Exercises

1. Find the finite rotation matrix for a 2×2 representation of the rotation group[1].

2. Find the additive parameter φ of the velocity boost matrix (along the x-axis) and the corresponding generator[2].

3. Using the additive parameter $\varphi = \frac{1}{2}\ln\frac{1+v}{1-v}$ find the velocity-boost matrices for the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the Lorentz group.

4. (Extra) Prove that generators of a Lie group have a Lie algebra

$$[I_l, I_m] \equiv I_l I_m - I_m I_l = iC^k_{lm} I_k \ ,$$

where C^k_{lm} are some (structure) constants[3].

[1] Hint: $\vec{I} = \frac{1}{2}\vec{\sigma}$, and $(\vec{\sigma}\vec{n})^2 = 1$, where $\vec{\sigma}$ are Pauli matrices.

[2] Hint: $v = \tanh\varphi$

[3] Hint: consider a group element

$$g(\beta') = g(\alpha)g(\beta)g^{-1}(\alpha)$$

in the limit $\beta \to 0$ and $\alpha \to 0$, namely:

$$\lim_{\beta\to 0}\beta'_k = f_{kl}(\alpha)\beta_l \ ,$$

where $f_{kl}(\alpha)$ are some functions with

$$\lim_{\alpha\to 0} f_{kl}(\alpha) = \delta_{kl} + C^k_{lm}\alpha_m \ .$$