Sir Arthur Stanley Eddington:
The Internal Constitution of the Stars
1926

At first sight it would seem that the deep interior of the sun and stars is less accessible to scientific investigation than any other region of the universe.

Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is hidden behind substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?
Asteroseismology

The study of stellar interiors from observations of stellar oscillations

• Oscillation frequencies can be determined with extremely high precision
• Frequencies are sensitive to internal structure and rotation
• Mode amplitudes and lifetimes are sensitive to near-surface physics, including convective dynamics
Stellar structure

- Convective zone
- Radiative zone
- Core
- Sunspots
- Tachocline

Stellar structure and oscillations

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Velocity (m/s)

SOI / MDI

Stellar structure and oscillations

- Convective zone
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2 Neutrons 2 Protons Helium Light!
Solar light spectrum

Doppler effect

\[ \frac{\Delta \lambda}{\lambda} = \frac{v}{c} \]

Stellar structure and oscillations

Single Dopplergram

SOI / MDI

Velocity (km/s)
Stellar structure and oscillations

Average Dopplergram Minus Polynomial Fit
45 images averaged (30-Mar-96 19:26 to 30-Mar-96 20:17)
SOI / MDI

Velocity (m/s)

SOI / MDI

Stellar structure and oscillations

Single Dopplergram Minus 45 Images Average
(30-Mar-96 19:54:00)
SOI / MDI

Velocity (m/s)

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Velocity (m/s)

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Oscillation example

Oscillating mass point

Approach:
1) Search for equilibrium solution (x_0)
2) Solve oscillation problem

1) Equilibrium: acceleration = 0:
\[ \ddot{x} = 0 \rightarrow F_{\text{net}} = 0 = mg - kx_0 \]
\[ \rightarrow x_0 = \frac{mg}{k} \]

2) movement about equilibrium state:
variable transformation:
\[ y = x - x_0 \]
\[ x = y + x_0 \]
\[ \rightarrow m\ddot{y} = mg - kx_0 - ky = -ky \]
\[ \ddot{y} + \frac{k}{m}y = 0 \]
\[ \rightarrow y = \sin \left( \frac{k}{m} t \right) \]
Stellar time scales (order of magnitudes)

Dynamical time scale (free fall):
- movement in a gravitational field
- consider star with mass $M$ and radius $R$

Gravitational acceleration:
- time it takes for a mass point to "fall" along distance $h$ in a gravitational field:

$$t_{\text{dyn}} := \left( \frac{R^3}{GM} \right)^{3/2} = \left( \frac{2hr^2}{GM} \right)^{1/2}$$

- $t_{\text{dyn}}$ is also the time a body orbits the star.

Release of gravitational energy (thermal time scale):
- after "switching off" the nuclear reactions: how long will star still shine?
- star contract as a result of decreasing gas and radiation pressure; star will radiate with luminosity $L$ ($L_\odot = 3.8 \times 10^{33} \text{ erg s}^{-1}$).

- gravitational potential:
  - gravitational binding energy (potential gravitational energy):
    $$\Omega = -\frac{GM^2}{R}$$
  - becomes more "negative" during contraction; potential energy will be released.

- Kelvin Helmholtz time:
  $$t_{\text{KH}} := \frac{GM^2}{BL}$$

$$t_{\text{KH}} = 30 \text{ million years}$$

Stellar time scales (order of magnitudes)

nuclear time scale (order of magnitudes):
- nuclear reactions take place within $\sim 10\%$ of stellar mass.
- estimate energy production through hydrogen burning:

$$\text{mass defect:}$$

- He : $4 \times 1.008 = 4.032 \text{ m}_n$
- He : $-4.033 \text{ m}_n$
- $2e^-$ : $-0.001 \text{ m}_n$
- $\Delta m : \frac{0.028 \text{ m}_n}{0.028/4} = 0.7\%$

- nuclear time:

$$t_{\text{nuc}} := \frac{\Delta mc^2}{L} = 7 \times 10^{-11} \frac{Mc^2}{10L}$$

$$t_{\text{nuc}} = 10^{10} \text{ years}$$

Stellar time scales (order of magnitudes)

nuclear time scale (of nuclear transmutations):
- estimate energy production through hydrogen burning:

$$\text{mass defect:}$$

- He : $4 \times 1.008 = 4.032 \text{ m}_n$
- He : $-4.033 \text{ m}_n$
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Stellar time scales (order of magnitudes)

dynamical time scale (free fall):

$$t_{\text{dyn}} := 30 \text{ million years} \left( \frac{M}{M_\odot} \right)^{3/2} \left( \frac{R}{R_\odot} \right)^{-1/2} \left( \frac{L}{L_\odot} \right)^{-1}$$

$$t_{\text{dyn}} << t_{\text{KH}} << t_{\text{nuc}}$$
Terminology:

a) linear ↔ nonlinear:
   e.g.: harmonic oscillator: \[ \ddot{y} + \omega^2 y = 0 \]
   real systems are not harmonic, but for small enough amplitudes the
   harmonic (linearized) approximation is sufficient.
   (Sun: displacement: \( (10^{-5} - 10^{-6})R_\odot \)).

b) adiabatic ↔ nonadiabatic:
   processes are “fast” enough for neglecting heat losses
   (e.g.: air pump -> fast compression (friction neglected)).
   The heat losses in the Sun are very slow (\( \dot{Q}_H \gg \)) -> “5-minute”
   pulsations are relatively fast -> adiabatic.
   adiabatically there is neither excitation (of a pulsation mode) nor damping.
   All oscillations live for “ever”.
   questions, such as why a particular pulsation period can be observed, or why a
   pulsation is appearing/disappearing, belongs to nonadiabatic theorie.

c) damped ↔ excited: can only be determined nonadiabatically.

Stellar oscillations

Stellar pulsations

e) Longitudinal ↔ transversal oscillations:
   Sound waves are longitudinal waves (Rubens’ flame tube)
Solar oscillations

Single Dopplergram Minus 45 Images Average
(30-MAR-96 19:54:00)

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-500. -400. -300. -200. -100. 0. 100. 200. 300. 400. 500.

Velocity (m/s)

acoustic mode
(standing wave)

Stellar oscillations

f) radial ↔ nonradial:
   - pure geometrical (exact) classification
   - radial case \( l = m \) is easiest to treat
   (mass is independent variable)

\[ n \text{ radial order} \]

\[ l \text{ spherical degree} \]

\[ m \text{ azimuthal degree} \]

\(-l \leq m \leq l\)
Spherical harmonics

\[ Y^{lm}(\theta, \phi) = (-1)^m \sqrt{\frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!}} P^m_l(\cos \theta) \exp(i m \phi) \]
acoustic mode (standing wave)

spherical degree \( l = 20 \) azimuthal order \( m = 16 \)

SOLAR OSCILLATIONS

The \( l=20 \ m=16 \) mode

Time-series of a decomposed oscillation observation

Acoustic spectrum (power spectral density) of the Sun (SOHO/GOLF)
Acoustic spectrum (power spectral density) of the Sun (SOHO/GOLF)

Solar spectrum
Computed solar frequencies

Solar spectrum

Solar spectrum
Asteroseismic HR diagram

Asteroseismic (Δν,δν) diagram

Christensen-Dalsgaard (1994)