**Exercises**

1. (Higgs mechanism)

   (a) Show that excitations of the Higgs field around zero are tachyons – particles with imaginary mass.

   (b) For our $U(1)$ Higgs model calculate all the terms which in the lecture note are denoted as the "interaction terms" and interpret them. Check that I haven't forgotten any mass terms.

2. (Standard model)

   (a) Show that the fields $A$, $Z$, and $W^{\pm}$ are eigenstates of the charge operator $\hat{Q}$ and find the corresponding eigenvalues.

   Hints:

   i. **How to define an abstract operator $\hat{I}$ which represents an infinitesimal generator $I$?**
   The action of an infinitesimal group element $1 + iI\alpha$, acting on the relevant object $\varphi$, is by definition

   $$\varphi \to (1 + iI\delta\alpha)\varphi \equiv \varphi + \delta\varphi \,,$$

   where $\delta$ denotes group covariant differential

   $$\delta\varphi = iI\delta\alpha\varphi \,.$$

   Analogously, the action of an abstract operator $\hat{I}$, representing the generator $I$ in the space of the object $\Phi$ (which the operator $\hat{I}$ acts upon), can be defined through the covariant differential $\delta\Phi$ of the object under the infinitesimal group transformation,

   $$\hat{I}\Phi = \frac{1}{i}\left.\frac{\delta\Phi}{\delta\alpha}\right|_{\alpha=0} \,.$$

   ii. **Action of the abstract operator $\hat{Y}_W$ on the gauge fields $B$ and $W$:**
   Under the gauge transformation generated by the week hyper-charge $U(1)_W$ the covariant part of the transformation of the fields $B$ and $W$ is zero, thus

   $$\frac{\hat{Y}_W}{2}B = \frac{\hat{Y}_W}{2}W = 0 \,.$$

   The fields $B$ and $W$ are thus eigenstates of $\hat{Y}_W$ with the eigenvalue 0.

   iii. **How the abstract operator $\hat{T}_3$ acts on the gauge fields $B$ and $W$:**
   Under the infinitesimal $SU(2)_L$ gauge transformation,

   $$\delta B = 0 \,,\ \delta W = i[\alpha, W] \,,$$

   where $\alpha = \alpha_j T_j$, $W = W_j T_j$, $T_j = \frac{1}{2}\tau_j$. Thus

   $$\hat{T}_3 B = 0 \,,$$

   $$\hat{T}_3 W = [T_3, W]$$

   Then, apparently,

   $$\hat{T}_3 W_3 = 0 \,,$$

   $$\hat{T}_3 W^{\pm} = \pm W^{\pm} \,,$$

   where

   $$W^{\pm} = \frac{1}{\sqrt{2}}\left(W_1 \mp iW_2\right) \,.$$

   iv. **How the abstract operator $\hat{Q}$ acts on the gauge fields $A$, $Z$ and $W^{\pm}$:**
   From the definition $\hat{Q} = \frac{1}{2}\hat{Y}_W + \hat{T}_3$ it immediately follows that

   $$\hat{Q}A = 0 \,,$$

   $$\hat{Q}Z = 0 \,,$$

   $$\hat{Q}W^{\pm} = \pm W^{\pm} \,.$$