Exercises

1. Show that the Lagrangians

$$\mathcal{L} = -\frac{1}{8\pi}\partial_a A^b \partial^a A_b$$

and

$$\mathcal{L} = -\frac{1}{16\pi}F^{ab}F_{ab}$$

are equivalent under the Lorenz condition $\partial_a A^a = 0$.

Hint: the difference is a full derivative, which does not contribute to the variation of the action.

2. Show that the Lagrangian

$$\mathcal{L} = -\frac{1}{8\pi}\partial_a A^b \partial^a A_b - j^a A_a$$

is gauge invariant if j^a is a conserved current (that is, $\partial_a j^a = 0$).

3. Consider a scalar field with the vacuum $|0\rangle$[1].

Argue, that the state $a^\dagger_{\mathbf{k}}|0\rangle$ describes a particle with momentum $\mathbf{k}$.

Argue, that the state $b^\dagger_{\mathbf{k}}|0\rangle$ describes an anti-particle with momentum $\mathbf{k}$.

4. Argue that a state with two particles $a^\dagger_{\mathbf{k}} a^\dagger_{\mathbf{k}'}|0\rangle$ is symmetric (antisymmetric) for bosons (fermions) under the exchange of the particles.

[1] vacuum is the state with the lowest energy and no particles: $\hat{n}_{\mathbf{k}}|0\rangle = 0$, where $\hat{n}_{\mathbf{k}}$ is the operator for the number of particles in the state with momentum $\mathbf{k}$.