Opacity κ

(Gray) **Diffusion approximation to radiative transport**:
(assumes isotropy → valid only in the deep stellar interior)

$$F = -k_{\rm rad}\nabla T = -\frac{4ac}{3}\frac{T^3}{\kappa\varrho}\frac{\partial T}{\partial r}$$

- opacity is a function of frequency ν (wave length λ).
- aim: to reduce the (rather complex) nongray problem of radiative transfer to the the gray problem.
- we need an appropriate mean opacity (averaged over frequency) for solving the gray radiative transport equation, such as the diffusion approximation to radiative transport.
- There are various ways for taking such a mean, non of which are correct for all requirements.

Opacity κ

(1) The Rosseland mean opacity (valid only in the deep stellar interior – isotropy!):

requirement:

$$F = \int_0^\infty F_\nu\,\mathrm{d}\nu = -\frac{4\pi}{3\rho}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}r}\mathrm{d}\nu = -\frac{4\pi}{3\rho}\frac{\mathrm{d}T}{\mathrm{d}r}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}T}\mathrm{d}\nu$$

$$F = -\frac{4acT^3}{3\rho\kappa_{\rm R}}\frac{\mathrm{d}T}{\mathrm{d}r} \longrightarrow \frac{1}{\kappa_{\rm R}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}T}\mathrm{d}\nu}{\int_0^\infty \frac{\mathrm{d}B_\nu}{\mathrm{d}T}\mathrm{d}\nu} = \frac{\pi}{acT^3}\int_0^\infty \frac{1}{\kappa_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}T}\mathrm{d}\nu$$

(2) The Planck mean opacity (most physically relevant mean near surface):

requirement (aim): radiative equilibrium: $\int_0^\infty (\kappa_\nu - \kappa_{\rm P})(J_\nu - B_\nu)\mathrm{d}\nu = 0$

near surface: $(J_\nu - B_\nu) \simeq -B_\nu/2 \longrightarrow \kappa_{\rm P} = \frac{\int_0^\infty \kappa_\nu B_\nu \mathrm{d}\nu}{\int_0^\infty B_\nu \mathrm{d}\nu}$

Opacity κ

Basic physical processes that contribute to opacity

(1) Electron scattering (Thomson scattering)

(2) Absorption due to free-free transitions (Bremsstrahlung)

(3) Absorption due to bound-free transitions (Photoionization)

(4) Due to bound-bound transitions (spectral-line absorption)

(5) The negative hydrogen ion (H^-)

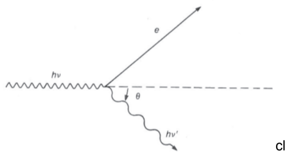
(6) Electron conduction

(7) Molecular opacities

Opacity κ

(1) Electron scattering (Thomson scattering)

- Electromagnetic (EM) wave (light) makes e^- to oscillate.
- Oscillating e^- behaves like a dipole radiating in other direction → scattering.



classical e^- radius

cross section/ν/gramm

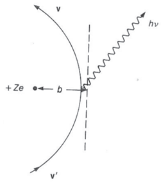
Weakening of 'original' EM wave (classically): $\kappa_\nu = \frac{8\pi}{3}\frac{r_{\rm e}^2}{\mu_{\rm e}m_{\rm u}} = 0.20\,(1+X)$
(for full ionization and non-degeneracy)

- For electron velocity $v \sim 0.1c$ the momentum exchange between photon and e^- becomes important → Compton scattering ($T > 10^8$ K).

Opacity κ

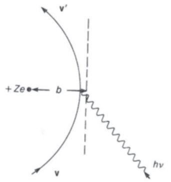
(2) Absorption due to free-free transitions (Bremsstrahlung)

- A free e$^-$ in the field of an atom or ion can:
  (a) emit a photon (bremsstrahlung) (b) absorb a photon (inverse bremsstrahlung)



Energy conservation:
$$\frac{1}{2}mv'^2 = h\nu + \frac{1}{2}mv^2$$

Most energy absorbed during time:
$t \sim b/v \propto T^{-1/2}$



Classical approach (Kramers absorption coeff. a_ν):

$$\underset{\text{charge nr. of ion}}{a_\nu \sim Z^2\nu^{-3}T^{-1/2}} \rightarrow \kappa_\nu \sim Z^2\rho T^{-1/2}\nu^{-3}$$

Rosseland mean for fully ionized mixture (ν^α in κ_ν $\rightarrow$ T^α in κ_R):

$$\kappa_\mathrm{ff} = 3.8\times10^{22}(1+X)[(X+Y)+B]\varrho T^{-7/2} \quad ; \; B = \sum_i \overset{\text{mass fraction of heavy elements}}{\frac{X_i Z_i^2}{A_i}}$$

Opacity κ

(3) Absorption due to bound-free transitions (Photoionization)

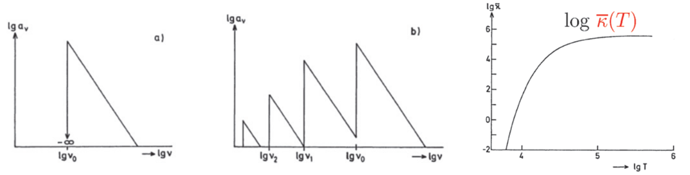
Neutral H atom in ground state can be ionized if: $h\nu > \chi_0 \quad (= 13.6\,\mathrm{eV})$

Energy conservation: $h\nu = \chi_0 + \frac{1}{2}m_\mathrm{e}v^2$ v ... released e$^-$ velocity

Absorption coefficient per ion: $\boxed{a_\nu} \equiv \kappa_\nu\rho/n_\mathrm{ion} = 0$ for $\nu < \chi_0/h$

$\boxed{\sim \nu^{-3}}$ for $\nu > \chi_0/h$ (Kramers)

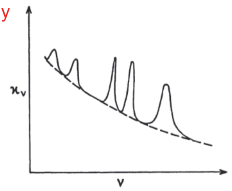
Major contribution from neutral H atom: $\kappa_\mathrm{bf} = X(1-x)\overline{\kappa}(T)$ x... degree of ionization



Opacity κ

(4) Due to bound-bound transitions (spectral-line absorption)

- energy absorption by an e$^-$ makes e$^-$ jump to higher bound state; the so-gained energy will later be re-emitted in an arbitrary direction.
- bound-bound opacities can only be computed numerically, since it involves the summation of *millions of individual absorption lines*.
- absorption lines in stars are strongly broadened by collisions.
- bound-bound absorption major contribution to total opacity for $T < 10^6$ K (can increase κ by up to a factor 2).
- For $T > 10^7$ K contribution is small (~ 10%).



Opacity κ

(5) The negative hydrogen ion (H$^-$)

- A free e$^-$ passing a neutral H atom can produce a dipole moment in the atom; the atom attracts the e$^-$ and can bind it to the 'system', forming an H$^-$ ion.
- ion very fragile – has an ionization energy of only 0.754 eV (compared to 13.6 eV) and a large cross section of absorption, either through
  (a) **bound-free absorption**
$$H^- + h\nu \rightleftharpoons H + e^-(v)$$
  (b) **free-free absorption**
$$H \quad + e^-(v) + h\nu \rightleftharpoons H \quad + e^-(v') \quad .$$

Using Saha's equation to estimate n_{-1} with χ = 0.754 eV, u$_{-1}$=1, u$_0$=2 and $n_0 = (1-x)\rho X/m_\mathrm{u}$, we find with $\kappa_\nu \; = \; a_\nu n_{-1}/\varrho$

Rosseland mean: $\kappa_{\mathrm{H}^-} = \frac{1}{4}\frac{h^3}{(2\pi m_\mathrm{e})^{3/2}(kT)^{5/2}m_\mathrm{u}}P_\mathrm{e}(1-x)X\,a(T)\mathrm{e}^{\chi/kT} \quad \propto n_\mathrm{e} \propto X$

Does not follow Kramer's opacity law, but (very roughly):

$$\kappa_{\mathrm{H}^-} \sim 2.5\times10^{-31}(Z/0.02)\,\rho^{1/2}T^9\ \mathrm{cm}^2\ \mathrm{g}^{-1} \qquad \propto Z\ (\mathrm{metals})$$

Opacity κ

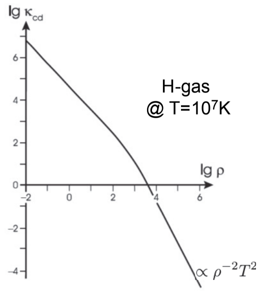
(6) Electron conduction

- e^- conduction important in dense, **degenerate regions** (interior of evolved stars).

- because all quantum cells (energy states) are occupied up to p_F (E_F), free e^- will not exchange momentum with ions, thereby increasing their free mean path.

- e^- conduction $\kappa_{\rm cd}$ is added to Rosseland mean $\kappa_{\rm R}$ as a ‘transparancy’, i.e.:

$$\frac{1}{\kappa} = \frac{1}{\kappa_{\rm R}} + \frac{1}{\kappa_{\rm cd}}\,.$$



H-gas
@ T=10^7K

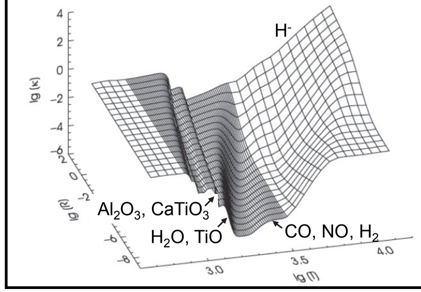
Opacity κ

(7) Molecular opacities

- Formation of molecules become increasingly (very) important for T< 10000K (<5000K).

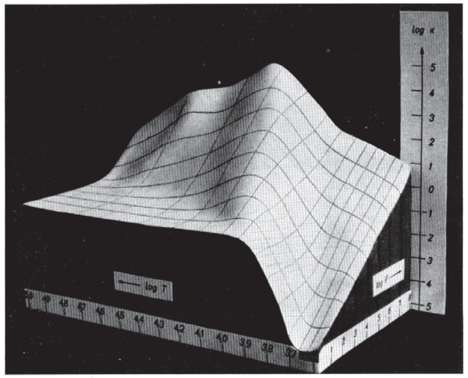
- because of their rich system of energy levels (rotational, vibrational excitation) they are important absorbers.

- mean opacity depends primarily on absorption properties and less on abundance: e.g. Ti is 3-orders of magnitude less abundances as O and yet TiO dominates, along with H_2O molecules, the opacity at low T. If C dominates over O, C_2, CN or C_2H_2 dominate.



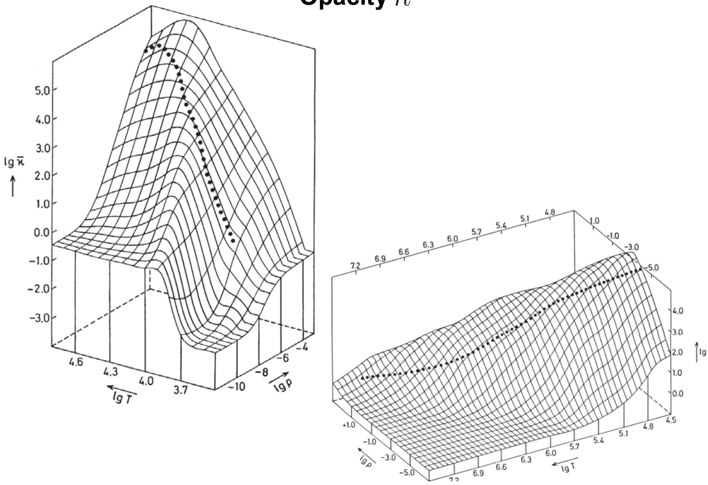
Note: even in cool stars with surface convection zones is the opacity important, because very outer convective layers are superadiabatic, i.e. convection is inefficient and radiation starts to become important again.

Opacity κ

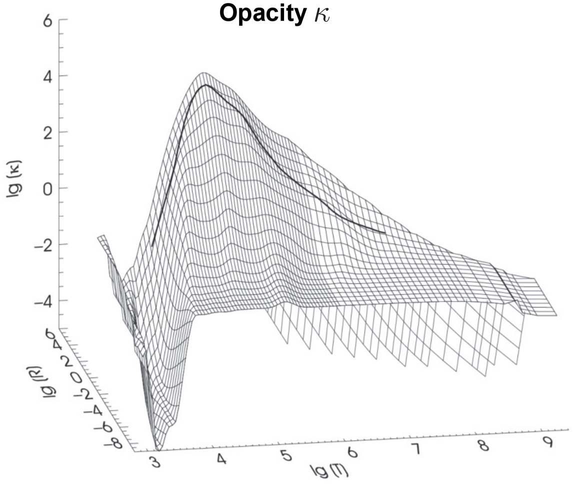


N. Baker & R. Kippenhahn: *Z. Astroph.* **54**, 114 - 151 (1962)

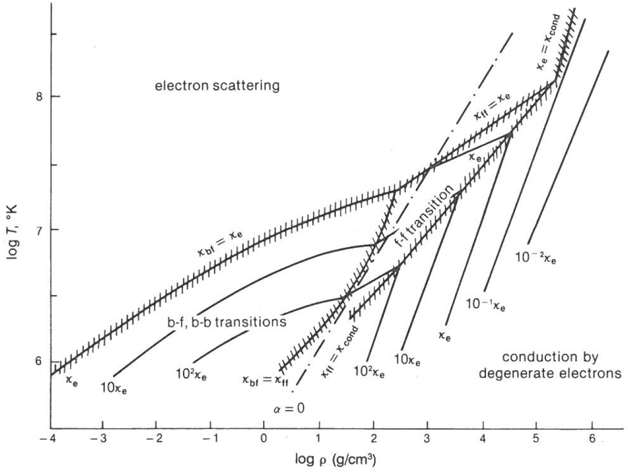
Opacity κ



Opacity κ

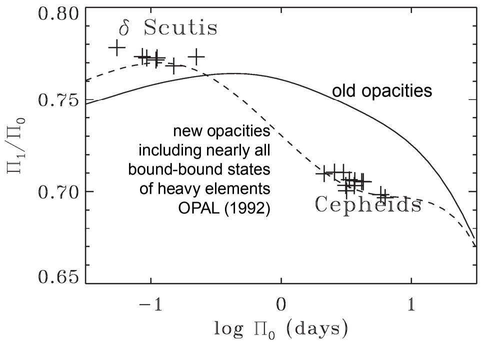


Opacity κ



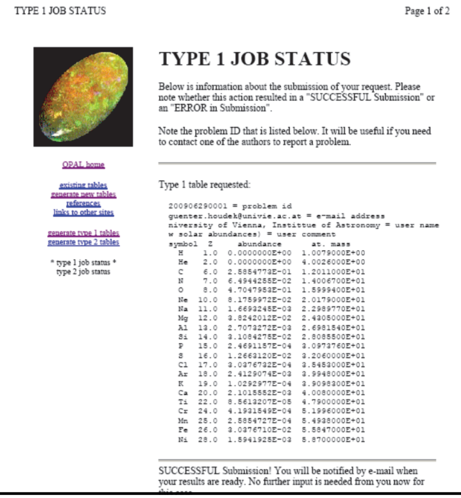
Opacity κ

How stellar evolution and pulsation can improve macrophysics



Christensen-Dalsgaard, Peterson (1992)

Opacity κ



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Page 1 of 2

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type 2 job status

TYPE 1 JOB STATUS

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guenter.houdek@univie.ac.at = e-mail address
niversity of Vienna, Instittue of Astronomy = user name
w solar abundances) = user comment
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  He   2.0  0.0000000E+00  4.0026000E+00
  C    6.0  2.5854773E-01  1.2011000E+01
  N    7.0  6.4944255E-02  1.4006700E+01
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  S   16.0  1.2663120E-02  3.2060000E+01
  Cl  17.0  3.0376732E-04  3.5453000E+01
  Ar  18.0  2.4129074E-03  3.9948000E+01
  K   19.0  1.0292977E-04  3.9098300E+01
  Ca  20.0  2.1015552E-03  4.0080000E+01
  Ti  22.0  8.5613207E-05  4.7900000E+01
  Cr  24.0  4.1931549E-04  5.1996000E+01
  Mn  25.0  2.5854727E-04  5.4938000E+01
  Fe  26.0  3.0376710E-02  5.5847000E+01
  Ni  28.0  1.5941925E-03  5.8700000E+01
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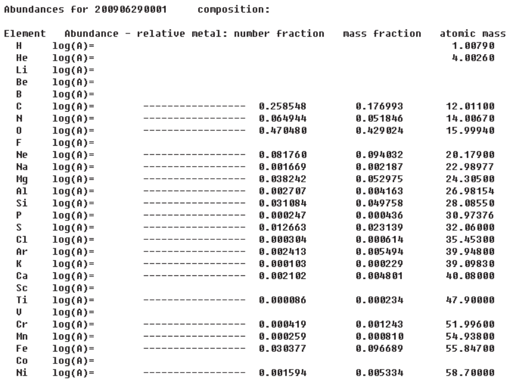
Opacity κ

Solar abundance of heavy elements

Asplund et al. (2009) solar composition $\longrightarrow$ $Z_{sun} = 0.0134$



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Abundances for 200906290001     composition:

Element   Abundance - relative metal: number fraction   mass fraction   atomic mass
  H     log(A)=                                                          1.00790
  He    log(A)=                                                          4.00260
  Li    log(A)=
  Be    log(A)=
  B     log(A)=
  C     log(A)=        -----------------  0.258548       0.176993       12.01100
  N     log(A)=        -----------------  0.064944       0.051846       14.00670
  O     log(A)=        -----------------  0.470480       0.429024       15.99940
  F     log(A)=
  Ne    log(A)=        -----------------  0.081760       0.094032       20.17900
  Na    log(A)=        -----------------  0.001669       0.002187       22.98977
  Mg    log(A)=        -----------------  0.038242       0.052975       24.30500
  Al    log(A)=        -----------------  0.002707       0.004163       26.98154
  Si    log(A)=        -----------------  0.031084       0.049758       28.08550
  P     log(A)=        -----------------  0.000247       0.000436       30.97376
  S     log(A)=        -----------------  0.012663       0.023139       32.06000
  Cl    log(A)=        -----------------  0.000304       0.000614       35.45300
  Ar    log(A)=        -----------------  0.002413       0.005494       39.94800
  K     log(A)=        -----------------  0.000103       0.000229       39.09830
  Ca    log(A)=        -----------------  0.002102       0.004801       40.08000
  Sc    log(A)=
  Ti    log(A)=        -----------------  0.000086       0.000234       47.90000
  V     log(A)=
  Cr    log(A)=        -----------------  0.000419       0.001243       51.99600
  Mn    log(A)=        -----------------  0.000259       0.000810       54.93800
  Fe    log(A)=        -----------------  0.030377       0.096689       55.84700
  Co    log(A)=
  Ni    log(A)=        -----------------  0.001594       0.005334       58.70000
```

Opacity κ

Solar abundance of heavy elements

Asplund et al. (2009) solar composition $\longrightarrow$ $Z_{sun} = 0.0134$

		Z_s	Z_s/X_s
revised:	Caffau et al. (2009)	0.0156 ± 0.0011	0.0213
	Houdek & Gough (2011)	0.0142 ± 0.0005	0.0186
	Asplund et al. (2009)	0.0134	0.0181
old:	Grevesse & Noels (1993)	0.0179	0.0244
	Grevesse & Sauval (1998)	0.0169	0.0231

Different values for chemical composition and therefore *Z* & opacity have severe consequences on stellar models

Opacity κ

Solar abundance of heavy elements

(Sound speed)^2 difference
Sun - model

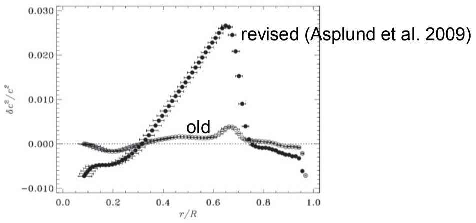

Fig. 1. Inferred relative differences in squared sound speed, in the sense (Sun) − (model), from inversion of the "Best Set" of observed frequencies of Basu et al. (1997). The open circles show results for Model S of Christensen-Dalsgaard et al. (1996), using the old solar composition, while the filled circles show results for a corresponding model, assuming the revised composition. The horizontal bars indicate the resolution of the inversion, while the vertical bars (hardly visible on this scale) show the 1-σ errors in the inferences.

Christensen-Dalsgaard et al. (2010)

Opacity κ

Solar abundance of heavy elements

Required opacity change when
using revised abundances

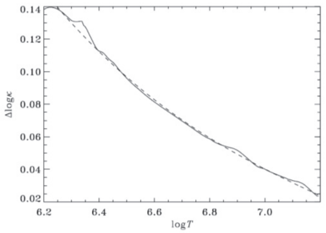

Fig. 2. The solid curve shows the intrinsic opacity change $\Delta \log \kappa = f(\log T)$ determined from Eq. (4), on the basis of models with the old (Model S) and the revised (Model Asp) composition. The dashed curve shows the cubic fit $f_{\rm approx}$ to $\Delta \log \kappa$, given by Eq. (5).

Christensen-Dalsgaard et al. (2010)

Opacity κ

4D - Interpolation
in (ρ, T, X, Z)