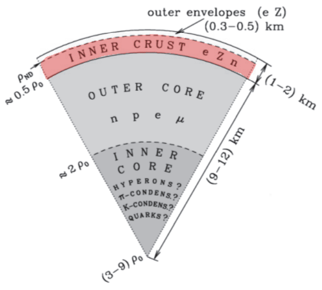
element	Z	N	Z/A	$\rho_{\max}$ (g cm^{-3})
^{56}Fe	26	30	0.4643	7.96 10^6
^{62}Ni	28	34	0.4516	2.71 10^8
^{64}Ni	28	36	0.4375	1.30 10^9
^{66}Ni	28	38	0.4242	1.48 10^9
^{86}Kr	36	50	0.4186	3.12 10^9
^{84}Se	34	50	0.4048	1.10 10^{10}
^{82}Ge	32	50	0.3902	2.80 10^{10}
^{80}Zn	30	50	0.3750	5.44 10^{10}
^{78}Ni	28	50	0.3590	9.64 10^{10}
^{126}Ru	44	82	0.3492	1.29 10^{11}
^{124}Mo	42	82	0.3387	1.88 10^{11}
^{122}Zr	40	82	0.3279	2.67 10^{11}
^{120}Sr	38	82	0.3167	3.79 10^{11}
^{118}Kr	36	82	0.3051	(4.33 10^{11})

neutron drip point →

- with increasing ρ the balance between attractive and repelling forces in isolated nuclei (Z,A) is shifted to heavier and neutron-enriched nuclei, because replacing a proton by a neutron decreases the repulsive Coulomb force inside the nucleus, and resulting β decay is inhibited by Fermi sea (E_F~$\rho^{2/3}$).

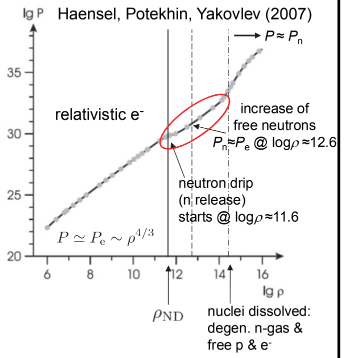
Neutron stars

Inner crust (neutron drip ND)



$\rho_0 \simeq 2.8\times 10^{14}\,\mathrm{g\,cm^{-3}}$

Haensel, Potekhin, Yakovlev (2007)

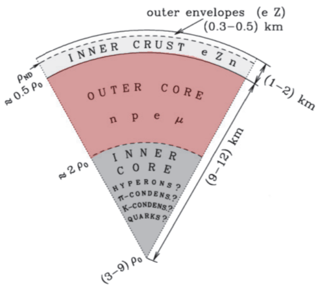


Onset of ND has severe consequences for EOS: increase in dρ mainly increases n_n at the expense of n_e (which provides the pressure) such that dP increases less → EOS becomes more compressible, i.e. the EOS gets softer. At even higher ρ the increasingly freed neutrons contribute increasingly to P.

Note: protons are never released, but the number of free neutrons increases with density!

Neutron stars

Outer core (above neutron drip)



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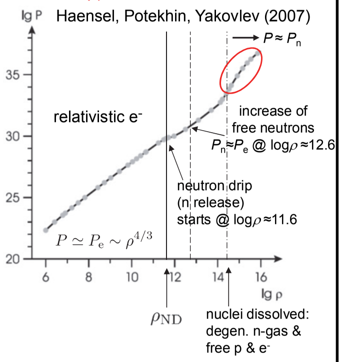
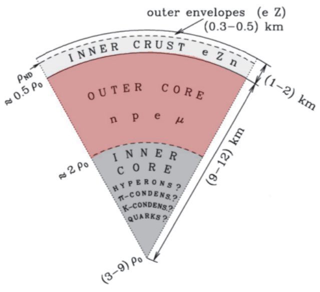


Nucleons (n,p) form a strongly interacting Fermi liquid, while e⁻ and muons (μ) constitute a nearly ideal Fermi gases. The relative numbers of n and p will be determined by the conditions of beta equilibrium (equilibrium w.r.t. to the weak-interaction processes):

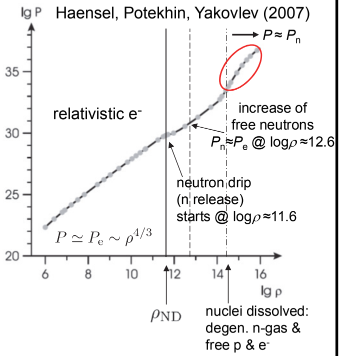
$$n \longrightarrow p + e + \bar{\nu}_e \,, \qquad p + e \longrightarrow n + \nu_e \,,$$
$$n \longrightarrow p + \mu + \bar{\nu}_\mu \,, \qquad p + \mu \longrightarrow n + \nu_\mu \,.$$

Neutron stars

Outer core (above neutron drip)



$\rho_0 \simeq 2.8 \times 10^{14}\,\mathrm{g\,cm^{-3}}$



This (beta) equilibrium is obtained by minimizing the (free) energy (only parameter is baryon density n_b) w.r.t. to the constraints:

fixed baryon density: $n_n + n_p - n_b = 0$

electrical neutrality: $n_e + n_\mu - n_p = 0$

& relations between chem. potentials $\mu_n = \mu_p + \mu_e \,, \quad \mu_\mu = \mu_e \,,$

} form a closed system of equations, which determines **equilibrium composition of** ***npeμ*** **matter.**

Neutron stars

Hyperon softening of EOS (**inner core**)

Cameron 1959 - hyperons could be present as stable particles in neutron-star cores

rapid nucleons converted into slow more massive hyperons

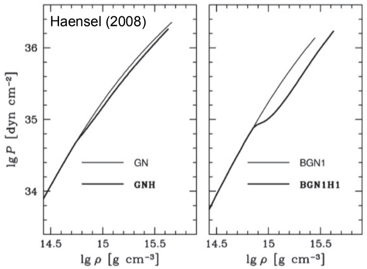


Figure: thick lines: with hyperons
thin lines: without hyperons

The formation of hyperons softens EOS because high-energy neutrons are replaced by more massive low-energy hyperons (producing lower pressure).

Hyperons appear due to strangeness-changing weak interactions.

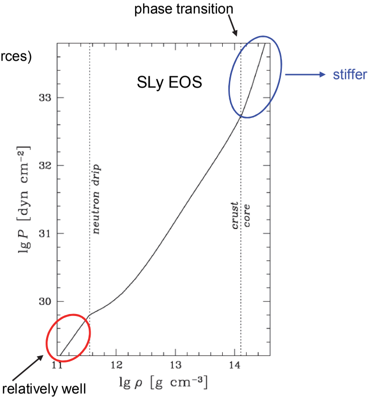
Strangeness *S* is a property of particles, expressed as a quantum number, for describing decay of particles in strong & electromagnetic reactions, which occur in a short period of time.

$$S = -(\underset{\text{\# of strange quarks}}{n_s} - \underset{\text{\# of strange aniquarks}}{n_{\bar{s}}})$$

Neutron stars

EOS above ρ_{ND}

SLy: (Skyrme-Lyon NN forces)



EOS relatively well know thanks to experimental data (ground state, cold catalyzed matter).

→ ρ

properties of nuclei affected by ambient n-gas, which contributes increasingly to P
→ EOS becomes uncertain.

Neutron stars

NS models (**special relativity effects**: mass-energy density)

Neutron pressure $P_n \approx 0.8P$ @ $\rho \geq 1.5\text{x}10^{13}\text{gcm}^{-3}$, i.e. P_n essentially determines total *P*.

EOS for ideal, fully degenerate neutrons similar to e⁻ EOS, because n are also fermions, but m_e is replaced by m_n and μ_e by 1 (i.e. one nucleon per fermion):

$$P_n = K_{\gamma'}\varrho_0^{\gamma'} \quad \text{with} \quad \gamma' = \frac{5}{3} \;, \quad K_{5/3} = \frac{1}{20}\left(\frac{3}{\pi}\right)^{2/3}\frac{h^2}{m_n^{8/3}} \;, \quad \text{for } \varrho_0 \ll 6\times10^{15}$$

$$\gamma' = \frac{4}{3} \;, \quad K_{4/3} = \frac{1}{8}\left(\frac{3}{\pi}\right)^{1/3}\frac{hc}{m_n^{4/3}} \;, \quad \varrho_0 \gg 6\times10^{15}\ \text{g cm}^{-3}$$

Above we have used the rest-mass density $\rho_0 = n_n m_n$, but for relativistic configurations one has, instead of ρ_0 to use the total mass-energy density

$$\rho = \rho_0 + u/c^2 \,.$$

This distinction was not necessary for e⁻ gas, because ρ_0 was always determined mainly by the non-degenerate nuclei (ions) and was therefore always larger than the energy density u/c^2 of the degenerated e⁻ gas.

Here, however, both ρ_0 and u/c^2 are now provided by the degenerate neutrons!

Neutron stars

NS models (special relativity effects: mass-energy density)

For non-relativistic neutrons $\rho_0 \gg u/c^2$ and $\rho \approx \rho_0$.

For relativistic neutrons $\rho \approx u/c^2$.

For a relativistic gas we also have

internal energy per unit volume

$$P_n = \frac{1}{3}u = \frac{1}{3}\rho c^2 \,,$$

and consequently the pressure provided by the degenerated neutrons can be written as

$$P_n \sim \varrho^\kappa \,,$$

$$\kappa = 5/3 \quad (\text{non}-\text{relativistic}) \,,$$

$$\boxed{\kappa = 1 \quad (\text{relativistic}) \,.}$$

Also, the squared sound speed is $v_s^2 = (dP/d\varrho)_{ad} = c^2/3 \longrightarrow v_s = 0.577c.$

Non-ideal effects (e.g. electrostatic interactions) become important already for $\rho_0 < 6\text{x}10^{15}\ \text{gcm}^{-3}$.

Neutron stars

NS models (including GR)

Hydrostatic equilibrium in **General Relativity** (GR):

Note: $\frac{r_g}{R_{NS}} \simeq 0.25, ..., 0.5 \longrightarrow$ GR becomes important

r, θ, φ - Schwarzschild coordinates in ref. frame at ∞ $\qquad e^{\lambda/2} = (1 - 2Gm/c^2 r)^{-1/2}$

$$ds^2 = e^\nu c^2 dt^2 - e^\lambda dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) \quad \text{with } \nu = \nu(r), \lambda = \lambda(r).$$

Black holes

GR (Field equation)

Field equation in Newtonian theory: $\nabla^2\phi = 4\pi G\rho \,,$

or: $$\nabla^2 g_{00} = \frac{2}{c^2}\nabla^2\phi = \frac{8\pi G}{c^2}\rho$$

Newton: mass density

source of gravity

Einstein: mass-energy density

In general, i.e. GR: $\nabla^2 g_{00} \xrightarrow{\text{(replace)}} G_{\mu\nu} = f(g_{\mu\nu}, \nabla g_{\mu\nu}, \nabla^2 g_{\mu\nu}) \,,$

Field equation (GR): $$\longrightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \,, \qquad T_{\mu\nu} = \begin{pmatrix} \rho c^2 & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} .$$

stress energy tensor

$$\left\{\begin{matrix}\text{measure of}\\ \text{curvature of}\\ \text{spacetime}\end{matrix}\right\} = \frac{8\pi G}{c^4}\left\{\begin{matrix}\text{measure of}\\ \text{energy density}\end{matrix}\right\}$$

Matter tells space how to curve.

Curved space tells matter how to move (Geodesic equation).

Neutron stars

NS models (including GR)

Hydrostatic equilibrium in **General Relativity** (GR):

Note: $\frac{r_g}{R_{\rm NS}} \simeq 0.25, ..., 0.5 \longrightarrow$ GR becomes important

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$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2 (d\vartheta^2 + \sin^2 \vartheta \, d\varphi^2) \quad \text{with } \nu = \nu(r), \lambda = \lambda(r).$$

Tolman-Oppenheimer-Volkoff (TOV) equation:

$$\frac{dP}{dr} = -\frac{Gm}{r^2} \varrho \left(1 + \frac{P}{\varrho c^2}\right) \left(1 + \frac{4\pi r^3 P}{mc^2}\right) \left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

together with mass-conservation equation (gravitational mass!):

$$m = \int_0^r 4\pi r^2 \varrho \, dr$$

Note: ρ = mass-energy density!
$\rho = \rho_0 + u/c^2$

The apparent Newtonian formula for m is misleading, because
$dV = 4\pi r^2 [1 - 2Gm/(rc^2)]^{-1/2} dr > (dV)_{\rm Newt.}$. Space-time is curved along r and t !

With an EOS the NS equations can be solved numerically to construct NS models.

Neutron stars

NS models (including GR)

Hydrostatic equilibrium in **General Relativity** (GR):

Note: $\frac{r_g}{R_{\rm NS}} \simeq 0.25, ..., 0.5 \longrightarrow$ GR becomes important

r, θ, φ - Schwarzschild coordinates in ref. frame at ∞ $\quad e^{\lambda/2} = (1 - 2Gm/c^2 r)^{-1/2}$

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2 (d\vartheta^2 + \sin^2 \vartheta \, d\varphi^2) \quad \text{with } \nu = \nu(r), \lambda = \lambda(r).$$

Gravity is enhanced by coupling to pressure in the general relativistic description.

Tolman-Oppenh

$$\frac{dP}{dr} = -\frac{Gm}{r^2} \varrho \left(1 + \frac{P}{\varrho c^2}\right) \left(1 + \frac{4\pi r^3 P}{mc^2}\right) \left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

together with mass-conservation equation (gravitational mass!):

$$m = \int_0^r 4\pi r^2 \varrho \, dr$$

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The apparent Newtonian formula for m is misleading, because
$dV = 4\pi r^2 [1 - 2Gm/(rc^2)]^{-1/2} dr > (dV)_{\rm Newt.}$. Space-time is curved along r and t !

With an EOS the NS equations can be solved numerically, to construct NS models.

Neutron stars

NS models (including GR)
Gravitational mass

The total stellar mass *M* is always the "gravitational mass", which is the value measured from an outside observer, i.e. it is the mass that would be detected through Kepler's law for the orbital motion if the star were a component of a well-separated binary system.

In the Newtonian limit $\quad m(r) = 4\pi \int_0^r \rho(r) r^2 {\rm d}r \, ,$

can be unambiguously interpreted as the mass contained within the radius r.

For relativistic stars (including space-time curvature) we have

$$m(r) = m_0(r) + U(r)/c^2 + \Omega(r) = m_0(r) + W(r)/c^2 \, ,$$

with

$$m_0 = \int_0^r \rho_0 {\rm d}V = 4\pi \int_0^r \left(1 - 2mG/c^2 r\right)^{-1/2} \rho_0 r^2 {\rm d}r \, ,$$

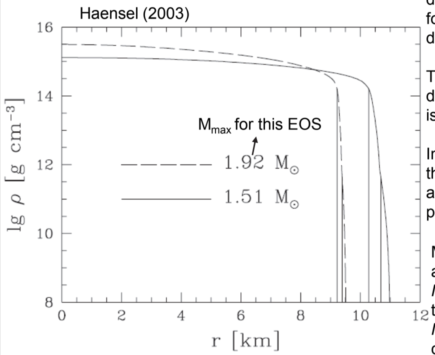
$$U/c^2 = \int_0^r (\rho - \rho_0) {\rm d}V = 4\pi \int_0^r \left(1 - 2mG/c^2 r\right)^{-1/2} (\rho - \rho_0) r^2 {\rm d}r \, ,$$

$$\longrightarrow \quad \frac{W}{c^2} = m - m_0 = 4\pi \int_0^r \left[1 - \left(1 - \frac{2mG}{c^2 r}\right)^{-1/2} \frac{\rho_0}{\rho}\right] \rho r^2 {\rm d}r \; < \; 0 \; .$$

⟶ The "gravitational mass" *M*=*m*(*R*) is less than the proper (baryon) rest mass *M*$_0$=*m*$_0$(*R*) !

Neutron stars

NS models



density within core rather uniform for *M*=1.2,…,1.5 M$_\odot$; most pronounced drop occurs near the crust bottom edge.

There is a steepening in the density drop near the neutron drip, where EOS is 'softer'.

Increasing M leads to a sharper drop of the density profiles and to thinner crust as a result of the increasing gravitational pull, most pronounced at *M*=*M*$_{\rm max}$.

Medium-stiff BBB2 EOS (core) leads to a crust containing 1.4% & 10.8% for *M*$_{\rm grav}$ = 1.24 & 1.51 M$_\odot$ with a crust thickness of 1.01km & 0.72km. For *M*$_{\rm max}$, *M*$_{\rm crust}$/*M*$_{\rm max}$ = 0.2% & 0.29km crust thickness.

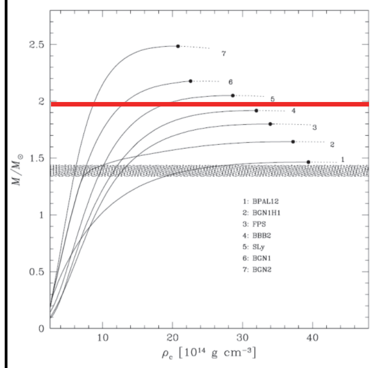
For softer (core) EOS the crust mass and thickness are smaller.

Fig. 4. Mass density *versus* radial coordinate for neutron-star models of gravitational mass 1.51 $M_\odot$ (solid line) and 1.92 $M_\odot$ (long dash-dotted line). Calculations are performed for the BBB2 EOS of the core, for which 1.92 $M_\odot$ is the maximum allowable mass. The positions of the crust-core interface and the neutron-drip point are indicated by thin vertical lines.

Neutron stars

Haensel (2003)

NS models (EOS effect)



ρ_c is (single) parameter in NS model family.

Curves exhibit maximum mass, which strongly depend on EOS, and is reached at central densities of several ρ_0. Configurations with $M>M_{max}$ are unstable and collapse to BH.

Note: replacing EOS by a free (non-interacting) Fermi gas of n (extremely soft EOS) leads to M_{max} = 0.72M$_\odot$, the values obtained by O&V (1939). This is in contraction with the measured mass of the binary radio pulsars of ~1.44 M$_\odot$ (shaded horizontal band) & ~1.97 M$_\odot$ (red line).

This implies that nucleon-nucleon (NN) interaction are sufficiently repulsive at supra-nuclear densities to lift M_{max} by more than 100% from the non-interacting nucleon gas value.

But: Hyperon-nucleon (H-N) & (H-H) interactions soften EOS and reduce M_{max} value.

Fig. 6. Gravitational mass M *versus* central density ρ_c of static neutron-star models for different EOSs. A maximum on a $M - \rho_c$ curve is indicated by a filled circle. Configurations to the right of the maximum are unstable with respect to small radial perturbations, and are denoted by dotted lines. The shaded band corresponds to the range of precisely measured masses of binary radio pulsars (Sect. 14).

Neutron stars

NS models (EOS effect)

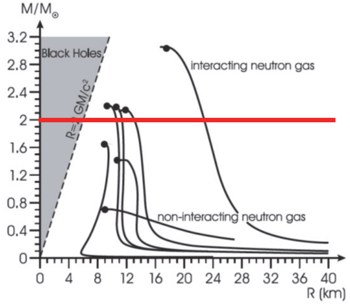
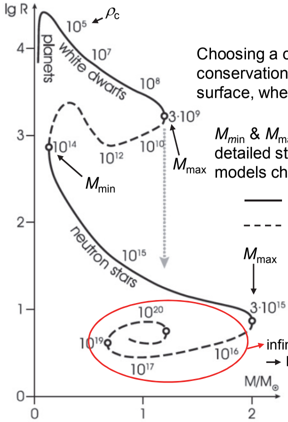


Fig. 38.1. The relation M against R of neutron-star models calculated using 7 different equations of state. The maximum mass is indicated in each case by the solid dot. Two extreme assumptions are specifically indicated: a non-interacting neutron gas (Oppenheimer & Volkoff 1939), which leads to an extremely soft equation of state and a very low maximum mass, and one where interactions are included and which leads to a maximum neutron star mass above $3M_\odot$ (After Fig. 10.3 in Weber, Negreiros, Rosenfield (2009), where also details about the other 5 equations of state can be found)

Neutron stars

NS models



Choosing a central density ρ_c the TOV equation, together with the mass conservation equation and an EOS, can be integrated from ρ = 0 to the surface, where ρ = 0 (P=0) ⟶ $R(\rho_c)$, $M(\rho_c)$.

M_{min} & M_{max} are critical points in the model computations at which a detailed stability analysis shows that the stability of the equilibrium models changes (dynamical stability):

—— $dM/d\varrho_c \ > \ 0$…. stable

- - - - $dM/d\varrho_c \ < \ 0$…. unstable

$P \propto \rho^{\gamma_{ad}} \longrightarrow$ Newtonian limit: $\gamma_{ad} = \gamma_{cr} = 4/3$.

infinite number of max. & min.

→ limiting point reached for $\rho_c \rightarrow \infty$.

for (small) GR effects ($GM/Rc^2 << 1$):

$$\gamma_{cr} = \frac{4}{3} + \Lambda \frac{GM}{Rc^2}$$

$$\Lambda = O(1) .$$

$$\downarrow$$

$$\gamma_{cr} > 5/3 \text{ (even up to 2)}$$

$$\downarrow$$

GR makes star more unstable: only the stiffest EOS would lead to stability!

Fig. 38.3. Schematic mass-radius relation (R in km) for configurations of cold catalyzed matter, from the planetary regime to ultradense neutron stars. Some values of ϱ_c (in g cm^{-3}) are indicated along the curve. At the extrema of M (*open circles*) the stability problem has a zero eigenvalue. Solid branches are stable, dashed branches are unstable. The grey, vertical arrow indicates the collapse of a white dwarf exceeding the maximum stable mass to a neutron star

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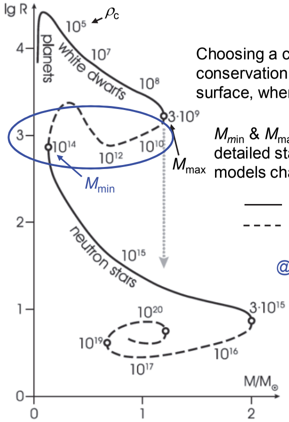
Neutron stars

NS models



Choosing a central density ρ_c the TOV equation, together with the mass conservation equation and an EOS, can be integrated from ρ = 0 to the surface, where ρ = 0 (P=0) ⟶ $R(\rho_c)$, $M(\rho_c)$.

M_{min} & M_{max} are critical points in the model computations at which a detailed stability analysis shows that the stability of the equilibrium models changes (dynamical stability):

—— $dM/d\varrho_c \ > \ 0$…. stable

- - - - $dM/d\varrho_c \ < \ 0$…. unstable

@M_{min}: curve leads to WD; this instability is a result from lowering γ' in connection with neutron drip: release of free n from nuclei (CCM in ground state) leads to $\gamma' < 4/3$ for $\rho \approx 4\text{x}10^{11}$ …. $7\text{x}10^{12}$ gcm^{-3}.
Typical models for the minimum mass of a stable NS suggest $M_{min} \approx 0.09M_\odot$, $R \approx$ 160km, $\rho_c \approx$ 1.5x1014 gcm^{-3}.

Fig. 38.3. Schematic mass-radius relation (R in km) for configurations of cold catalyzed matter, from the planetary regime to ultradense neutron stars. Some values of ϱ_c (in g cm^{-3}) are indicated along the curve. At the extrema of M (*open circles*) the stability problem has a zero eigenvalue. Solid branches are stable, dashed branches are unstable. The grey, vertical arrow indicates the collapse of a white dwarf exceeding the maximum stable mass to a neutron star

Neutron stars

Maximum mass of a NS

- NS are subject to a $M_{\max}$ (similarly as WD), because of (1) the degenerate behaviour of cold matter at ultrahigh densities and (2) General Relativity.

- $M_{\max}$ important also for identifying compact objects with $M > M_{\max}$ as black holes (BH).

- Simple argument for the existence of $M_{\max} << 5\,M_\odot$ by neglecting GR effects, but including special relativity (SR) effects (i.e. considering u/c^2 in addition to neutron (n) rest mass):

from hydrostatic support: $P \propto M^2/R^4$; with $\rho \propto M/R^3$ to replace $R \rightarrow\ P \propto M^{2/3}\rho^{4/3}$.

from the ideal (polytropic) EOS: $\rho \propto P^{1/\kappa}\ \rightarrow\ M \propto \rho^{3(\kappa-4/3)/2}$.

non-relativistic: $\kappa = 5/3\ \rightarrow\ M \propto \rho^{1/2}\ \rightarrow\ \mathrm{d}M/\mathrm{d}\rho > 0\,,$

relativistic: $\kappa = 1\ \rightarrow\ M \propto \rho^{-1/2}\ \rightarrow\ \mathrm{d}M/\mathrm{d}\rho < 0\,,$

somewhere in between these limits $\mathrm{d}M/\mathrm{d}\rho = 0$ $\rightarrow\ M_{\max}$ must exist when n start to become relativistic, i.e. $u/c^2 \geq$ rest mass ρ_0.

Note: $$M_{\rm Ch} = 0.197\left[\left(\frac{hc}{G}\right)^{3/2}\frac{1}{m_{\rm u}}\right]\frac{1}{1} \simeq 5.83\,{\rm M}_\odot$$

one nucleon per fermion

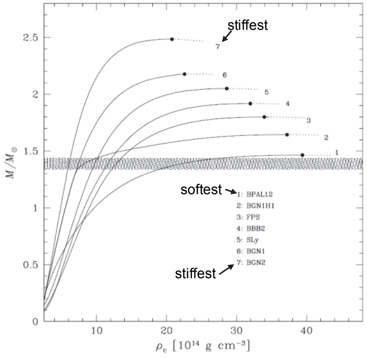
was obtained with neglecting the energy density term (SR effect) u/c^2, i.e. $\rho = \rho_0$!

$$\longrightarrow\quad M_{\max} < M_{\rm Ch} \simeq 5.83\,{\rm M}_\odot\,.$$

Note: $M_{\rm Ch}$ does also not depend on rest mass!

Neutron stars

Maximum mass of a NS (effect of EOS)



ρ_c is (single) parameter in NS model family.

Curves exhibit maximum mass, which strongly depend on EOS, and is reached at central densities of several ρ_0. Configurations with $M > M_{\max}$ are unstable and collapse to BH.

Increasing the 'stiffness' (1→7) makes matter less compressible; for given M one expects a larger R and a smaller ρ_c.
Or, for given ρ_c more mass can be put on top until reaching the surface with $\rho = 0$. This lowers the gravity inside the model and $M_{\max}$ is higher.

Neutron stars

Maximum mass of a NS

- upper mass limit for NS depends on EOS (soft/stiff).
- is there a way to estimate an upper limit that is independent of EOS?
- YES: Rhodes & Ruffini (1974) reported such a way with the help of the "causal condition":

(1) adopt GR (&SR), i.e. use the TOV equation for hydrostatic support

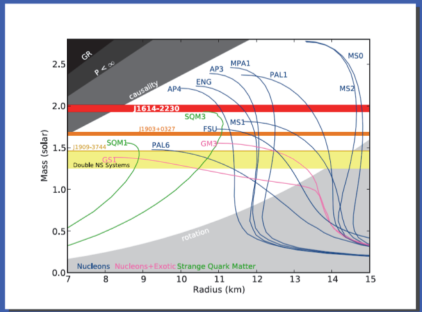
(2) EOS satisfies "microscopic stability" condition: $\frac{\mathrm{d}P}{\mathrm{d}\rho} \geq 0\,,$

(3) EOS satisfies "causality" condition: $\frac{\mathrm{d}P}{\mathrm{d}\rho} \leq c^2\,,$

$$\left.\begin{array}{l}\frac{\mathrm{d}P}{\mathrm{d}\rho} \geq 0\,,\\ \frac{\mathrm{d}P}{\mathrm{d}\rho} \leq c^2\,,\end{array}\right\}\quad 0 \leq \frac{\mathrm{d}P}{\mathrm{d}\rho} \leq c^2\,.$$

(4) EOS is known below some, i.e. $\rho < \rho_u$, "matching density" ρ_u .

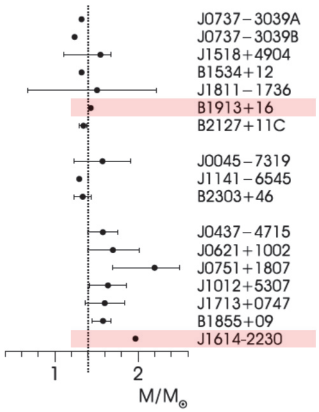
Constraints on the Mass–Radius Relation (Lattimer and Prakash 2004)



- Schwarzschild limit (GR): $R > 2GM = R_s$
- causality limit for EoS: $R > 3GM$
- mass limit from PSR J1614-2230 (red band): $M = (1.97 \pm 0.04)M_\odot$

Neutron stars

Maximum mass of a NS



binary ms pulsar

binary ms pulsar