Spontaneous symmetry breaking and the Higgs mechanism

In the standard model the Higgs mechanism is the way the massless gauge bosons acquire effective mass through coupling to an auxiliary field, called Higgs field. The mechanism is based on spontaneous symmetry breaking.

Spontaneous symmetry breaking

Spontaneous symmetry breaking takes place when the system's Lagrangian has certain symmetry while the ground state (the vacuum) does not.

Here is a simple model: a complex self-interacting scalar field ϕ with the $U(1)$ symmetric Lagrangian

$$\mathcal{L} = \partial_\mu\phi^\star\partial^\mu\phi - V(\phi^\star\phi) \tag{1}$$

where the potential energy at the vicinity of the minimum has the form

$$V = \frac{\lambda^2}{4}\left(\phi^*\phi - m^2\right)^2 , \tag{2}$$

where $\frac{\lambda^2}{4}$ (usually denoted as $\frac{\lambda}{2}$) is a dimensionless constant and m (usually denoted as v) is a constant with the dimension of mass[1].

The ground state of the system is reached for a finite (non-zero) field ϕ_0 (often called Higgs condensate or vacuum expectation value) such that $\phi_0^\star\phi_0 = m^2$. Due to the symmetry of the Lagrangian there are infinitely many vacuum states $\phi_0 = me^{i\varphi}$, $0 \le \varphi < 2\pi$.

Breaking the symmetry means choosing a particular vacuum state, e.g. $\phi_0 = m$. Let us look at small fluctuations,

$$\phi = (m+\eta)e^{i\theta/m} , \tag{3}$$

around this vacuum state. In terms of the new fields η and θ the Lagrangian becomes (up to the second order terms)

$$\mathcal{L} = \partial_\mu\eta\partial^\mu\eta - \lambda^2m^2\eta^2 + \partial_\mu\theta\partial^\mu\theta. \tag{4}$$

This Lagrangian apparently describes a massless boson θ (called Goldstone boson) and a massive boson η (to be called Higgs boson in the standard model) with the mass $m_H = \lambda m$.

There is a theorem, called Goldstone theorem, which states that whenever a continuous symmetry is spontaneously broken, a new massless (or light, if the symmetry was not exact) scalar particle appears in the spectrum of the system. It corresponds to excitations along the "symmetry direction".

[1] Hypothetical particles with negative square mass are called "tachyons".

Higgs mechanism

Higgs mechanism is spontaneous symmetry breaking in a gauge field theory.

Let us couple our scalar field ϕ to a gauge field B_μ,

$$\mathcal{L} = D_\mu^\star\phi^\star D^\mu\phi - V(\phi^\star\phi) + L_{YM}, \tag{5}$$

where $D_\mu = \partial_\mu + igB_\mu$ is the gauge-group covariant derivative, and L_{YM} is the Yang-Mills Lagrangian for the free gauge field.

Let us look for small fluctuations of the Higgs field from the condensate, $\phi = (m+\eta)e^{i\theta/m}$.

The easiest way to interpret the resulting Lagrangian is to fix the gauge:

$$\begin{aligned}\phi &\to e^{-i\theta/m}\phi & (6)\\ B_\mu &\to B_\mu + \frac{1}{gm}\partial_\mu\theta . & (7)\end{aligned}$$

Up to second order in fields the Lagrangian becomes

$$\begin{aligned}\mathcal{L} = \partial_\mu\eta\partial^\mu\eta - \lambda^2m^2\eta^2 + L_{YM} + g^2m^2B_\mu B^\mu \\ +(\text{interaction terms}). \quad (8)\end{aligned}$$

Before the symmetry breaking there were 4 degrees of freedom: two in the complex Higgs field ϕ and two in the real massless (and hence transverse) gauge field B_μ.

After the symmetry breaking a massive real scalar boson η (Higgs boson) has appeared; the gauge boson acquired a finite mass gm; the θ field has turned into the third polarization component of the gauge field and disappeared from the Lagrangian: in total again 4 degrees of freedom.

The standard model

The standard model is a Yang-Mills field theory with the symmetry $U(1)\times SU(2)\times SU(3)$. It consists of the electroweak sector, $SU(2)_L\times U(1)$; the strong sector, $SU(3)$, called quantum chromodynamics (QCD); and the Higgs sector.

The QCD Lagrangian

The QCD sector is a usual Yang-Mills Lagrangian with $SU(3)$ symmetry,

$$\begin{aligned}\mathcal{L}_{\text{QCD}} &= \sum_q \bar{q}\left(i\gamma^\mu D_\mu - m_q\right)q - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} & (9)\\ D_\mu &= \partial_\mu + ig_sI^aA^a_\mu & (10)\\ F^a_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - g_sf_{abc}A^b_\mu A^c_\nu & (11)\end{aligned}$$

where $q \in \{u,d,s,c,t,b\}$ is the quark field of specific flavour (Lorentz bispinor and color triplet), I^a are the generators of the $SU(3)$ group usually represented as $\frac{1}{2}\lambda^a$ where λ^a are Gell-Mann matrices, f_{abc} are the structure constants of the $SU(3)$ group, g_s is the QCD coupling constant.

Electroweak sector

The electroweak sector is a $U(1) \times SU(2)_L$,

$$\begin{aligned}\mathcal{L}_{EW} &= \sum_\psi \bar{\psi}\gamma^\mu \left(i\partial_\mu - g'\frac{Y_W}{2}B_\mu - g\vec{T}_L\vec{W}_\mu \right)\psi + \\ &+\mathcal{L}_{YM}(B_\mu) + \mathcal{L}_{YM}(\vec{W}_\mu), \end{aligned} \tag{12}$$

where B_μ is the $U(1)$ gauge field; $\vec{W}_\mu$ is the three-component $SU(2)$ gauge field; g' and g are coupling constants; Y_W is the *weak hyper-charge* – the generator of the $U(1)$ group; $\vec{T} = \frac{1}{2}\vec{\tau}$ (where $\vec{\tau}$ are the Pauli matrices in the weak isospin $SU(2)$ space) are the infinitesimal generators of the $SU(2)$ group, subscript $_L$ indicates that they only act on the left fermions ψ_L, which are weak doublets while the right fermions ψ_R are weak singlets, e.g. for the leptons of the first generation,

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},\ \psi_R = e_R\ . \tag{13}$$

and for the quarks of the first generation,

$$\psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ \psi_R = u_R,\ d_R\ . \tag{14}$$

The charge Q is equal

$$Q = T_3 + \frac{1}{2}Y_W\ . \tag{15}$$

The gauge field that interacts with the third component of the weak isospin τ_3 but does not interact with the charge Q is called Z-boson,

$$Z_\mu = \frac{g}{\sqrt{g^2+g'^2}}W^3_\mu - \frac{g'}{\sqrt{g^2+g'^2}}B_\mu\ . \tag{16}$$

The orthogonal combination, which interacts with Q but not with τ_3, becomes the electromagnetic field A_μ,

$$A_\mu = \frac{g'}{\sqrt{g^2+g'^2}}W^3_\mu + \frac{g}{\sqrt{g^2+g'^2}}B_\mu\ . \tag{17}$$

Introducing the weak angle $\tan\theta_W = g'/g$ the transformation can be conveniently written as

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W^3 \end{pmatrix} \tag{18}$$

With the notations $e = g\sin\theta_W$, $W^\pm = \frac{1}{\sqrt{2}}\left(W^1 \mp iW^2\right)$, $T^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2)$ the Lagrangian takes the standard form,

$$\begin{aligned}\mathcal{L}_{EW} &= -e\sum_\psi \bar{\psi}\gamma^\mu Q\psi A_\mu \\ &-\frac{g}{\cos\theta_W}\sum_\psi \bar{\psi}\gamma^\mu\left(\frac{1}{2}\tau^3_L - Q\sin^2\theta_W\right)\psi Z_\mu \\ &-\frac{g}{2\sqrt{2}}\sum_\psi \bar{\psi}\gamma^\mu\left(T^+_L W^+ + T^-_L W^-\right)\psi \\ &+\mathcal{L}_{YM}(B) + \mathcal{L}_{YM}(W)\end{aligned} \tag{19}$$

Higgs sector

The Higgs field must provide masses to three gauge fields – hence, together with the Higgs boson, there should be minimum 4 components in the Higgs field. In the standard model it is a complex spinor of the group $SU(2)_L$,

$$\varphi = \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}\ , \tag{20}$$

where the indexes $^+$ and ^{0} indicate the Q-charges of the components; the Y_W-charge of both components is equal 1.

The Higgs sector Lagrangian is

$$\begin{aligned}\mathcal{L}_H = \varphi^\dagger &\left(\overleftarrow{\partial}_\mu - ig'\frac{1}{2}Y_W B_\mu - ig\frac{1}{2}\vec{\tau}\vec{W}_\mu\right)\cdot \\ &\cdot\left(\overrightarrow{\partial}_\mu + ig'\frac{1}{2}Y_W B_\mu + ig\frac{1}{2}\vec{\tau}\vec{W}_\mu\right)\varphi \\ &-\frac{\lambda^2}{4}\left(\varphi^\dagger\varphi - v^2\right)^2\end{aligned} \tag{21}$$

After the spontaneous symmetry breaking the fields Z and $W^\pm$ get masses, $m_w^2 = \frac{1}{4}v^2g^2$, $m_Z^2\cos^2\theta_w = m_w^2$, while the field A remains massless.

exercises

1. The last exercise on note 12.