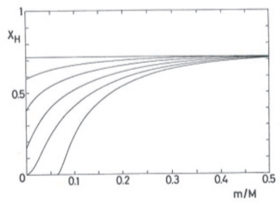
Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Low-mass stars have typically degenerate cores after H burning, and only small or no convective cores.



→ produce growing He core starting with zero mass → smooth transition from central to shell H-burning.

Central density of low-mass stars typically >100 g cm^{-3} → boarder to degeneracy
→ Schönberg-Chandrasekhar (SC) limit not important (initial M_c < 0.1M) .

With outward burning shell source (H), M_c>0.1M, core contract and becomes degenerate (makes SC limit irrelevant), i.e. degenerate, isothermal He core develops in thermal equilibrium without the need of rapid core contraction!

→ no Hertzsprung gap (i.e. evolution on nuclear time scale)

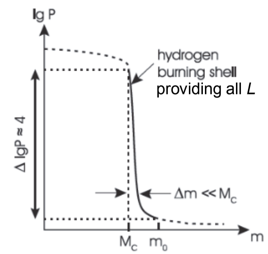
→ no core heating (of a contracting (ideal-gas) core as in massive stars)

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

- low-mass stars are located already close to Hayashi line (track), i.e. any envelope expansion (*mirror principle*) due to (rather slow) $M_c\uparrow$ (shell burning) is only possible with an L increase.

- Interestingly *L depends on the properties of the core only* (M_c, R_c) and is practically independent of the envelope mass (and therefore M), because the pressure drops so sharply outside the core, i.e. the extended envelope is nearly weightless, and has no influence on the burning shell.

→ models in this phase can be described by generalized *homology*.



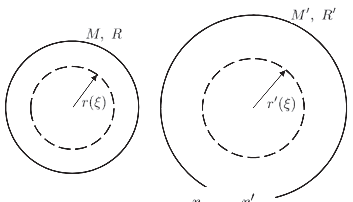
Homology relations

- Similar stellar models computed with the same assumptions (parameters and material functions) may differ in such a way that these differences could be described by simple analytical expressions.

- Only 'one' stellar model would then be necessary to compute and from which new stellar models could be derived using simple analytical expressions.

- Models with such similar properties are called '*homologous stars*'.

Homologous stars:



relative mass value (homologous points)

$$\xi := \frac{m}{M} = \frac{m'}{M'}$$

homology condition (for all ξ):

$$\frac{r(\xi)}{r'(\xi)} = \frac{R}{R'} = \frac{r(0.7)}{r'(0.7)} = \ldots$$

… this ratio for homologous mass shells is constant throughout the stars.

here (shell homology): $\frac{r}{R_c} = \frac{r'}{R'_c}$;

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

Model (complete equilibrium) with degenerate He core (M_c, R_c) surrounded by H-envelope with abundance X_H; M_c grows owing to H-shell burning, providing total luminosity L.

Material functions are: $\kappa = \kappa_0 P^a T^b, \quad \varepsilon = \varepsilon_0 \varrho^{n-1} T^\nu$

We adopt ideal gas EOS for (non-degenerate) regions outside (degenerate) core (P_{rad}=0): $P = \frac{\Re}{\mu}\varrho T$

We further assume following relations in H-burning shell ($R_c \le r \le R_c + \triangle r$):

basic parameters

$$\rho = \tilde{\rho}(r/R_c) M_c^{\varphi_1} R_c^{\varphi_2} \mu^{\varphi_3} \qquad P = \tilde{P}(r/R_c) M_c^{\tau_1} R_c^{\tau_2} \mu^{\tau_3}$$

$$T = \tilde{T}(r/R_c) M_c^{\psi_1} R_c^{\psi_2} \mu^{\psi_3} \qquad l = \tilde{l}(r/R_c) M_c^{\sigma_1} R_c^{\sigma_2} \mu^{\sigma_3}$$

at the homologous points: $\frac{r}{R_c} = \frac{r'}{R'_c}$;

From EOS: $\tau_1 = \varphi_1 + \psi_1$, $\tau_2 = \varphi_2 + \psi_2$, $\tau_3 = \varphi_3 + \psi_3 - 1$,

Evolution through helium burning – low-mass stars (<2.3 M$_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

For example, we can explicitly write:

$$\frac{\varrho}{\varrho'} = \left(\frac{M_c}{M_c'}\right)^{\varphi_1} \left(\frac{R_c}{R_c'}\right)^{\varphi_2} \left(\frac{\mu}{\mu'}\right)^{\varphi_3}$$
$$\frac{P}{P'} = \left(\frac{M_c}{M_c'}\right)^{\tau_1} \left(\frac{R_c}{R_c'}\right)^{\tau_2} \left(\frac{\mu}{\mu'}\right)^{\tau_3}$$

We insert these relations in to the stellar structure equations in the form:

$$-r^{-2}\mathrm{d}r = \mathrm{d}(1/r)$$

$$dP \sim M_c \varrho d(1/r) \,,$$
$$d(T^4) \sim \kappa \varrho l \, d(1/r) = \kappa_0 \varrho P^a T^b l \, d(1/r) \,,$$
$$dl \sim \varepsilon \varrho d(r^3) = \varepsilon_0 \varrho^n T^\nu d(r^3) \,,$$

$m \simeq M_c = \mathrm{const.}$ – good approximation in regions where *P* drops to ~0;

Evolution through helium burning – low-mass stars (<2.3 M$_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

$$dP \sim M_c \varrho d(1/r) \,,$$
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$$dl \sim \varepsilon \varrho d(r^3) = \varepsilon_0 \varrho^n T^\nu d(r^3) \,,$$

Integration of the 1st equation over the H-burning shell for a model (M_c, R_c) and a second model with (M_c', R_c') with subsequent comparison of the solutions leads to

$$P(r/R_c) \sim M_c^{\varphi_1+1} R_c^{\varphi_2-1} \mu^{\varphi_3}$$

i.e. by comparison with the definition of *P* we obtain:

$$\tau_1 = \varphi_1 + 1 \,, \tau_2 = \varphi_2 - 1 \,, \tau_3 = \varphi_3 \,.$$

Evolution through helium burning – low-mass stars (<2.3 M$_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

Similarly we obtain from integrating 2nd and 3rd equations a set of linear relations for all 12 exponents:

$$P(r/R_c) \sim \overbrace{M_c^{\varphi_1+1}}^{\tau_1} \overbrace{R_c^{\varphi_2-1}}^{\tau_2} \overbrace{\mu^{\varphi_3}}^{\tau_3} \longrightarrow \tau_1 = \varphi_1 + 1 \,, \tau_2 = \varphi_2 - 1 \,, \tau_3 = \varphi_3$$

$$T \sim M_c^{\psi_1} R_c^{\psi_2} \mu^{\psi_3} \longrightarrow \begin{cases} (4-b)\psi_1 = \varphi_1 + a\tau_1 + \sigma_1 \\ (4-b)\psi_2 = \varphi_2 + a\tau_2 + \sigma_2 - 1 \\ (4-b)\psi_3 = \varphi_3 + a\tau_3 + \sigma_3 \end{cases}$$

$$l \sim M_c^{\nu+n\varphi_1} R_c^{3-\nu+n\varphi_2} \mu^{\nu+n\varphi_3} \longrightarrow \begin{cases} \sigma_1 = n\varphi_1 + \nu\psi_1 \,, \quad \sigma_2 = n\varphi_2 + \nu\psi_2 + 3 \\ \sigma_3 = n\varphi_3 + \nu\psi_3 \end{cases}$$

Evolution through helium burning – low-mass stars (<2.3 M$_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

Similarly we obtain from integrating 2nd and 3rd equations a set of linear relations for all 12 exponents:

$$\varphi_1 = -\frac{\nu - 4 + a + b}{N} \,, \quad \varphi_2 = \frac{\nu - 6 + a + b}{N}$$
$$\varphi_3 = \frac{4 - b - \nu}{N} \,, \qquad N = 1 + n + a$$
$$\psi_1 = 1 \,, \qquad \psi_2 = -1 \,, \qquad \psi_3 = 1$$
$$\tau_1 = 1 + \varphi_1 \,, \quad \tau_2 = \varphi_2 - 1 \,, \quad \tau_3 = \varphi_3$$
$$\sigma_1 = \nu + n\varphi_1 \,, \quad \sigma_2 = 3 - \nu + n\varphi_2$$
$$\sigma_3 = \nu + n\varphi_3$$

Evolution through helium burning – low-mass stars (<2.3 M$_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

Similarly we obtain from integrating 2nd and 3rd equations a set of linear relations for all 12 exponents,

for the following expressions for *P*, *T* and *l*, which relate these quantities from one model (characterized by M_c, R_c, μ) to another model (characterized by M_c', R_c', μ') at each homologous point r/R_c:

$$P(r/R_c) \sim \overbrace{M_c^{\varphi_1+1}}^{\tau_1}\overbrace{R_c^{\varphi_2-1}}^{\tau_2}\overbrace{\mu^{\varphi_3}}^{\tau_3}$$

$$T \sim M_c^{\psi_1}\ R_c^{\psi_2}\ \mu^{\psi_3}$$

$$l \sim M_c^{\nu+n\varphi_1}\ R_c^{3-\nu+n\varphi_2}\ \mu^{\nu+n\varphi_3}$$

Evolution through helium burning – low-mass stars (<2.3 M$_\odot$)

Shell source homology: (relating properties between core and H-burning shell)

$$\longrightarrow \quad T \sim M_c^{\psi_1}\ R_c^{\psi_2} = M_c/R_c$$

For *a*=*b*=0 (electron scattering) and ν=13, *n*=2 (CNO cycle) we find:

$$\longrightarrow \quad \underline{P} \sim M_c^{\varphi_1+1}R_c^{\varphi_2-1}\ \mu^{\varphi_3} = \underline{M_c^{-2}R_c^{4/3}\mu^{-3}}\ ,$$

$$\longrightarrow \quad \underline{l} \sim M_c^{\nu+n\varphi_1}\ R_c^{3-\nu+n\varphi_2}\ \mu^{\nu+n\varphi_3} = \underline{M_c^{7}R_c^{-16/3}\mu^{7}}\ ,$$

which are independent of total stellar mass *M* !

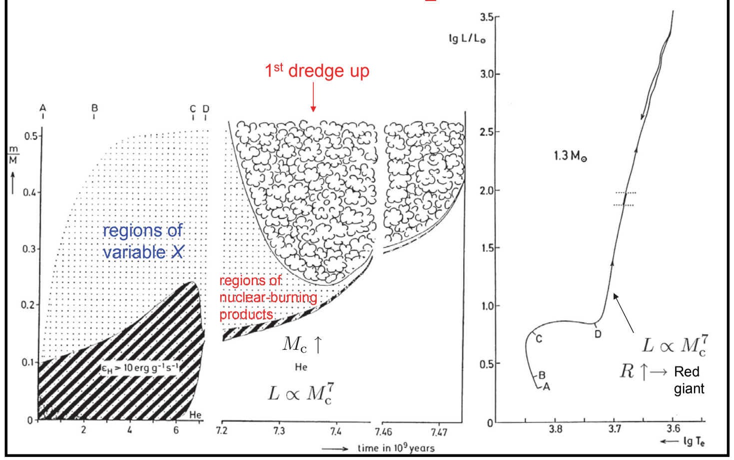
For μ = const. and for a degenerate core (note for a white dwarf we have $R_c \sim M_c^{-1/3}$) we have, *for example*, by using data from cold white-dwarf models :

$$L \propto M_c^{8..10}\ , \quad \text{and} \quad T \propto M_c^{\beta}\ , \quad \text{with } \beta \text{ somewhat larger than } 1\ .$$

i.e. the temperature *T* of the shell, and therefore also of the core T_c, increases with M_c.

Evolution through helium burning – low-mass stars (<2.3 M$_\odot$)

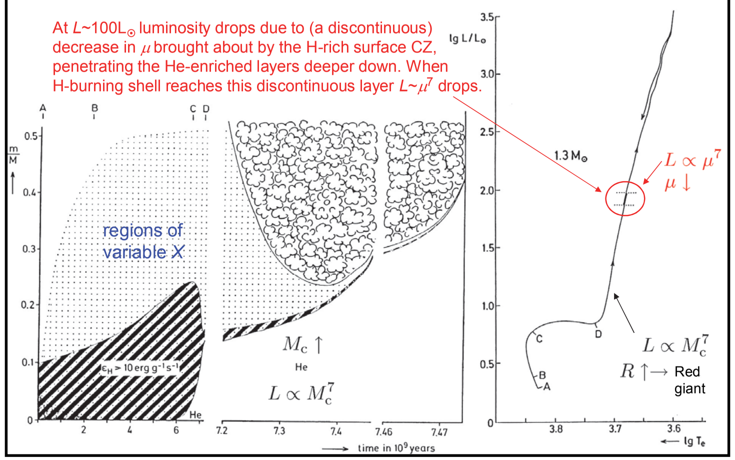
Evolution along the red-giant branch: 1.3 M$_\odot$



Evolution through helium burning – low-mass stars (<2.3 M$_\odot$)

Evolution along the red-giant branch

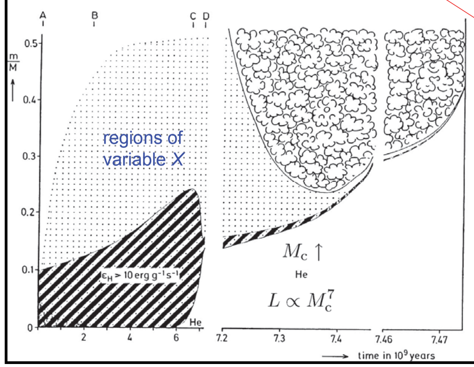
At $L \sim 100 L_\odot$ luminosity drops due to (a discontinuous) decrease in μ brought about by the H-rich surface CZ, penetrating the He-enriched layers deeper down. When H-burning shell reaches this discontinuous layer $L \sim \mu^7$ drops.

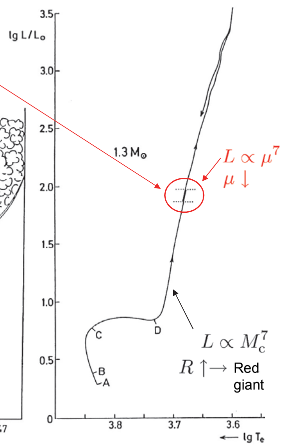


Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution along the red-giant branch

After the shell sources has passed the discontinuity, μ remains at its reduced value, but $L \sim M_c^7$ increases again with increasing M_c.



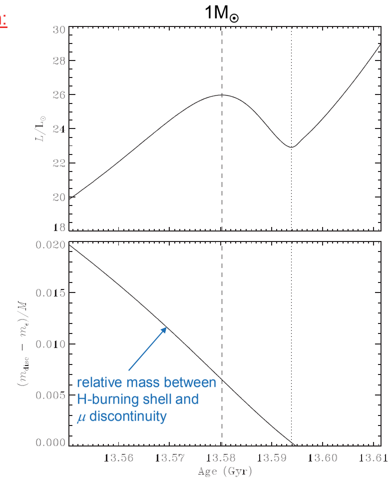


Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution along the red-giant branch:
"The hump"

Latest (unpublished) results for a 1$M_\odot$ computed by JCD:

The relevant weight of the layers above the shell starts to decrease with age when the H shell is close to but not at the discontinuity; that causes the decrease in the luminosity. When the shell crosses the discontinuity, the weight (or the contribution from changes in composition to the weight) does not change and hence the luminosity increases again.



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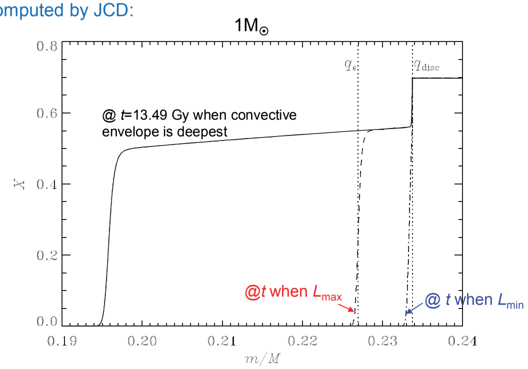


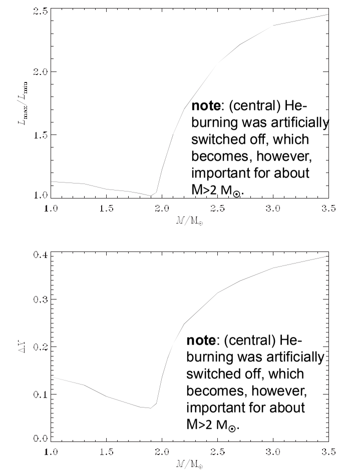
Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution along the red-giant branch:
"The hump"

Latest (unpublished) results for various models computed by JCD:

The ***position*** of the bump moves up in luminosity with increasing mass. However, the **ratio** L_{max}/L_{min} decreases with increasing mass up to a mass of around 2$M_\odot$. It is closely correlated with the step ΔX in hydrogen abundance left behind at the maximum extent (in mass) of the convective envelope in the first dredge-up. This decreases with increasing mass from 1 to 2$M_\odot$.

This is not fully understood yet, but it might be the result of hydrogen burning getting more centrally concentrated as the CNO cycle takes over.



Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

Evolution to the helium flash

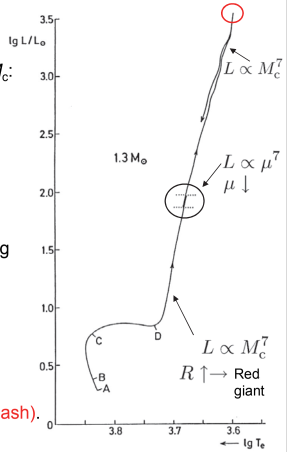
Core temperature T_c increases
(a) due to growing temperature in shell source with M_c:
(transported by radiation and conduction to core)

$$T \propto \frac{M_c}{R_c} \simeq M_c^\beta\,, \qquad \text{with } \beta \simeq 1\,,$$

(b) due to non-stationary terms: contracting core releases gravitational energy ε_g relatively fast, thereby heating the (non-degenerate) transition layer below the shell source and thereby also the whole (degenerate) core; all these is enhanced by increasing luminosity: $\dot M_c \propto L \propto M_c^7$.

Both these effects, controlled by $M_c\uparrow$, increase core temperature to about 10^8 K and He ignites. This happens when $M_c \approx 0.48\ M_\odot$, independently of M.

The core is highly degenerate and He burning is unstable (secular instability)→ thermal runaway (He flash).



Evolution through helium burning – low-mass stars (< 2.3 $M_\odot$)

The helium flash: secular stability behaviour of nuclear burning

– energy balance of a (central) sphere: $\varepsilon m_s - l_s = 0.$

energy/s generated inside sphere of mass m_s — energy/s leaving sphere

– energy perturbation occurs on a timescale τ such that $\tau_{hydr} < \tau < \tau_{KH}$; then

$$m_s d\varepsilon - dl_s = m_s \frac{dq}{dt} \equiv m_s c^* \frac{dT_c}{dt}\,.$$

– perturbation of radiative transfer (diffusion approximation) $l \sim \frac{T^3 r^4}{\kappa}\frac{dT}{dm}$,

$$\frac{dl_s}{l_s} = 4\vartheta_c + 4x - \kappa_P p_c - \kappa_T \vartheta_c$$

where an homologous change ($\vartheta = dT/T = \text{constant}$) for perturbation dT/dm, i.e.

$$d(dT/dm) = d(T\vartheta)/dm = \vartheta dT/dm.$$

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

The helium flash: secular stability behaviour of nuclear burning

– homologous reaction: (for each mass shell $m(r)$ is dr/r = constant)

$$\frac{d\varrho_c}{\varrho_c} = -3x\,,\qquad p_c := \frac{dP_c}{P_c} = -4x\,,\qquad \frac{d\varrho_c}{\varrho_c} = \alpha p_c - \delta\vartheta_c\,, \quad\Big\}\quad p_c = \frac{4\delta}{4\alpha-3}\vartheta_c\,. \qquad \vartheta_c := dT_c/T_c$$

– from 1[st] law of TD, using $dq = c_P dT - \frac{\delta}{\varrho}dP$ and $\nabla_{ad} \equiv \left(\frac{P}{T}\frac{dT}{dP}\right)_s = \frac{P\delta}{T\varrho c_P}$:

$$dq = du + Pdv = c_P T_c(\vartheta_c - \nabla_{ad} p_c) := c^* T_c \vartheta_c\,,$$

where $$c^* = c_P\left(1 - \nabla_{ad}\frac{4\delta}{4\alpha-3}\right)\,.$$

gravothermal specific heat $c^* < 0$ for ideal gas ($c^* > 0$ for nonrel. degenerated gas).

Evolution through helium burning – low-mass stars (< 2.3 $M_\odot$)

The helium flash: secular stability behaviour of nuclear burning

$$\frac{dl_s}{l_s} = 4\vartheta_c + 4x - \kappa_P p_c - \kappa_T\vartheta_c$$

$p_c := \frac{dP_c}{P_c} = -4x$, $p_c = \frac{4\delta}{4\alpha-3}\vartheta_c$ ⟶

$$\frac{dl_s}{l_s} = \left[4 - \kappa_T - \frac{4\delta}{4\alpha-3}(1+\kappa_P)\right]\vartheta_c\,.$$

$m_s d\varepsilon - dl_s = m_s\frac{dq}{dt}$ ⟶ ($l_s = \varepsilon m_s$)

$$\frac{m_s}{l_s}\frac{dq}{dt} = (m_s d\varepsilon - dl_s)l_s^{-1} = \varepsilon_T\vartheta_c + \varepsilon_P p_c - \frac{dl_s}{l_s}$$
$$= \left[(\varepsilon_T + \kappa_T - 4) + \frac{4\delta}{4\alpha-3}(\varepsilon_P + \kappa_P + 1)\right]\vartheta_c\,,$$

for nonrel. degenerate gas: $\delta = 0 \longrightarrow c^* = c_p > 0$

or $m_s\frac{dq}{dt} \equiv m_s c^*\frac{dT_c}{dt}$ ⟶

$$\frac{m_s c^* T_c}{l_s}\frac{d\vartheta_c}{dt} = \left[(\varepsilon_T + \kappa_T - 4) + \frac{4\delta}{4\alpha-3}(1+\varepsilon_P+\kappa_P)\right]\vartheta_c\,.$$

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

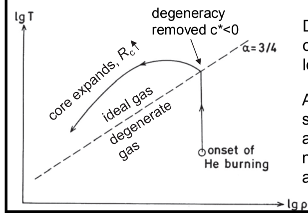
The helium flash

We assume He ignites around the centre in a small sphere of mass m_s with "luminosity" $l_s = \varepsilon m_s$ and with (positive) gravothermal specific heat $c^* = c_p$.

Small temperature perturbation $\vartheta_c (= dT_c/T_c)$ evolves as (secular stability problem): $\dot{\vartheta}_c = \frac{l_s}{c p m_s T_c}(\varepsilon_T + \kappa_T - 4)\vartheta_c$

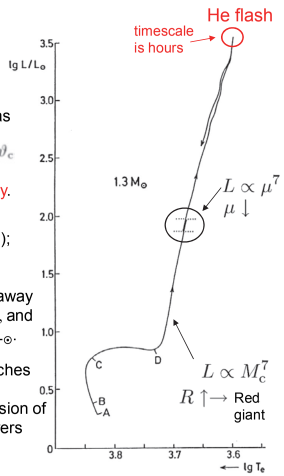
with $\varepsilon_T \approx 19$ (He burning) $\longrightarrow \dot{\vartheta}_c > 0 \longrightarrow$ thermal runaway.

Because of the strong degeneracy, the central density (and pressure) remains constant (i.e., independent of T !); all nuclear energy goes into internal energy.



During thermal runaway overproduction of ε, and local max(l) ~ $10^{11} L_\odot$.

Almost nothing reaches surface, since it is absorbed by expansion of non-degenerate layers above core.



Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

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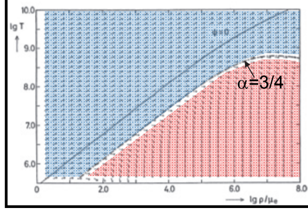


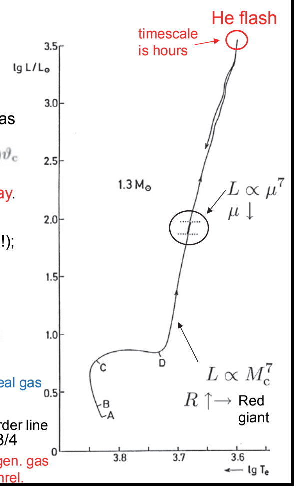
homologous contraction

$\frac{dT_c}{T_c} = \frac{4\alpha - 3}{3\delta}\frac{d\varrho_c}{\varrho_c}$

$\frac{dT_c}{T_c} = \frac{1}{3}\frac{d\varrho_c}{\varrho_c}$ …ideal gas

$\frac{dT_c}{T_c} \rightarrow 0$ …boarder line α=3/4

$d\rho_c = 0$ …degen. gas nonrel.



Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

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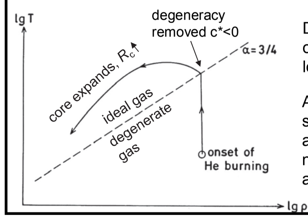

During thermal runaway overproduction of ε, and local max(l) ~ $10^{11} L_\odot$.

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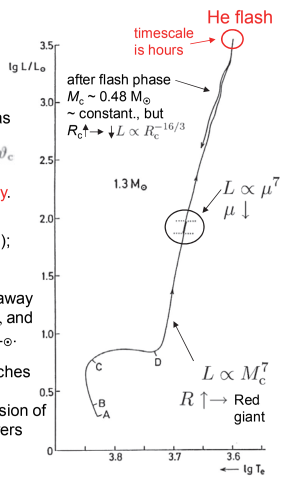
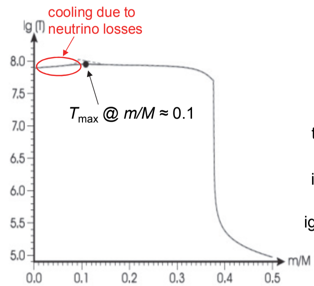


Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

The helium flash: effect of (cooling) neutrinos

1.2 $M/M_\odot$



energy flows from T_{max} in either directions (l>0, l<0).

energy released by core contraction in transition zone below H-burning shell.

transport mechanisms are radiation and conduction.

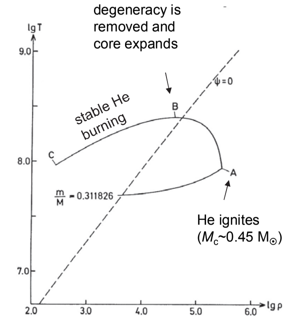
inward-going energy (l<0) is carried away by neutrinos.

ignition of He and flash not @ centre but in shell @ T_{max}.

for 0.7<$M/M_\odot$<2.2 He ignites @ ~ m/M≈0.17

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

The helium flash: detailed numerical calculation for a 1.3 $M_\odot$



after core expansion a non-degenerate phase follows with stable He burning @ ~ same T @ which flash started (A) but with much lower densities (C).

Evolution through helium burning – low-mass stars (<2.3 $M_\odot$)

The helium flash: detailed numerical calculation for a **0.85 $M_\odot$** model