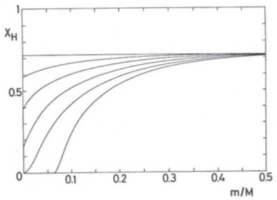
Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Low-mass stars have typically degenerate cores after H burning, and only small or no convective cores.



→ produce growing He core starting with zero mass → smooth transition from central to shell H-burning.

Central density of low-mass stars typically >100 g cm^{-3} → boarder to degeneracy
→ Schönberg-Chandrasekhar (SC) limit not important (initial $M_c < 0.1M$).

With outward-burning shell source (H), M_c>0.1M, core contract and becomes degenerate (makes SC limit irrelevant), i.e. degenerate, isothermal He core develops in thermal equilibrium without the need of rapid core contraction!

→ no Hertzsprung gap (i.e. evolution on nuclear time scale)

→ no core heating (of a contracting (ideal-gas) core as in massive stars)

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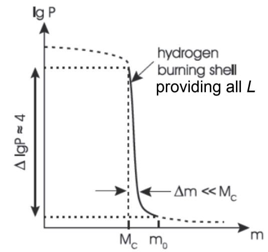
- low-mass stars are located already close to Hayashi line (track), i.e. any envelope expansion (*mirror principle*) due to (rather slow) M_c↑ (shell burning) is only possible with an L increase.

- Interestingly *L depends on the properties of the core only* (M_c, R_c) and is practically independent of the envelope mass (and therefore M), because the pressure drops so sharply outside the core, i.e. the extended envelope is nearly weightless, and has no influence on the burning shell.

→ models in this phase can be described by generalized ***homology***.



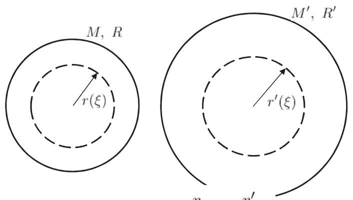
Homology relations

- Similar stellar models computed with the same assumptions (parameters and material functions) may differ in such a way that these differences could be described by simple analytical expressions.

- Only 'one' stellar model would then be necessary to compute and from which new stellar models could be derived using simple analytical expressions.

- Models with such similar properties are called '*homologous stars*'.

Homologous stars:



relative mass value (homologous masses)

$$\xi := \frac{m}{M} = \frac{m'}{M'}$$

homology condition (for all ξ):

$$\frac{r(\xi)}{r'(\xi)} = \frac{R}{R'} = \frac{r(0.7)}{r'(0.7)} = \ldots$$

… this ratio for homologous mass shells is constant throughout the stars.

here (shell homology): $\frac{r}{R_c} = \frac{r'}{R'_c}$;

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Shell source homology: (relating properties between core and H-burning shell)

Model (complete equilibrium) with degenerate He core (M_c, R_c) surrounded by H-envelope with abundance X_H; M_c grows owing to H-shell burning, providing total luminosity L.

Material functions are: $\kappa = \kappa_0 P^a T^b, \quad \varepsilon = \varepsilon_0 \varrho^{n-1} T^\nu$

We adopt ideal gas EOS for (non-degenerate) regions outside (degenerate) core (P_{rad}=0): $P = \frac{\Re}{\mu}\varrho T$

We further assume following relations in H-burning shell ($R_c \le r \le R_c + \triangle r$):

basic parameters

$$\rho = \tilde{\rho}(r/R_c) M_c^{\varphi_1} R_c^{\varphi_2} \mu^{\varphi_3} \qquad P = \tilde{P}(r/R_c) M_c^{\tau_1} R_c^{\tau_2} \mu^{\tau_3}$$

$$T = \tilde{T}(r/R_c) M_c^{\psi_1} R_c^{\psi_2} \mu^{\psi_3} \qquad l = \tilde{l}(r/R_c) M_c^{\sigma_1} R_c^{\sigma_2} \mu^{\sigma_3}$$

at the homologous points: $\frac{r}{R_c} = \frac{r'}{R'_c}$;

From EOS: $\tau_1 = \varphi_1 + \psi_1$, $\tau_2 = \varphi_2 + \psi_2$, $\tau_3 = \varphi_3 + \psi_3 - 1$,

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Shell source homology: (relating properties between core and H-burning shell)

For example, we can explicitly write:

$$\frac{\varrho}{\varrho'} = \left(\frac{M_c}{M_c'}\right)^{\varphi_1} \left(\frac{R_c}{R_c'}\right)^{\varphi_2} \left(\frac{\mu}{\mu'}\right)^{\varphi_3}$$
$$\frac{P}{P'} = \left(\frac{M_c}{M_c'}\right)^{\tau_1} \left(\frac{R_c}{R_c'}\right)^{\tau_2} \left(\frac{\mu}{\mu'}\right)^{\tau_3}$$

We insert these relations in to the stellar structure equations in the form:

$$-r^{-2}\mathrm{d}r = \mathrm{d}(1/r)$$

$$dP \sim M_c \varrho d(1/r) \,,$$
$$d(T^4) \sim \kappa \varrho l\, d(1/r) = \kappa_0 \varrho P^a T^b l\, d(1/r) \,,$$
$$dl \sim \varepsilon \varrho d(r^3) = \varepsilon_0 \varrho^n T^\nu d(r^3) \,,$$

$m \simeq M_c = \mathrm{const.}$

good approximation in regions where *P* drops to ~0;

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Shell source homology: (relating properties between core and H-burning shell)

$$dP \sim M_c \varrho d(1/r) \,,$$
$$d(T^4) \sim \kappa \varrho l\, d(1/r) = \kappa_0 \varrho P^a T^b l\, d(1/r) \,,$$
$$dl \sim \varepsilon \varrho d(r^3) = \varepsilon_0 \varrho^n T^\nu d(r^3) \,,$$

Integration of the 1st equation over the H-burning shell for a model (M_c, R_c) and a second model with (M_c', R_c') with subsequent comparison of the solutions leads to

$$P(r/R_c) \sim M_c^{\varphi_1+1} R_c^{\varphi_2-1} \mu^{\varphi_3}$$

i.e. by comparison with the definition of *P* we obtain:

$$\tau_1 = \varphi_1 + 1 \,, \tau_2 = \varphi_2 - 1 \,, \tau_3 = \varphi_3 \,.$$

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

Similarly we obtain from integrating 2nd and 3rd equations a set of linear relations for all 12 exponents:

$\Downarrow$

$$P(r/R_c) \sim \overbrace{M_c^{\varphi_1+1}}^{\tau_1} \overbrace{R_c^{\varphi_2-1}}^{\tau_2} \overbrace{\mu^{\varphi_3}}^{\tau_3} \longrightarrow \tau_1 = \varphi_1 + 1 \,, \tau_2 = \varphi_2 - 1 \,, \tau_3 = \varphi_3$$

$$T \sim M_c^{\psi_1} R_c^{\psi_2} \mu^{\psi_3} \longrightarrow \begin{cases} (4-b)\psi_1 = \varphi_1 + a\tau_1 + \sigma_1 \\ (4-b)\psi_2 = \varphi_2 + a\tau_2 + \sigma_2 - 1 \\ (4-b)\psi_3 = \varphi_3 + a\tau_3 + \sigma_3 \end{cases}$$

$$l \sim M_c^{\nu+n\varphi_1} R_c^{3-\nu+n\varphi_2} \mu^{\nu+n\varphi_3} \longrightarrow \begin{cases} \sigma_1 = n\varphi_1 + \nu\psi_1 \,, \quad \sigma_2 = n\varphi_2 + \nu\psi_2 + 3 \\ \sigma_3 = n\varphi_3 + \nu\psi_3 \end{cases}$$

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

Similarly we obtain from integrating 2nd and 3rd equations a set of linear relations for all 12 exponents:

$$\varphi_1 = -\frac{\nu - 4 + a + b}{N} \,, \quad \varphi_2 = \frac{\nu - 6 + a + b}{N}$$
$$\varphi_3 = \frac{4 - b - \nu}{N} \,, \qquad N = 1 + n + a$$
$$\psi_1 = 1 \,, \qquad \psi_2 = -1 \,, \qquad \psi_3 = 1$$
$$\tau_1 = 1 + \varphi_1 \,, \quad \tau_2 = \varphi_2 - 1 \,, \quad \tau_3 = \varphi_3$$
$$\sigma_1 = \nu + n\varphi_1 \,, \quad \sigma_2 = 3 - \nu + n\varphi_2$$
$$\sigma_3 = \nu + n\varphi_3$$

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

Similarly we obtain from integrating 2nd and 3rd equations a set of linear relations for all 12 exponents,

for the following expressions for P, T and l, which relate these quantities from one model (characterized by M_c, R_c, μ) to another model (characterized by M_c', R_c', μ') at each homologous point r/R_c:

$$P(r/R_c) \sim \overbrace{M_c^{\varphi_1+1}}^{\tau_1} \overbrace{R_c^{\varphi_2-1}}^{\tau_2} \overbrace{\mu^{\varphi_3}}^{\tau_3}$$

$$T \sim M_c^{\psi_1} \; R_c^{\psi_2} \; \mu^{\psi_3}$$

$$l \sim M_c^{\nu+n\varphi_1} \; R_c^{3-\nu+n\varphi_2} \, \mu^{\nu+n\varphi_3}$$

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

$$\longrightarrow \quad T \sim M_c^{\psi_1} \; R_c^{\psi_2} \, \mu^{\psi_3} = M_c/R_c \, \mu \quad ; \quad \psi_1 = 1, \; \psi_3 = 1, \quad \psi_2 = -1$$

For a=b=0 (electron scattering) and ν=13, n=2 (CNO cycle) we find:

$$\longrightarrow \quad P \sim M_c^{\varphi_1+1} R_c^{\varphi_2-1} \, \mu^{\varphi_3} = M_c^{-2} R_c^{4/3} \mu^{-3} \; ,$$

$$\longrightarrow \quad l \sim M_c^{\nu+n\varphi_1} \; R_c^{3-\nu+n\varphi_2} \; \mu^{\nu+n\varphi_3} \; = M_c^7 R_c^{-16/3} \mu^7 \; ,$$

which are independent of total stellar mass M !

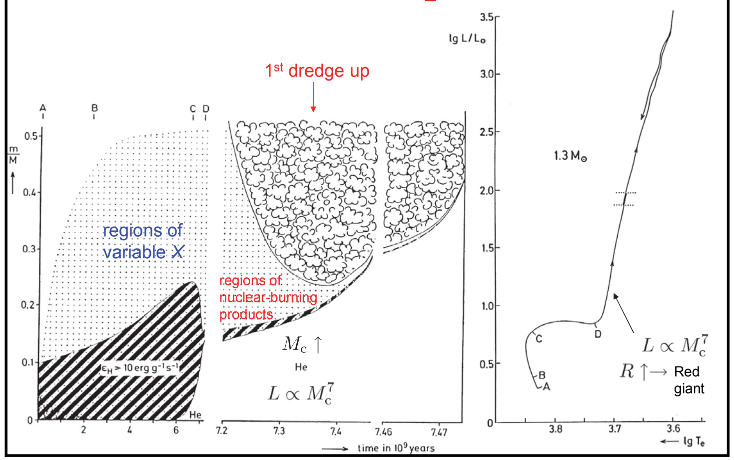
For μ = const. and for a degenerate core (note for a white dwarf we have R_c~$M_c^{-1/3}$) we have, *for example*, by using data from cold white-dwarf models :

$L \propto M_c^{7..10}$, and $T \propto M_c^{\beta}$, with β somewhat larger than 1 .

i.e. the temperature T of the shell, and therefore also of the core T_c, increases with M_c.

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

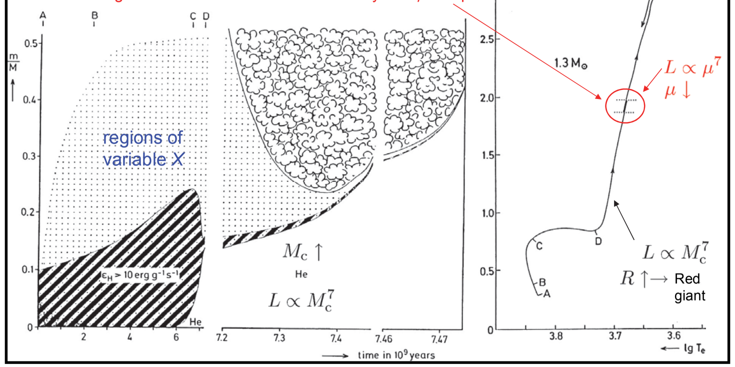
Evolution along the red-giant branch: 1.3 $M_\odot$



Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution along the red-giant branch

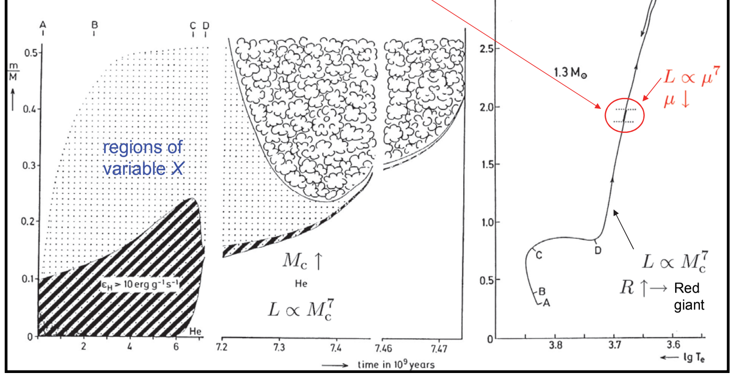
At L~100L$_\odot$ luminosity drops due to (a discontinuous) decrease in μ brought about by the H-rich surface CZ, penetrating the He-enriched layers deeper down. When H-burning shell reaches this discontinuous layer L~μ^7 drops.



Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution along the red-giant branch

After the shell source has passed the discontinuity, μ remains at its reduced value, but $L\sim M_c^7$ increases again with increasing M_c.

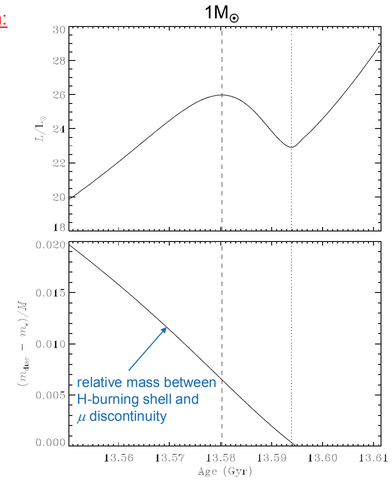


Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution along the red-giant branch:
"The hump"

Latest (JCD 2015) results for a 1$M_\odot$ computed by JCD:

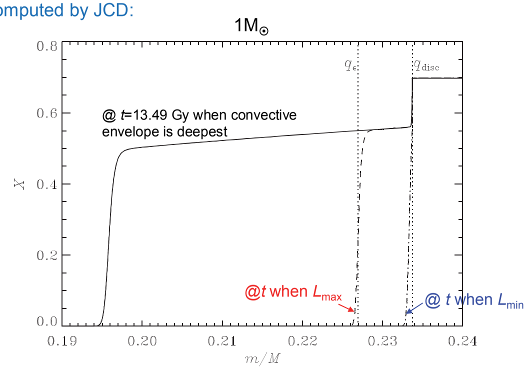
The relevant weight of the layers above the shell starts to decrease with age when the H shell is close to but not at the discontinuity; that causes the decrease in the luminosity. When the shell crosses the discontinuity, the weight (or the contribution from changes in composition to the weight) does not change and hence the luminosity increases again.



Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution along the red-giant branch:
"The hump"

Latest results (JCD, 2015) for a 1$M_\odot$ computed by JCD:



Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution along the red-giant branch:
"The hump"

Latest (JCD 2015) results for various models:

The ***position*** of the bump moves up in luminosity with increasing mass. However, the **ratio** L_{max}/L_{min} decreases with increasing mass up to a mass of around 2$M_\odot$. It is closely correlated with the step ΔX in hydrogen abundance left behind at the maximum extent (in mass) of the convective envelope in the first dredge-up. This decreases with increasing mass from 1 to 2$M_\odot$.

This is not fully understood yet, but it might be the result of hydrogen burning getting more centrally concentrated as the CNO cycle takes over.