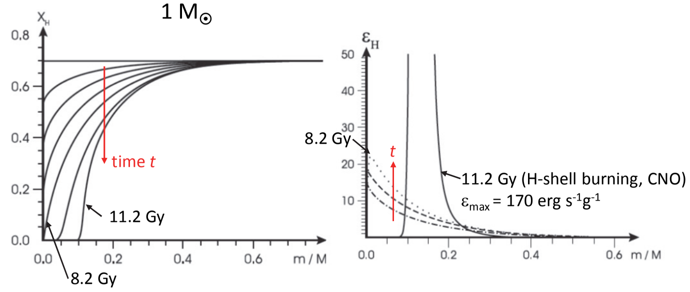
Evolution on the main sequence

Change in H-content

- change of central H content relatively simple for radiative cores ($0.1\,M_\odot \lesssim M \lesssim 1\,M_\odot$).

- for convective cores a somewhat larger volume is affected and dX_H/dt is obtained from the expression with "moving CZ boundaries" or the turbulent-diffusion-like expression.

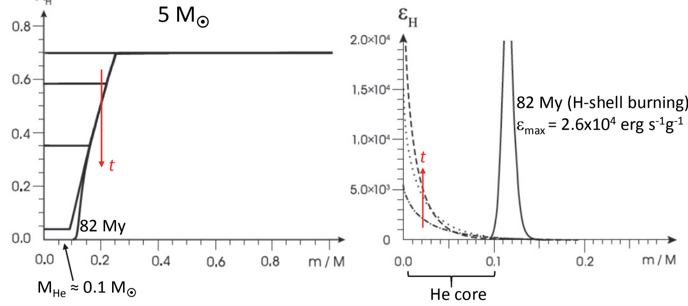


Evolution on the main sequence

Change in H-content

- In more massive stars ($5\,M_\odot$), H-burning is even more concentrated near core because of high T-dependence of CNO cycle; rapid (convective) mixing makes core, however, homogeneous.

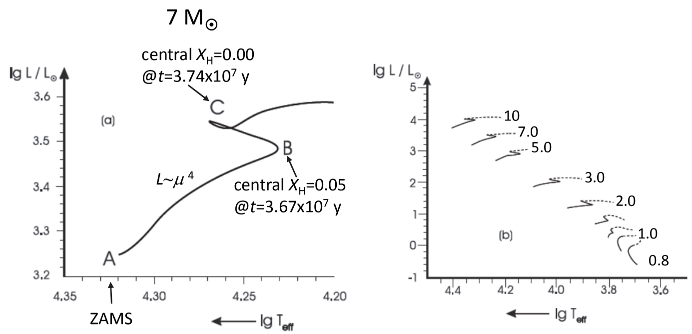
For larger stellar masses M, He-core size rather uncertain because of limited knowledge of convective overshooting and semiconvection.



Evolution on the main sequence

Evolution in H-R diagram

- During H-MS star evolves only slowly from ZAMS (A) until central H exhaustion (B,C) as shown below for a $7M_\odot$:



Evolution on the main sequence

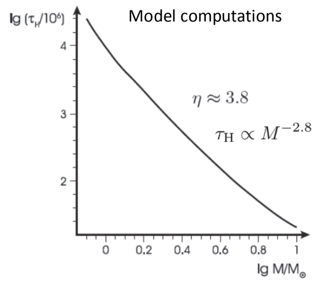
Timescales τ_H for central H-burning

$$E_H \sim M_H \sim M$$

$$\tau_H = \frac{E_H}{L} \propto \frac{M}{L} \sim M^{1-\eta}\,, \qquad L \sim M^\eta$$

With $\eta = 3.5 \rightarrow \tau_H \sim M^{-2.5}$,

i.e. a strong decrease of τ_H towards larger values of M.



Evolution on the main sequence

Convection: "overshooting"

- In any "local" convection model, such as the mixing-length model, convective elements would stop immediately at the upper border of a convective core, which is unrealistic because their inertia (momentum) cannot suddenly become zero at this border.
- Instead the convective elements will continue to penetrate into the radiative (convectively stable) layers for some distance, which will affect the size of the convective core and also the thermal structure in the radiative layers just outside the convective layers.

buoyancy force k_r acting on an element is

$$k_r \sim \nabla - \nabla_{\rm ad} ,$$

with a positive factor of proportionality.

within CZ: $\nabla - \nabla_{\rm ad}$ is extremely small, and positive $(\approx 10^{-6})$.

just outside CZ in radiative layers: $\nabla - \nabla_{\rm ad}$ soon reaches rather large and negative values: because ∇ drops very quickly (< 0.1 H_p) below $\nabla_{\rm ad}$.

⟶ Breaking (decelerating) of convective element within short (overshoot) distance.

Evolution on the main sequence

Convection: "overshooting"

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⟶ Breaking (decelerating) of convective element within short (overshoot) distance.

In this overshoot region $DT \sim \nabla - \nabla_{\rm ad}$ < 0 , i.e.

⟶ $F_{\rm con} \sim v \cdot DT$ < 0 !

Consequently F_{rad} has to increase and therefore also ∇, which reduces the magnitude of $\nabla - \nabla_{\rm ad}$ and therefore also of the "breaking" force $k_r \sim \nabla - \nabla_{\rm ad}$: ⟶ deeper penetration (larger overshooting).

Details are complicated and can only be described with a proper nonlocal convection model, which are very complicated and (numerically) time-consuming.

In practice one parameterizes the extent of overshooting.

Evolution on the main sequence

Convection: "overshooting"

In practice one parameterizes the extent of overshooting; 2 methods are commonly used:

(1) Parameterizing the extent of overshooting via an overshoot parameter l_{ov}:

$$l_{\rm ov} = \alpha_{\rm ov} H_P .$$

The parameter $\alpha_{\rm ov}$ is typically of order $0.1 \cdots 0.2$ for modern stellar models. It has no relation to the mixing-length parameter $\alpha_{\rm MLT}$, and is most often determined by fitting models to observed colour-magnitude diagrams.

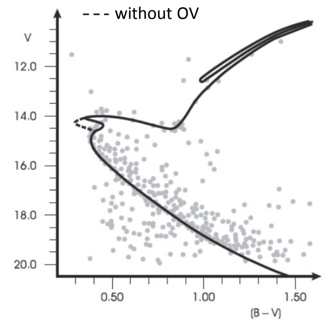
(2) Convective overshooting is treated as a diffusive process with a diffusion constant:

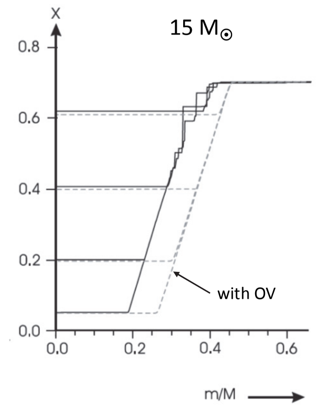
$$D(z) = D_0 \exp \frac{-2z}{f_{\rm ov} H_P},$$

where z is the radial distance from the formal Schwarzschild border, and $f_{\rm ov}$ the free parameter of this description. D_0 sets the scale of diffusive speed and is derived from the convective velocity obtained from mixing-length theory and taken below the Schwarzschild boundary.

Evolution on the main sequence

Convection: "overshooting"





Evolution on the main sequence

Convection: "semiconvection"

Layers in which $\nabla_\mu \equiv d\ln\mu/d\ln P > 0$, such as outside a retreating convective core which leaves a H-profile behind that increases with *r* (μ decreases), are dynamically (convectively) stable but vibrationally unstable.

Vibrational instability means that any so small perturbation to a mass element leads to an oscillatory movement with (slowly) increasing amplitude on the thermal timescale (non-adiabaticity). The mass element penetrates increasingly deeper into regions of different chemical composition. This slow mixing is called semiconvection.

Convection (dynamically unstable [homogeneous] layers) sets in according to (Karl) Schwarzschild if

$$\nabla_{\rm ad} < \nabla_{\rm rad}$$

and for a chemically inhomogeneous layer, according to Ledoux, if

$$\nabla_{\rm ad} + \frac{\varphi}{\delta}\nabla_\mu < \nabla_{\rm rad}$$

Semiconvection (slow mixing) occurs if

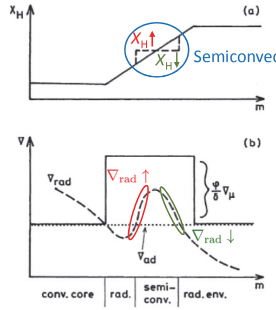
$$\nabla_{\rm ad} < \nabla_{\rm rad} < \nabla_{\rm ad} + \frac{\varphi}{\delta}\nabla_\mu \; .$$

Evolution on the main sequence

Convection: "semiconvection"

Semiconvection (slow mixing) occurs if

$$\nabla_{\rm ad} < \nabla_{\rm rad} < \nabla_{\rm ad} + \frac{\varphi}{\delta}\nabla_\mu \; .$$



Semiconvection mixes X_H

Since $\kappa \sim (1 + X_{\rm H})$

and $\nabla_{\rm rad} \sim \kappa$:

Lower part will eventually become convective, whereas radiative envelope will grow deeper into the formerly semiconvective layer.