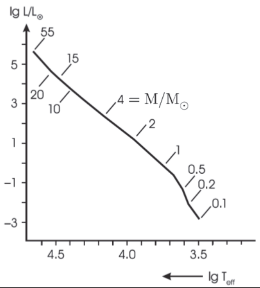
The (zero-age) Main Sequence

- we consider stellar models on the so-called *zero-age main sequence* (ZAMS).
- such stellar models just started central hydrogen burning, and are therefore still chemically homogeneous; they are also in mechanical and thermal equilibrium.

-this homogeneity is a result of the (nearly) fully convective structure during the pre-main sequence contraction phase.

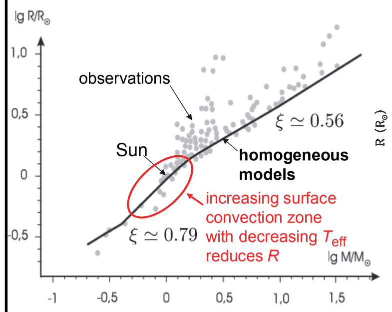


Hertzsprung-Russell diagram showing *zero-age main sequence, homogeneous,* models computed with X=0.7 and Y=0.28 ($Z = 1 - X - Y = 0.02$).

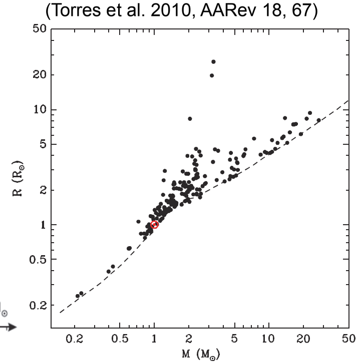
The (zero-age) Main Sequence: surface values

Mass-radius relations

$$R \propto M^{\xi}$$



Observations from eclipsing binaries (Torres et al. 2010, AARev 18, 67)

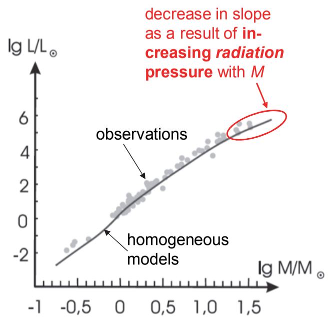


The (zero-age) Main Sequence: surface values

Mass-luminosity relations

$L \propto M^{\eta}$; over whole mass range: $\eta \simeq 3.37$
1,…, 10 M/M$_\odot$: $\eta \simeq 3.89$
1,…, 50 M/M$_\odot$: $\eta \simeq 3.35$

Note: from using Kramers' opacities for lower-mass stars: $L_s \propto T_{\rm eff}^{0.8} M^{4.4}$.



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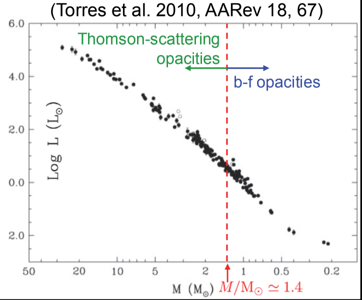
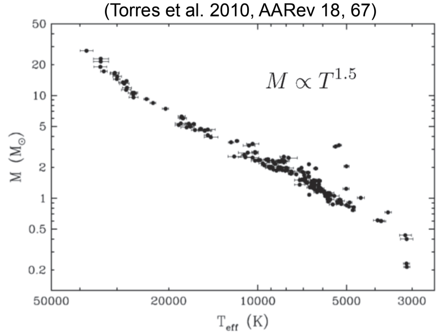


The (zero-age) Main Sequence: surface values

Mass-$T_{\rm eff}$ relations

Observations from eclipsing binaries (Torres et al. 2010, AARev 18, 67)



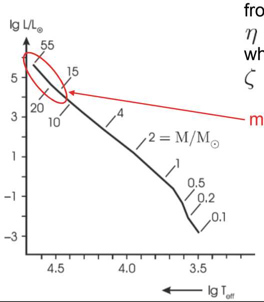
Homology (pp, ideal gas): $M \propto T^{(1-z_1)^{-1}} \simeq T^{1.66}$

The (zero-age) Main Sequence: surface values

H-R diagram: slope of MS

from $R \propto M^{\xi}$ and $L \propto M^{\eta}$ $\longrightarrow$ $R \propto L^{\xi/\eta}$.

using $L \propto R^2 T_{\rm eff}^4$ $\longrightarrow$ $L \propto T_{\rm eff}^{\zeta}$; $\zeta = \dfrac{4}{1 - 2\xi/\eta}$

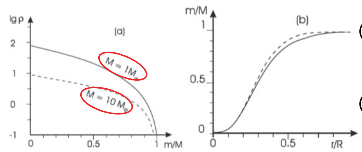


from previous figures we noticed that η decreases for larger stellar masses *M*, whereas ξ remains about constant, i.e ζ increases, or in other words

main sequence becomes steeper towards high *L*.

Hertzsprung-Russell diagram showing zero-age main sequence, homogeneous, models computed with *X*=0.7 and *Y*=0.28 (*Z* = 1 – *X* – *Y* = 0.02).

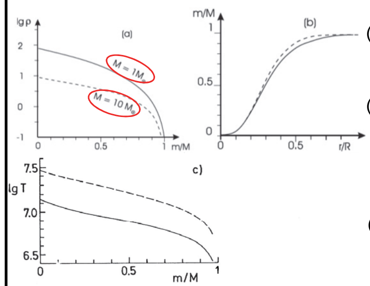
The (zero-age) Main Sequence: interior solutions



(a) 1M$_\odot$: ρ increases by ~ 10^9 from surface to centre. For 10M$_\odot$ ρ_c is smaller by a factor of ~10.

(b) Mass elements are increasingly concentrated towards the centre.
1M$_\odot$: inner ~30% of *R* (3% in *V*) contain 50% of *M*; outer ~50% of *R* (88% in *V*) contain only 15% of *M*.

The (zero-age) Main Sequence: interior solutions



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(c) 1M$_\odot$: temperature *T* increases by a factor of ~ 2400 from photosphere to centre.
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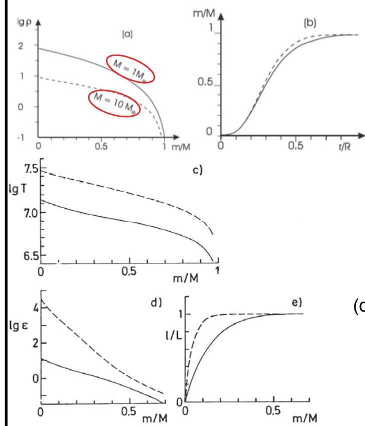
The (zero-age) Main Sequence: interior solutions



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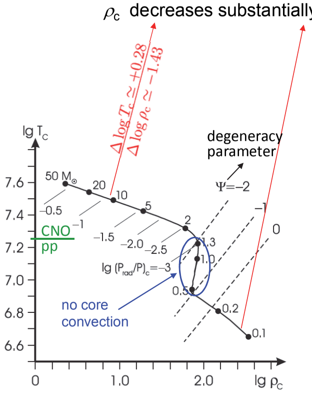
(d,e) 1M$_\odot$:energy generation rate $\varepsilon(T)$ drops by a factor of ~100 from centre to 0.6*M*; Fig. (e): 90%*L* is generated in inner 0.30 m/*M* (99%*L* @ 0.53 m/*M*).
10M$_\odot$:T_c~3x10^{7}K: CNO cycle dominates: $\varepsilon \propto T^{15} \rightarrow \varepsilon$ more concentrated near c. 90% *L* is generated within 10% *M*.

The (zero-age) Main Sequence: interior solutions

Central values of density and temperature

T_c increases only slightly with M
ρ_c decreases substantially with M

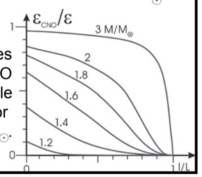


rapid change near $1M_\odot$:
related to changes in both centre and surface, in particular extension of convection zones; *core convection* has less pronounced mass concentration, as shown by both flat parts of the T_c-ρ_c curve.

low mass stars < $0.5M_\odot$ become degenerate near centre (high ρ_c), also Coulomb interaction important

$P_{rad} \propto T^4$ important for massive stars; it is about 1/3 of the total pressure in the centre of $50M_\odot$

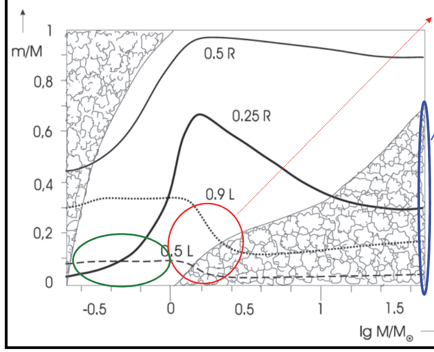
pp → CNO:
Integral under curve gives L fraction due to CNO burning; dominates whole energy generation for $1.8M_\odot$.



The (zero-age) Main Sequence: convective regions

Principally two types of models (zero-age MS):
(i) convective core + radiative envelope (upper MS)
(ii) radiative core + convective envelope (lower MS).

Transition is near 1 $M_\odot$.



transition from pp to CNO:
flux $l/4\pi r^2$, (l/m) , is large, i.e.
∇_{rad} is large → convective $(\nabla_{rad} > \nabla_{ad})$.

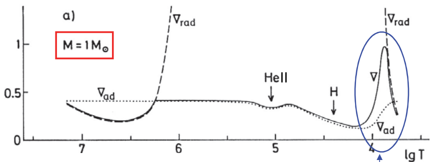
Inner ~65% of M is convective for $50M_\odot$, caused by increasing P_{rad} → $\nabla_{ad} \simeq 0.27$.

Increasing M even further will lead to fully convective stars (supermassive stars).

In less massive stars pp dominates, i.e. energy production over a much larger area → $(\nabla_{rad} < \nabla_{ad})$ (radiative).

The (zero-age) Main Sequence: convective regions

convection if $\nabla_{rad} > \nabla_{ad}$



stars with low T_{eff} (low M, see fig.) develop surface CZ because of the increasing opacity (increasing ∇_{rad}). ∇_{rad} increases steeply to > 10^5.

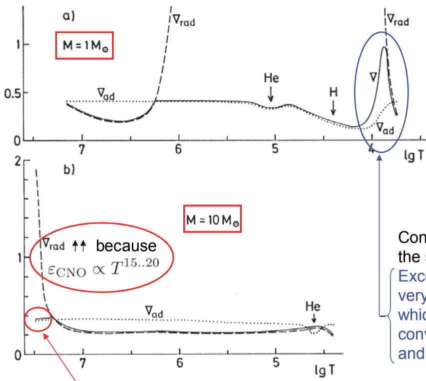
Partial ionization regions (H, He) also decrease $\nabla_{ad} < 0.4$ →contributes to forming a CZ ($\nabla_{ad} \propto c_p/c_v$; c_p & c_v increase, but c_p/c_v ratio decreases).

Convection is very efficient in most parts of the star, i.e. $\nabla \simeq \nabla_{ad}$ $(\nabla - \nabla_{ad} \simeq 10^{-6})$
Except near outer layers, where ρ becomes very small (i.e, less mass to move around, which is partially compensated by higher convective velocities → near supersonic!), and $\max(\nabla - \nabla_{ad}) \simeq 1$.

T_{eff} ↓↓ → depth of CZ increases → $0.25M_\odot$ stars are fully convective.

The (zero-age) Main Sequence: convective regions

convection if $\nabla_{rad} > \nabla_{ad}$



For massive stars ($50M_\odot$), increasing P_{rad} decreases $\nabla_{ad} \simeq 0.27$ → contribute to conv. core.

stars with low T_{eff} (low M, see fig.) develop surface CZ because of the increasing opacity (increasing ∇_{rad}). ∇_{rad} increases steeply to > 10^5.

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The Eddington luminosity L_E

is the maximum luminosity a star can achieve when there is balance between the force of radiation acting outwards and the gravitational force acting inwards (hydrostatic equilibrium).

For $L > L_E$ the star will initiate a very intensive radiation-driven stellar wind from its outer layers (the star becomes unbound).

$$P_{\rm rad} = \frac{a}{3}T^4 \quad \rightarrow \quad \frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}r} = \underbrace{\frac{4a}{3}T^3\frac{\mathrm{d}T}{\mathrm{d}r}}$$

from diffusion approximation to radiative transfer: $-\rho\frac{\kappa F_{\rm rad}}{c} = -\rho\frac{\kappa L_{\rm r}}{4\pi r^2 c}$

If total pressure is fully determined by $P_{\rm rad}$, hydrostatic equilibrium @R is given for

$$g = \frac{GM}{R^2} = -\frac{1}{\rho}\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}r}\Big|_{r=R} = \frac{\kappa L_{\rm r}}{4\pi R^2 c} \;:=\; \frac{\kappa L_{\rm E}}{4\pi R^2 c}$$

$$\longrightarrow \quad L_{\rm E} = \frac{4\pi c G M}{\kappa}\,.$$

With $\kappa \simeq \kappa_{\rm scatt} \simeq 0.20(1+X)$ and X=0.70

$$\frac{L_{\rm E}}{\mathrm{L}_\odot} \simeq 3.824 \times 10^4 \frac{M}{\mathrm{M}_\odot}\,.$$