The formation of protostars
Free-fall collapse of a homogeneous sphere

Jeans criteria (derived from linear perturbation theory) provides only the conditions under which equilibrium will exponentially grow, but does not provide any information about the fully developed (nonlinear) collapse or its final state. This can only be done with higher-order perturbation theory, i.e. within the nonlinear regime.

Simple case of free collapse of a homogeneous (ρ_0=constant) sphere

- after cloud has become unstable (according to Jeans criteria) GM/R^2 dominates over $\rho^{-1}\nabla P$, i.e.

$$\left|\frac{1}{\varrho}\frac{\partial P}{\partial r}\right| \approx \frac{P}{\varrho R} \approx \frac{\Re}{\mu}\frac{T}{R} \quad \longrightarrow \quad g/|\rho^{-1}\nabla P| \propto M/RT\,.$$

→ we can neglect $\rho^{-1}\nabla P$ in the momentum equations, which leads to

$$\ddot{r} = -\frac{Gm}{r^2}\,,$$ and which becomes with $m = 4\pi r_0^3 \rho_0/3$ (ρ_0 = constant!)

$$\frac{1}{2}\dot{r}^2 = \frac{4\pi r_0^3}{3r} G\varrho_0 + \text{constant}$$

after both sides have been multiplied by $\dot{r}$.

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$$\frac{1}{2}\dot{r}^2 = \frac{4\pi r_0^3}{3r} G\varrho_0 + \text{constant} \qquad (\rho_0 = \text{constant!})$$

Choosing integration constant such that $\dot{r} = 0$ at begin of collapse when $r = r_0$

$$\frac{\dot{r}}{r_0} = \pm\left[\frac{8\pi G}{3}\varrho_0\left(\frac{r_0}{r} - 1\right)\right]^{1/2} .$$

real values only for $r < r_0$, i.e.
(-) provides relevant solution

The variable substitution $\cos^2\zeta = \frac{r}{r_0}$ i.e. $\frac{\dot{r}}{r_0} = -2\,\dot{\zeta}\cos\zeta\,\sin\zeta\,, \quad \frac{r_0}{r} - 1 = \frac{\sin^2\zeta}{\cos^2\zeta}\,,$

leads to

$$\zeta + \frac{1}{2}\sin 2\zeta = \left(\frac{8\pi G\varrho_0}{3}\right)^{1/2} t$$

where we used the identity $2\,\dot{\zeta}\cos^2\zeta = \frac{d}{dt}\left(\zeta + \frac{1}{2}\sin 2\zeta\right)$, and $r = r_0$ (ζ=0) @ t=0.

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$$\zeta + \frac{1}{2}\sin 2\zeta = \left(\frac{8\pi G\varrho_0}{3}\right)^{1/2} t$$

$$\cos^2\zeta = \frac{r}{r_0}$$
$$\frac{\dot{r}}{r_0} = -2\,\dot{\zeta}\cos\zeta\,\sin\zeta\,,$$
$$\frac{r_0}{r} - 1 = \frac{\sin^2\zeta}{\cos^2\zeta}\,,$$

Note: because ρ_0 = constant, solution $\zeta(t)$ is same for all mass shells, i.e. for given t, r/r_0 and $\dot{r}/r_0$ are the same for all mass shells → homologous contraction! Moreover, because $\dot{r}/r_0$ is independent of r_0 → sphere remains **homogeneous**.

Time to reach centre r =0, or $\zeta = \pi/2$ is

$$t_{\rm ff} = \left(\frac{3\pi}{32 G\varrho_0}\right)^{1/2} .$$

With $\varrho_0 = 4\times 10^{-23}$ g/cm^3, corresponding to a slightly enhanced interstellar density, one obtains $t_{\rm ff} \approx 10^7$years. For a typical protostellar clump with $\varrho_0 = 4\times 10^{-19}$ g/cm^3, results in $t_{\rm ff} \approx 2\times 10^5$ years.

The formation of protostars
Collapse onto a condensed object

- As collapsing cloud becomes opaque, heating will start in central regions, because radiation can more easily escape from surface → collapse will stop first in central regions.
- We consider core already in hydrostatic equilibrium surrounded by still free-falling cloud (we ignore angular momentum transfer, i.e. circumstellar [accretion] disk).
- Simplest case of steady state: core of mass M surrounded by infinite reservoir of matter from which a steady-state flow of velocity v rains down (**mass flow d*M*/d*t* = constant**):

→ $\dot{M} = 4\pi r^2 \varrho v$ = **constant in time and space**, i.e. differentiation leads to continuity equ.

$$\frac{2}{r} + \frac{1}{\varrho}\frac{d\varrho}{dr} + \frac{1}{v}\frac{dv}{dr} = 0\,.$$

If we set for the free-fall velocity $v = v_{\rm ff} = [GM/(2r)]^{1/2}$ and assume M≈constant

$$\frac{1}{\varrho}\frac{d\varrho}{dr} = -\frac{3}{2r}\,, \qquad \text{or} \qquad \varrho(r) = \frac{\text{constant}}{r^{3/2}}\,.$$

The formation of protostars
Collapse onto a condensed object

- The matter falling onto the core with radius R (and mass M) is stopped at its surface (R).
- The kinetic energy is transformed into heat, part of which is used to heat up the core, the rest being radiated away. If we ignore the core heating, radiation losses are

$$L_{\rm accr} = \frac{1}{2} v_{\rm ff}^2(R)\dot{M} = \frac{1}{4}\frac{GM}{R}\dot{M}.$$

accretion luminosity

Note: we assumed for $v_{\rm ff}$ core mass M to be constant, i.e. mass increase within $\tau_{\rm ff}$ is negligible ($\dot{M} <<$) or

$$\tau_{\rm accr} := M/\dot{M} >> \tau_{\rm ff} \ .$$

The formation of protostars
A collapse calculation

- We have 5 equations for 5 unknowns, $m(r,t)$, $v(r,t)$, $P(r,t)$, $T(r,t)$ and $l(r,t)$, and material functions $\rho\,(P,T)$, $\kappa\,(P,T)$, and $u(P,T)$.
- Outer BC @ R: $v(R)=0$.
- Initial conditions: $v(r,0)=0$, $P(r,0)=10^{-19}$gcm^{-3} = constant, $T(r,0)$ = 10K = constant ⟶ $l(r,0)=0$.
- Instability was found numerically for $R < 0.46\ GM\mu/\mathcal{R}T$, which is very similar to the value obtained for a homologous collapse, $R_m=(4/9)\ GM\mu/\mathcal{R}T$.
- free-fall time $\tau_{\rm ff} \approx$ 210 000 y , obtained for a collapse of a homogeneous sphere, $\tau_{\rm ff} = (3\pi/32G\rho_0)^{1/2}$ for $\rho_0 = 10^{-19}$ gcm^{-3} .

The formation of protostars
A collapse calculation

- Discussion of first collapse calculations of spherical, homogeneous cloud of 1M$_\odot$ by Larson (1969).

  BC: surface R of sphere is fixed; the equations to be solved numerically are

$$\frac{\partial m}{\partial t} + 4\pi r^2 v\varrho = 0 \ ,$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{Gm}{r^2} + \frac{1}{\varrho}\frac{\partial P}{\partial r} = 0 \,,$$

specific internal energy

$$\frac{\partial u}{\partial t} + P\frac{\partial}{\partial t}\left(\frac{1}{\varrho}\right) + v\left[\frac{\partial u}{\partial r} + P\frac{\partial}{\partial r}\left(\frac{1}{\varrho}\right)\right] + \frac{1}{4\pi\varrho r^2}\frac{\partial l}{\partial r} = 0 \,,$$

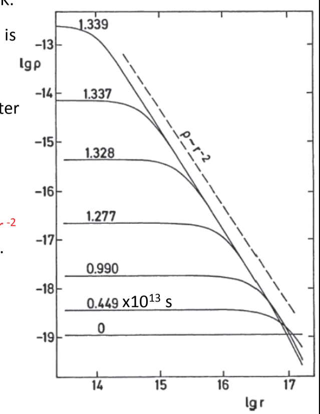
Note: $d/dt = \partial/\partial t + v\partial/\partial r$

$$\frac{\partial m}{\partial r} = 4\pi r^2\varrho \ ,$$

$$l = -\frac{16\pi acr^2}{3\kappa\varrho}T^3\frac{\partial T}{\partial r} \ .$$

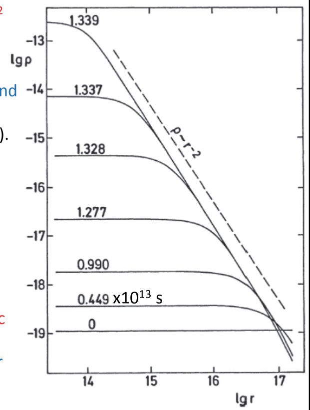
The formation of protostars
A collapse calculation: optically thin phase & core formation

- Very first phase: optically thin, isothermal with T=10 K.
- Later, with BCs keeping outer layers @ R=fixed, there is a non-homologous contraction of the inner surface layers, i.e. non-linear regime.
- $\rho\uparrow$ rapidly @ central layers, whereas $\rho\approx$const. @ outer layers.
- because $\tau_{\rm ff} \simeq (G\bar{\rho})^{-1/2}$ & $\bar{\rho}\uparrow$ towards centre, central layers fall faster than the outer layers ⟶ $\bar{\rho}_c\uparrow\uparrow$.
- calculations show that (starting from ρ=const.) $\rho\sim r^{-2}$ ($\neq r^{-3/2}$!) over gradually increasing parts of the cloud.



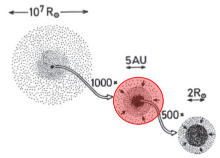
The formation of protostars
A collapse calculation: optically thin phase & core formation

- calculations show that (starting from ρ =const.) $\rho \sim r^{-2}$ ($\neq r^{-3/2}$!) over gradually increasing parts of the cloud.
- an increasingly smaller homogeneous (central) mass collapses ever more rapidly, continuously leaving behind more matter in the inhomogeneously contracting envelope, where τ_{ff}>> (because ρ << & ∇P break infall).
- collapse of homogeneous central part stops when matter becomes opaque ($\rho \approx 10^{-13}$gcm^{-3}) ⟶ adiabatic increase of T!
- a core in hydrostatic equilibrium forms (M=10^{31}g, R=6x10^{13}cm), surrounded by still-falling envelope.
- v_{ff}≈75kms^{-1} @ core surface, which becomes supersonic with M_c↑& R_c↓, ⟶ spherical shock front separates supersonic "rain" from hydrostatic core; falling matter stops and releases energy ⟶ L_{accr} of accreting core.



The formation of protostars
A collapse calculation: core collapse

(a) After ~ 1.3x10^{13} s core becomes optically thick, and v_{ff} becomes supersonic @ core surface with M↑ & R↓ , ⟶ 1st **spherical shock front** separates supersonic "rain" from hydrostatic core; infalling matter stops and releases energy ⟶ L_{accr} of accreting core.



Dynamical stability:

adiabatic change: $P \propto \rho^{\gamma_{ad}}$

homologous compression: $\rho \propto R^{-3}$

$\Rightarrow P \propto R^{-3\gamma_{ad}}$

From hydrostatic equilibrium: $$P = \int_m^M \frac{Gm}{4\pi r^4} dm$$

adiabatic **and** homologous change: $R^{-3\gamma_{ad}} \propto R^{-4}$: homologous change

⟶ for $\gamma_{ad} \geq 4/3$ pressure on left-hand-side increases faster than "weight" on the right ⟶ stable.

for $\gamma_{ad} < 4/3$ ⟶ unstable.

The formation of protostars
A collapse calculation: core collapse

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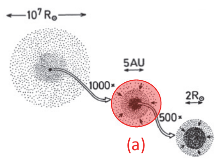
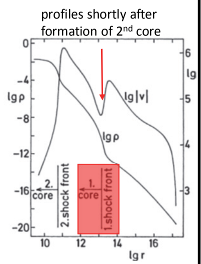




The formation of protostars
A collapse calculation: core collapse

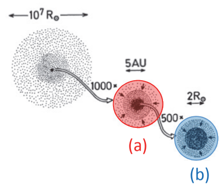
(a) After ~ 1.3x10^{13} s core becomes optically thick, and v_{ff} becomes supersonic @ core surface with M↑ & R↓ , ⟶ 1st **spherical shock front** separates supersonic "rain" from hydrostatic core; infalling matter stops and releases energy ⟶ L_{accr} of accreting core.

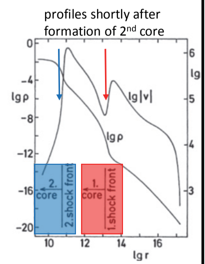
(b) Accreting hydrostatic core heats up. Gas consists mainly of molecular H_2 , which starts to dissociate @ T≈ 2000K.

Molecular H_2 has f=5 degrees of freedom (3 trans., 2 rot.) ⟶ γ_{ad} = (f+2)/f = 7/5 = 1.40 is already close to critical 4/3!

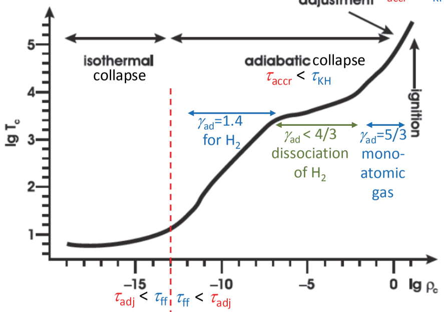
For γ_{ad} < 4/3 is "hydrostatic equilibrium" dynamically unstable. This is about when M_{core}~1/2 M_{init}.
⟶ **2nd collapse .**

When most of H_2 is dissociated into H (atomic form) ⟶ γ_{ad} > 4/3 (approaching 5/3 of an ideal mono-atomic gas) and a stable subcore develops ⟶ called ***protostar:*** M_{proto}~1.5x10^{-3}M☉, R_{ini}~1.3R☉, ρ_c~2x10^{-2}gcm^{-3}, T_c~2x10^4K.
@ surface ⟶ **2nd shock front.**





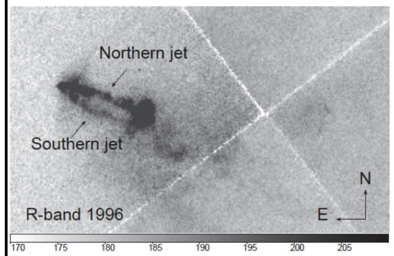
The formation of protostars
A collapse calculation: core collapse
Central evolution



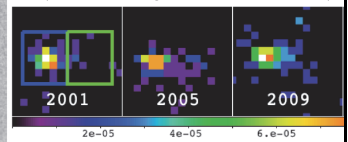
The formation of protostars
A collapse calculation: core collapse
Central evolution

The nearest X-ray emitting protostellar jet (HH 154) observed with *Hubble**

R. Bonito[1,2,3], C. V. M. Fridlund[4], F. Favata[5], G. Micela[2], G. Peres[1], A.A. Djupvik[6], and R. Liseau[7]
A&A (2008, 2011)



X-ray emission image (Chandra observatory)



The formation of protostars
A collapse calculation: core collapse
Central evolution

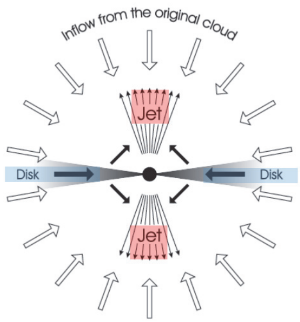
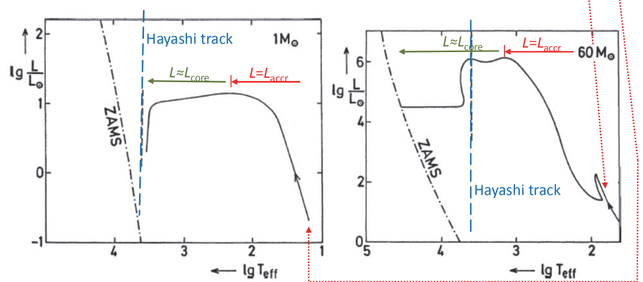


Fig. 27.4. Sketch of the mass flow within a collapsing protostellar sphere. From the original cloud matter is accreted onto the protostar that sits at the centre of the figure. Because of angular momentum conservation most of it accumulates in an accretion disk. Part of the matter finally falling onto the star is however ejected in a bipolar jet along the axis of rotation. The jet also may gain additional material directly from the disk, due to heating of the inner disk. (according to Zinnecker & Yorke 2007)

The formation of protostars
Evolution in the H-R diagram

- radiation by core is absorbed by in-falling (opaque) envelope and reemitted in the infrared.
- defining R at optical depth τ=2/3 ⟶ $L = 4\pi R^2 \sigma T_{eff}^4$.
  ⟶models are right of Hayashi track; note: envelopes are **not in hydrostatic equilibrium**.
- "thinning out" of envelope moves τ=2/3 to deeper layers until it has reached hydrostatic core ⟶ R↓⟶T_{eff}↑ to radiate energy, L=L_{accr}~$\dot{M}$, away.
- with decreasing $\dot{M}$⟶ L_{accr}↓ and L is increasingly provided by contracting core.



The formation of protostars

Bate, M. R., 2010, MNRAS, 404, L79-83.

http://www.astro.ex.ac.uk/people/mbate/Animations/prestellar_discs.html

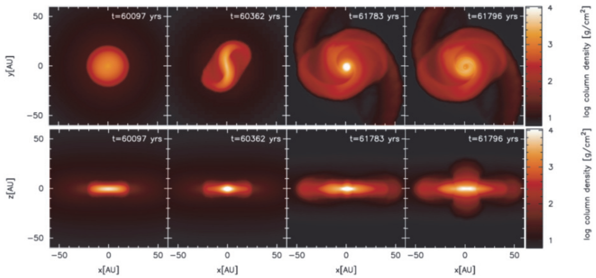
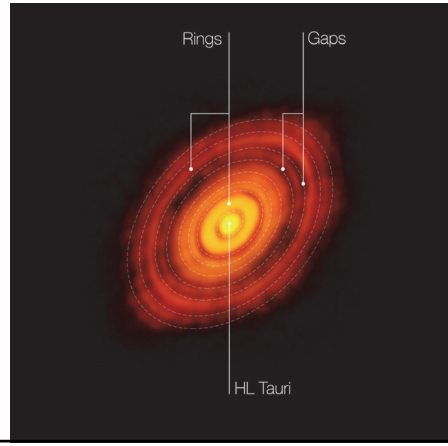


Figure 2. Column density images of the evolution of the first core, shock wave and outflow for the $\beta = 0.005$ case (using 3×10^6 SPH particles). The rapidly rotating first core is highly oblate (left-hand panels) and undergoes a dynamical bar-mode instability (centre-left panels) which transports angular momentum out of the centre, transforms the core into a disc, and triggers the formation of the stellar core. The energy released by stellar core formation heats the inner regions of the disc and drives a shock wave through the disc (upper right two panels) and a bipolar outflow perpendicular to the disc (lower right two panels). Animations can be found at http://www.astro.ex.ac.uk/people/mbate/Animations/Stellar/.

The formation of protostars

ALMA observations: HL Tauri

http://www.eso.org/public/news/eso1436/



The formation of protostars

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