Higgs mechanism

The *Higgs mechanism* enables the massless gauge bosons in a gauge theory to acquire effective mass through interaction with auxiliary fields (called the Higgs fields). The mechanism is based on the phenomenon of spontaneous symmetry breaking.

In the standard model the Higgs mechanism generates the masses of the $W^{\pm}$ and Z weak gauge bosons.

Spontaneous symmetry breaking

Spontaneous symmetry breaking takes place when the Lagrangian of a system is symmetric while the ground state isn't.

Here is the canonical example: a complex self-interacting scalar (Higgs) field φ with a globally $U(1)$ symmetric Lagrangian,

$$\mathcal{L} = \partial_a \varphi^\star \partial^a \varphi - V_{\mathrm{MH}}(\varphi^\star \varphi) \tag{1}$$

where the potential energy V_{MH} has the "Mexican hat" form (see fig. 1),

$$V_{\mathrm{MH}}(\varphi^\star \varphi) = \frac{\lambda^2}{4}\left(\varphi^\star \varphi - m^2\right)^2 , \tag{2}$$

where λ is a dimensionless constant and m is a constant with the dimension of mass[1].

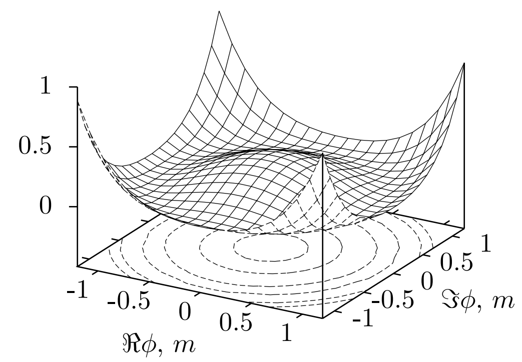

Figure 1: "Mexican hat" potential (2) in arbitrary units as function of the real, $\Re\varphi$, and imaginary, $\Im\varphi$, parts of the complex scalar field φ.

The lowest energy state of the system is reached for a finite (non-zero) field φ_0 (often called Higgs condensate or vacuum expectation value) such that $\varphi_0^\star \varphi_0 = m^2$. Due to the symmetry of the Lagrangian there are infinitely many vacuum states $\varphi_0 = me^{i\xi}$, where $0 \le \xi < 2\pi$.

[1]hypothetical particles with negative square mass are called *tachyons*.

Breaking the symmetry means choosing a particular vacuum state, e.g. $\varphi_0 = m$. Let us look at small fluctuations around this vacuum state in the form

$$\varphi = (m + \eta)e^{i\theta/m} . \tag{3}$$

In terms of the new fields η and θ the Lagrangian becomes (up to the second order terms)

$$\mathcal{L} = \partial_a \eta \partial^a \eta - \lambda^2 m^2 \eta^2 + \partial_a \theta \partial^a \theta. \tag{4}$$

This Lagrangian apparently describes a massless boson θ (called *Goldstone boson*) and a massive boson η (to be called Higgs boson in the standard model) with the (positive) mass $m_H = \lambda m$.

This simple model actually illustrates the *Goldstone theorem* which states that whenever a continuous symmetry is spontaneously broken, a massless (or light, if the symmetry was not exact) scalar particle appears in the spectrum of the system. It corresponds to excitations along the "symmetry direction".

Higgs mechanism

Higgs mechanism is spontaneous symmetry breaking in a gauge theory.

Let us make our model (1) gauge invariant by coupling our Higgs field φ to a gauge field B_a,

$$\mathcal{L} = D_a^\star \varphi^\star D^a \varphi - V_{\mathrm{MH}}(\varphi^\star \varphi) + \mathcal{L}_{\mathrm{YM}} , \tag{5}$$

where $D_a = \partial_a + igB_a$ is the gauge-group covariant derivative, and $\mathcal{L}_{\mathrm{YM}}$ is the Yang-Mills Lagrangian for the free gauge field B_a.

Again let us look for small fluctuations of the Higgs field from the condensate, $\varphi = (m+\eta)e^{i\theta/m}$.

The easiest way to interpret the resulting Lagrangian is by fixing the gauge:

$$\varphi \rightarrow e^{-i\theta/m}\varphi \tag{6}$$

$$B_a \rightarrow B_a + \frac{1}{gm}\partial_a \theta . \tag{7}$$

Up to second order terms in the new fields the Lagrangian becomes

$$\mathcal{L} = \partial_a \eta \partial^a \eta - \lambda^2 m^2 \eta^2 + L_{\mathrm{YM}} + g^2 m^2 B_a B^a + (\text{interaction terms}) , \tag{8}$$

and describes a massive scalar Higgs boson η and a massive vector boson B_a (with mass $gm = g\varphi_0$).

Before the symmetry breaking there were 4 degrees of freedom: two in the complex Higgs field φ and two in the real massless (and hence transverse) gauge field B_a.

After the symmetry breaking there appeared a massive real scalar boson η (Higgs boson); the gauge boson acquired a finite mass gm and therefore lost its transverse character; the θ field has turned into the longitudinal polarization component of the gauge field and disappeared from the Lagrangian: in total again 4 degrees of freedom.

The standard model

The standard model is a Yang-Mills field theory with the symmetry $U(1) \times SU(2) \times SU(3)$. It consists of the electroweak sector[2], $U(1) \times SU(2)_L$; the strong sector, $SU(3)$, called quantum chromodynamics (QCD); and the Higgs sector.

The QCD sector

The QCD Lagrangian is an ordinary Yang-Mills Lagrangian with $SU(3)$ symmetry,

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{q}\left(i\gamma^a D_a - m_q\right) q + \mathcal{L}_{\mathrm{YM}} \tag{9}$$

where

$$\begin{aligned} D_a &= \partial_a + i g_s I^j A_a^j\,, &(10)\\ \mathcal{L}_{\mathrm{YM}} &= -\frac{1}{4} F^j_{ab} F^{ab}_j\,, &(11)\\ F^j_{ab} &= \partial_a A_b - \partial_b A_a - g_s f^j_{kl} A^k_a A^l_b\,, &(12)\end{aligned}$$

where $q \in \{u, d, s, c, t, b\}$ is the quark field of specific flavour (Lorentz bispinor and color triplet); I^j are the generators of the $SU(3)$ group, usually represented as $\frac{1}{2}\lambda^j$ where λ^j are Gell-Mann matrices; f_{jkl} are the structure constants of the $SU(3)$ group; and g_s is the QCD coupling constant.

Electroweak sector

The electroweak sector is a "weakly mixed" $U(1) \times SU(2)_L$,

$$\mathcal{L}_{\mathrm{EW}} = \sum_\psi \bar{\psi}\gamma^a\left(i\partial_a - g'\frac{Y_W}{2}B_a - g\vec{T}_L\vec{W}_a\right)\psi + \mathcal{L}_{\mathrm{YM}}(B_a) + \mathcal{L}_{\mathrm{YM}}(\vec{W}_a), \tag{13}$$

where B_a is the $U(1)$ gauge field; $\vec{W}_a$ is the three-component $SU(2)$ gauge field; g' and g are coupling constants; Y_W is the *weak hyper-charge* – the generator of the $U(1)$ group; $\vec{T} = \frac{1}{2}\vec{\tau}$ (where $\vec{\tau}$ are the Pauli matrices in the weak isospin $SU(2)$ space) are the infinitesimal generators of the $SU(2)$ group, subscript $_L$ indicates that they only act on the left fermions ψ_L, which are weak doublets while the right fermions ψ_R are weak singlets. For example, for the leptons of the first generation,

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},\ \psi_R = e_R\,. \tag{14}$$

The second singlet, the right neutrino, if exists, is "sterile" (if massless) as it does not interact at all. For the quarks of the first generation,

$$\psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ \psi_R = u_R,\ d_R\,. \tag{15}$$

The weak hypercharge Y_W is defined such that the electric charge Q is equal

$$Q = (T_L)_3 + \frac{1}{2}Y_W\,. \tag{16}$$

The gauge field that interacts with the third component of the weak isospin τ_3 but does not interact with the electric charge Q is called Z-boson,

$$Z_a = \frac{gW_a^{(3)} - g'B_a}{\sqrt{g^2 + g'^2}}\,. \tag{17}$$

The orthogonal combination, which interacts with Q but not with τ_3, becomes the electromagnetic field A_a,

$$A_a = \frac{g'W_a^{(3)} + gB_a}{\sqrt{g^2 + g'^2}}\,. \tag{18}$$

Introducing the *weak mixing angle*[3], $\tan\theta_W = g'/g$, the transformation can be conveniently written as an orthogonal transformation,

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} B \\ W^3 \end{pmatrix}, \tag{19}$$

which apparently preserves the "kinetic energy" terms in the Lagrangian.

With the notations $e = g\sin\theta_W$, $W^\pm = \frac{1}{\sqrt{2}}\left(W^1 \mp iW^2\right)$, $T^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2)$ the fermionic part of the Lagrangian takes the standard form,

$$\begin{aligned}\mathcal{L}^{(\mathrm{F})}_{\mathrm{EW}} &= \sum_\psi \bar{\psi}\gamma^a i\partial_a\psi - e\sum_\psi \bar{\psi}\gamma^a Q\psi A_a \\ &- \frac{g}{\cos\theta_W}\sum_\psi \bar{\psi}\gamma^a\left(\frac{1}{2}\tau^3_L - Q\sin^2\theta_W\right)\psi Z_a \\ &- \frac{g}{2\sqrt{2}}\sum_\psi \bar{\psi}\gamma^a\left(T^+_L W^+ + T^-_L W^-\right)\psi\,. \end{aligned} \tag{20}$$

Higgs sector

The Higgs field must provide masses to three gauge fields – hence, together with the Higgs boson, there

[2] the subscript $_L$ means that only left bispinors take part in the interaction.

[3] also called Weinberg angle.

should be minimum 4 components in the Higgs field. In the standard model it is a complex spinor of the group $SU(2)_L$,

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} , \qquad (21)$$

where the indexes $^+$ and ^{0} indicate the Q-charges of the components; the Y_W-charge of both components is equal 1.

The Higgs sector Lagrangian is

$$\mathcal{L}_H = \varphi^\dagger \left(\overleftarrow{\partial}_a - ig'\frac{1}{2}Y_W B_a - ig\frac{1}{2}\vec{\tau}\vec{W}_a \right) \cdot \\ \cdot \left(\overrightarrow{\partial}_a + ig'\frac{1}{2}Y_W B_a + ig\frac{1}{2}\vec{\tau}\vec{W}_a \right) \varphi \\ -\frac{\lambda^2}{4}\left(\varphi^\dagger\varphi - v^2\right)^2 \qquad (22)$$

After the spontaneous symmetry breaking the fields Z and $W^\pm$ get masses, $m_w^2 = \frac{1}{4}v^2g^2$, $m_Z^2 \cos^2\theta_w = m_w^2$, while the field A remains massless.

Exercises

Show that the fields A, Z, and $W^\pm$ are eigenstates of the charge operator $\hat{Q}$ and find the corresponding eigenvalues.

Hints:

1. **How to define an abstract operator $\hat{I}$ which represents an infinitesimal generator I?**

   The action of an infinitesimal group element $0 + iI\alpha$, acting on the relevant object φ, is by definition

   $$\varphi \to (1 + iI\delta\alpha)\varphi \equiv \varphi + \delta\varphi ,$$

   where δ denotes group covariant differential

   $$\delta\varphi = iI\delta\alpha\varphi .$$

   Analogously, the action of an abstract operator $\hat{I}$, representing the generator I in the space of the object Φ (which the operator $\hat{I}$ acts upon), can be defined through the covariant differential $\delta\Phi$ of the object under the infinitesimal group transformation,

   $$\hat{I}\Phi = \frac{1}{i}\left.\frac{\delta\Phi}{\delta\alpha}\right|_{\alpha=0} .$$

2. **Action of the abstract operator $\hat{Y}_W$ on the gauge fields B and W:**

   Under the gauge transformation generated by the week hyper-charge $U(1)_W$ the covariant part of the transformation of the fields B and W is zero, thus

   $$\frac{\hat{Y}_W}{2}B = \frac{\hat{Y}_W}{2}W = 0 .$$

   The fields B and W are thus eigenstates of $\hat{Y}_W$ with the eigenvalue 0.

3. **How the abstract operator $\hat{T}_3$ acts on the gauge fields B and W:**

   Under the infinitesimal $SU(2)_L$ gauge transformation,

   $$\delta B = 0 \ , \ \delta W = i[\alpha, W] ,$$

   where $\alpha = \alpha_j T_j$, $W = W_j T_j$, $T_j = \frac{1}{2}\tau_j$. Thus

   $$\hat{T}_3 B = 0 ,$$

   $$\hat{T}_3 W = [T_3, W]$$

   Then, apparently,

   $$\hat{T}_3 W_3 = 0 ,$$

   $$\hat{T}_3 W^\pm = \pm W^\pm ,$$

   where

   $$W^\pm = \frac{1}{\sqrt{2}}\left(W_1 \mp iW_2\right) .$$

4. **How the abstract operator $\hat{Q}$ acts on the gauge fields A, Z and $W^\pm$:**

   From the definition $\hat{Q} = \frac{1}{2}\hat{Y}_W + \hat{T}_3$ it immediately follows that

   $$\hat{Q}A = 0 ,$$

   $$\hat{Q}Z = 0 ,$$

   $$\hat{Q}W^\pm = \pm W^\pm .$$