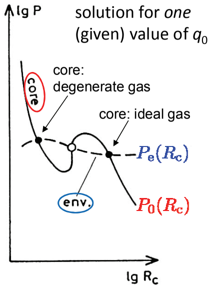
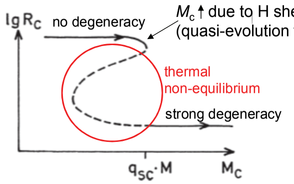
Evolution through helium burning: crossing the Hertzsprung gap



Integration of (isothermal) *core and envelope models* in (**complete = thermal and hydrostatic**) equilibrium

'fitting' conditions at m=M_c for **given** M and M_c ,i.e. $\boldsymbol{q_0}$:

$$r_e = R_c, \quad P_e = P_0, \quad T_e = T_0, \quad l_e = l_0 = 0$$



$M = 3M_\odot$

Resulting solutions for a linear set of models for fixed M and *varying M_c*, i.e varying q_0=M_c/M *(quasi-evolution)*:

→ we have 3 branches (2 stable, 1 unstable)

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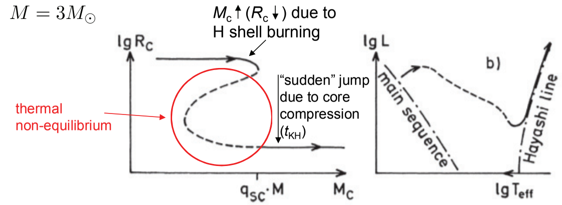


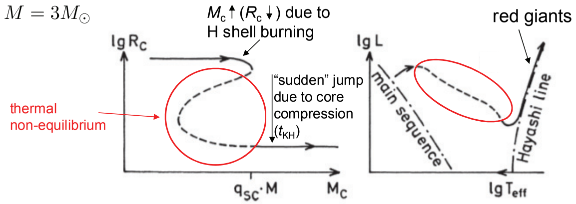
Calculations (simple quasi-evolution) assumed complete equilibrium:

hydrostatic equ.: $$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

thermal equ.: $$\frac{\partial l}{\partial m} = \varepsilon_n - \varepsilon_\nu$$

Evolution through helium burning: crossing the Hertzsprung gap

$M = 3M_\odot$



BUT if stellar structure *changes* occur on a *time scale << than nuclear time scale* we have also to consider:

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

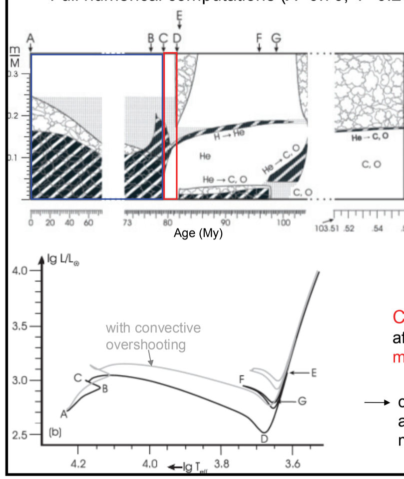
dynamical time scale t_{dyn}

$$\frac{\partial l}{\partial m} = \varepsilon_n - \varepsilon_\nu + \varepsilon_g \quad ; \; \varepsilon_g := -T\frac{\partial s}{\partial t} = -c_P\frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t}$$

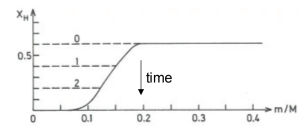
typically Kelvin- Helmholtz time scale t_{KH} ← gravitational energy released from contracting core

Evolution through helium burning : intermediate-mass stars (>2.5 $M_\odot$)

Full numerical computations (X=0.70, Y=0.28, Z=0.02) for M = 5 $M_\odot$



A-C:
central H burning exhausted after ~ 79 mill.y; convection also stops. At ~ same time H shell burning starts, initially in rather broad shell, where X_H increases with radius, left by shrinking convective core:



C-D:
after C evolution accelerates→abscissa scale!
model no longer in thermal equ.. i.e.$|\varepsilon_g| > 0$!

→ core shrinks below and envelope expands above H shell source → radial motion of mass elements

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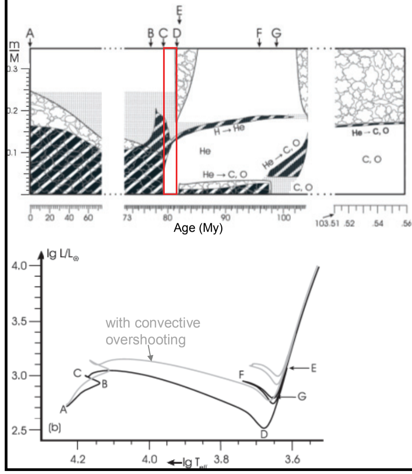
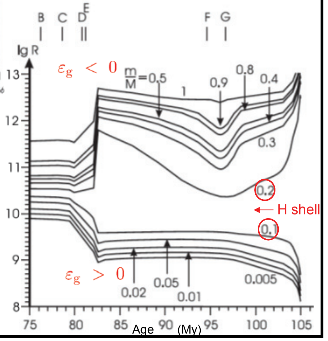


C-D:
radial motion: core contraction & envelope expansion around H shell @ $m \simeq 0.14M$ → **mirror principle.**



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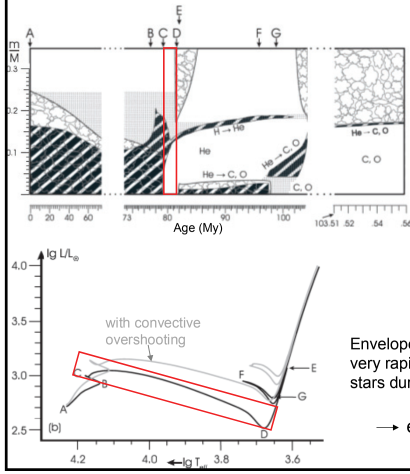


C-D:
radial motion: core contraction & envelope expansion around H shell @ $m \simeq 0.14M$ → **mirror principle.**

Such massive star starts @ MS with relatively low central density → core remains non-degenerate during present contraction; this leads to heating, because of $\varepsilon_g > 0$.

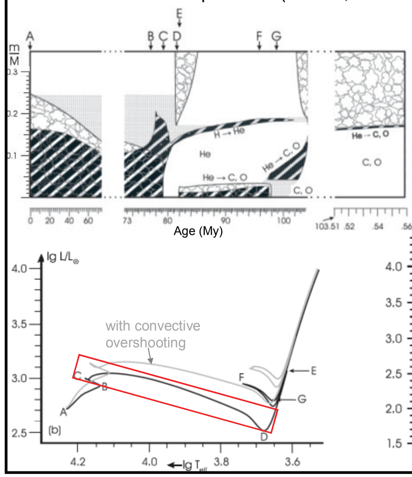
If $T_c > 10^8\ K$ helium is ignited in non-degenerate core → rapid contraction stops. Core contraction (C-D) time ~ t_{KH} (~3 million years).

Envelope expansion transform star into a red giant very rapidly (t_{KH}) → little chance to observe massive stars during this short phase of evolution:

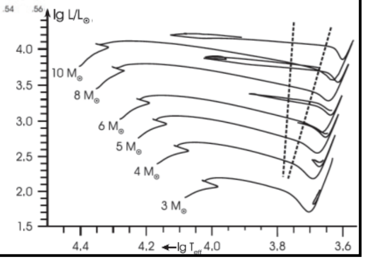
→ explains Hertzsprung gap

Evolution through helium burning : intermediate-mass stars (>2.5 $M_\odot$)

Full numerical computations (X=0.70, Y=0.28, Z=0.02) for M = 5 $M_\odot$



Evolution is qualitatively similar for all stars in which He is ignited in non-degenerate core ($2.5M_\odot<M<10M_\odot$).



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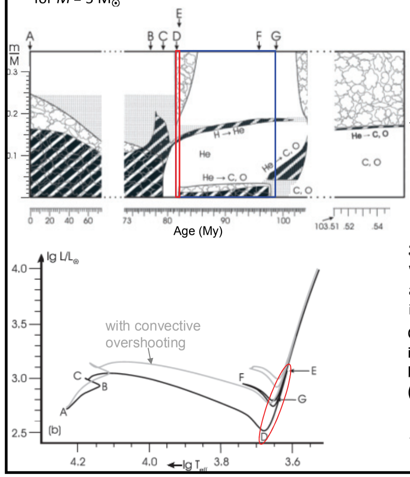


D-G: central He burning

D-E: red-giant region in H-R diagram: close to **Hayashi line**; very deep surface CZ down to $m/M \approx 0.17$ (deeper with ↑M). Reaches layers, where μ has been modified (dotted areas) by earlier H burning → products (heavier elements such as those from the CNO cycle) are mixed and transported to surface → 1st dregde up.

High T-dependence of He burning leads to convective core with only ~ 4% M.

At first dominant reaction is triple-a: $3\alpha \rightarrow {}^{12}\mathrm{C}$. Later ${}^{12}\mathrm{C} + \alpha \rightarrow {}^{16}\mathrm{O}$ takes over. With ${}^{4}\mathrm{He}$ becoming rare, depletion of ${}^{12}\mathrm{C}$ on account of ${}^{16}\mathbf{O}$ is larger than production of ${}^{12}\mathrm{C}$. i.e. ${}^{12}\mathrm{C}$ decreases again after maximum.

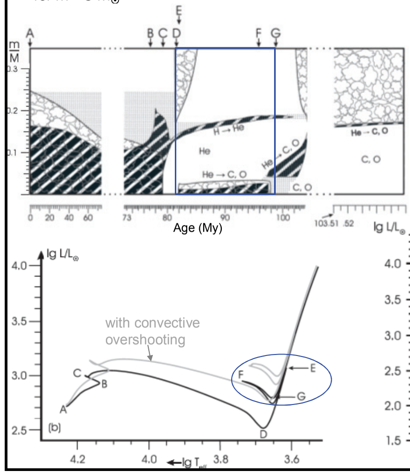
Central He burning lasts ~ 16 My, which is ~ 20% of H-MS phase; this is rather long because most of L is from H-burning shell (ε_{He}=7%@E, 26%@F and 42%@G).

Core radius R_{He} changes rather little with time for $Y_c > 0.3$, but is affected by convective overshooting (which enlarges R_{He}).

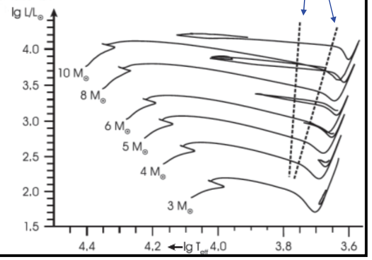
Evolution through helium burning : intermediate-mass stars (>2.5 $M_\odot$)

Full numerical computations (X=0.70, Y=0.28, Z=0.02)
for M = 5 $M_\odot$

D-G: central He burning



E-F-G: after E star moves first down Hayashi line, and then to the left (blue). The "bluest" point F is reached after ~ 14 mill y (~88% of central He-burning phase); the track leads back to G. These loops cross the classical instability strip (IS) for stars with M>5$M_\odot$, where δ Cepheids (pulsating stars) are observed.



Evolution through helium burning : intermediate-mass stars (>2.5 $M_\odot$)

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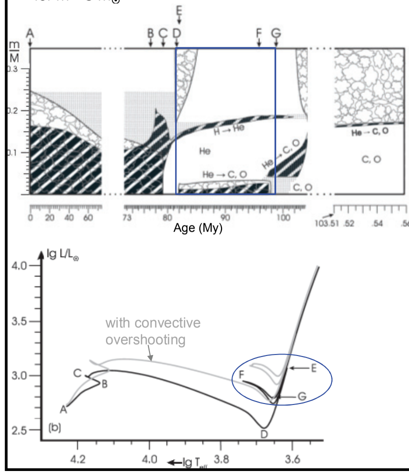

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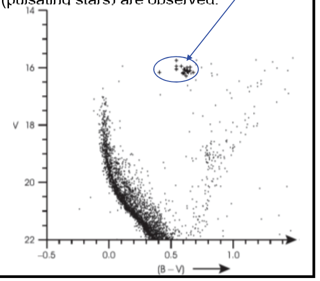

Evolution through helium burning : intermediate-mass stars (>2.5 $M_\odot$)

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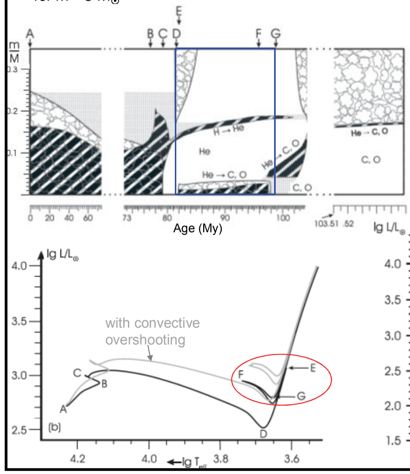
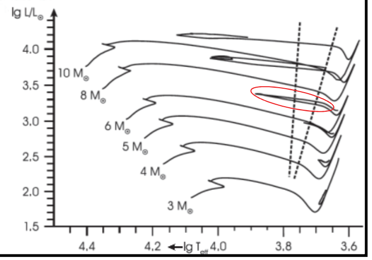
D-G: central He burning



E-F-G: these loops occur during the rather slow nuclear evolution phase of central He burning, i.e. there is a good chance to observe such pulsating red-giant stars in this phase of evolution.



Evolution through helium burning : intermediate-mass stars (>2.5 $M_\odot$)

Full numerical computations (X=0.70, Y=0.28, Z=0.02) for **M = 5 $M_\odot$**

The Cepheid Phase: [(C)-E-G]

typical giant stars located within the IS, which is only about 600-1000 K in width.

pulsate with large amplitude in *typically* one (radial) pulsation mode (breathing mode).

pulsations are driven by Carnot-like heat engine in the He ionization layers (κ mechanism).

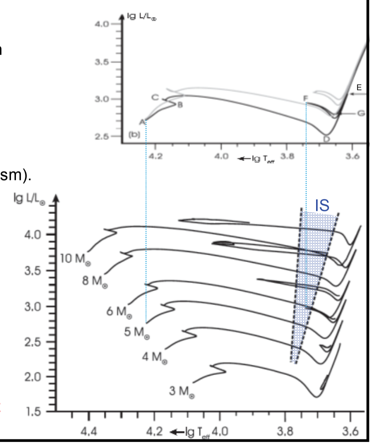
in our case stellar model crosses IS 3 times: C-D (too fast), (E-F), (F-G).

theory predicts for pulsation period Π:

$$\Pi \propto (\overline{\rho})^{-1/2} \propto R^{3/2}/M^{1/2} \simeq \mathrm{O}(t_{\rm hyd})$$

E-F: R↓ → Π↓ F-G: R↑ → Π↑

for L↑ → R↑ → Π↑ → Π–L relation
↓
important for distant measurements



Evolution through helium burning : intermediate-mass stars (>2.5 M$_\odot$)

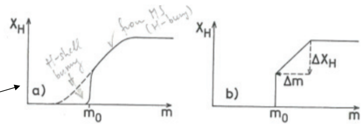
To loop or not to loop (E-G)

Evolution through loops (**for $M \leq 7$ M$_\odot$**) rather slow → use models in complete equilibrium.

Consider separate models for He core (M_c, R_c, l_0) and H-rich envelope; H-shell burning provides additional L-l_0.

central He burning weak during 1st loop

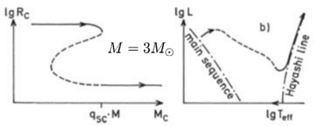
Envelope:
inner BC: R_c, l_0=0 @ m=M_c
solution: L, T_{eff}, P_0, T_0
solution form 2-parameter set: R_c, M_c
next we need simple description for X_H profile
4-parameter set: R_c, M_c, Δm, ΔX_H



need simple non-monotonic function(R_c,M_c,Δm,ΔX_H) for distance measure from Hayashi line.

Hint given by right figure: envelopes vary monotonically with M_c/R_c, whereas cores move non-monotonically through all 3 branches.

$\varphi := h(\Delta m, \Delta X)\dfrac{M_c}{R_c}$ .... effective (core) surface potential

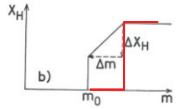


Evolution through helium burning : intermediate-mass stars (>2.5 M$_\odot$)

To loop or not to loop (E-G)

surface potential $\varphi := h(\Delta m, \Delta X)\dfrac{M_c}{R_c}$

we normalize h=1 for (Δm=0, ΔX=0 → step function)



→ φ is indeed indicator of distance from Hayashi line.

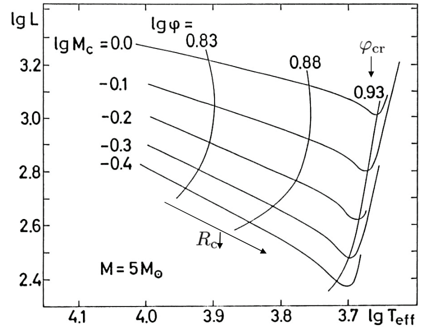
for $\varphi > \varphi_{cr}$ all envelope solutions move upwards.

From numerical calculations:

$h = e^{\text{constant}\cdot\Delta m\cdot\Delta X}$

$M = 5M_\odot$

Five sequences of **envelope solutions** for h=1 & with constant M_c,but varying R_c.



Evolution through helium burning : intermediate-mass stars (>2.5 M$_\odot$)

To loop or not to loop (E-G)

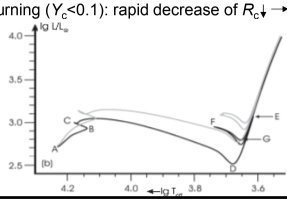
Envelope: $\varphi := h(\Delta m, \Delta X)\dfrac{M_c}{R_c}$ $\qquad h = e^{\text{constant}\cdot\Delta m\cdot\Delta X}$

Core: *still need relations between R_c and M_c from core solutions.*
Envelope solution, specified by (R_c, M_c, h), provide $P_0(R_c)$ & $T_0(R_c)$ for each M_c, which are used as outer BC for core solution → provides required $R_c(M_c)$.

$M_c/R_c \propto M_c^{0.4}$ for $\varphi < \varphi_{cr}$
$M_c/R_c \propto M_c^{0.25}$ for $\varphi > \varphi_{cr}$
factor increases φ when shell source burns outwards (M_c↑).

However, we have, additionally to this factor, an effect from the chemical core evolution affecting R_c, because R_c drops after central He burning (Y_c<0.1): rapid decrease of R_c↓ → φ↑ .

→ Both effect, the increase of M_c (H-shell burning outwards) and decrease of R_c shifts the model to the right in the HR diagram, and may therefore finish a loop (F→ G), but they can never start it, i.e. decrease φ .



Evolution through helium burning : intermediate-mass stars (>2.5 M$_\odot$)

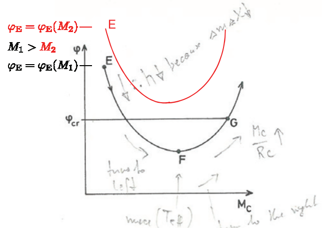
To loop or not to loop (E-G)

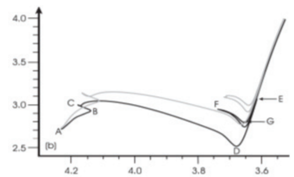
$\varphi := h(\Delta m, \Delta X)\dfrac{M_c}{R_c}$ $\qquad h = e^{\text{constant}\cdot\Delta m\cdot\Delta X}$

↓

factor h has to be responsible for the onset of the loop (decrease of φ):

When H-shell source "burns" farther into H-profile, h decreases and outweighs the increase of the factor M_c/R_c in the first phase after the evolution point E → φ↓





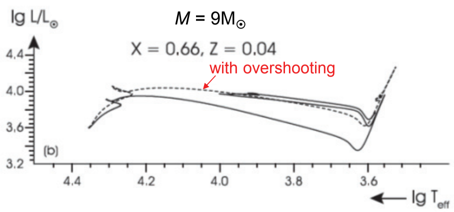
→ found non-monotonic variation of φ .

Evolution through helium burning : intermediate-mass stars (>2.5 $M_\odot$)

To loop or not to loop (E-G)

φ_{crit} depends on:

- timescales of H-shell and central He-burning phase.
- convection and in particular convective overshooting, which affects M_c (increases convective core), modifies φ_{crit} .



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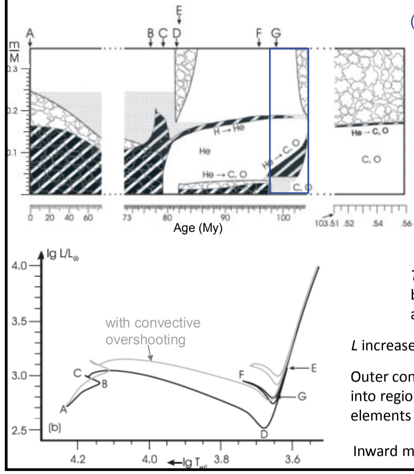

(F)G-: after central He burning

He burning stops in central core if ^4He is completely processed to ^{12}C, ^{14}O, & ^{20}Ne. Burning continues in a shell around the exhausted core.

While He shell burns outwards, the C-O core increases in mass and contracts. Star has now two shell sources:

Mirror principle: core contract, He region between shell sources expand, envelope contracts.

T drops in outward-moving H shell → H-burning stops: core contracts, env. expands around He-burning shell.

L increases rapidly with increasing C-O core mass.

Outer convective envelope reaches down and penetrates into region where H-burning shell had produced heavier elements (CNO) → 2nd dredge up.

Inward motion of CZ reaches T~2x10^{8}K → H reignites.