• Determination of the phase

• Accuracy

• Data sampling

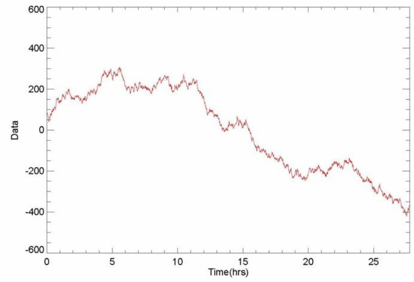
• Statistical weights

• non-white noise

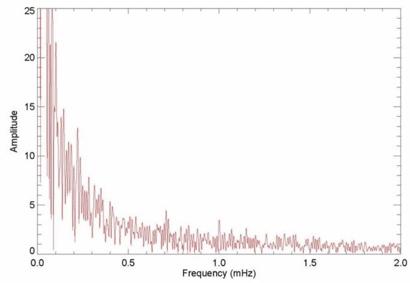
• non-coherent oscillations

In the present lecture I will first discuss issues related to non-white noise sources and non-coherent oscillations (oscillations that are not described as a simple harmonic oscillator).

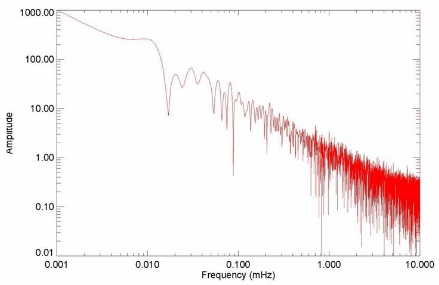
The aim is to understand the power spectrum for non-white noise and non-coherent oscillations.



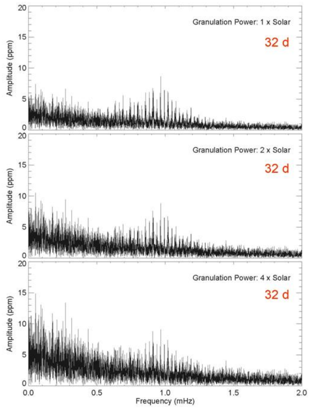
Non-white noise sources are noise sources that do not show the same noise level (power) at all frequencies. The most common non-white noise source is a drift where the noise source show more power at low frequency than at high frequency.



The power spectrum of the time series above is shown here.

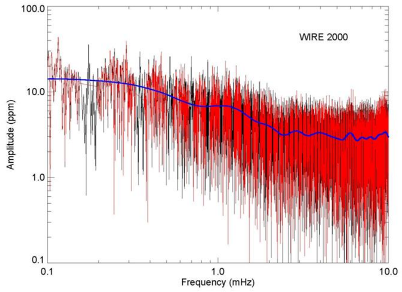


Plotting this spectrum in a log-log plot shows that the present non-white noise source has a 1/f shape in amplitude ($1/f^2$ in power). Those noise sources are therefore named 1/f noise.



The present plot shows a simulated background noise that is well described by a 1/f noise.

WIRE



Buzasi, Bruntt, Kjeldsen, Christensen-Dalsgaard… 2004

In this case (in log-log) I show the background noise spectrum for the star Procyon observed by the NASA WIRE satellite.

In order to see is a noise source is non-white one shall plot the power spectrum and check if the spectrum is flat or not.

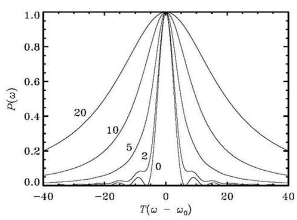
- Determination of the phase
- Accuracy
- Data sampling
- Statistical weights
- non-white noise
- non-coherent oscillations

We will now discus the power spectrum for a non-coherent oscillation.

$$v(t) = a_0 \cdot \cos(\omega_0 t - \delta_0) \cdot e^{-\eta t}$$

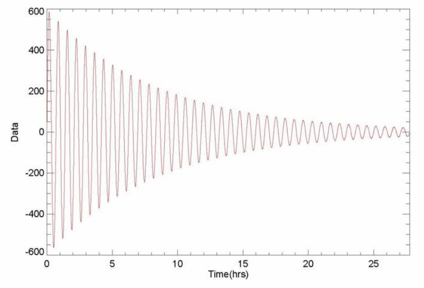
$$\eta = \frac{1}{T_{life}}$$

$$P(\omega) = \frac{1}{4} \frac{a_0^2}{(\omega - \omega_0)^2 + \eta^2}$$



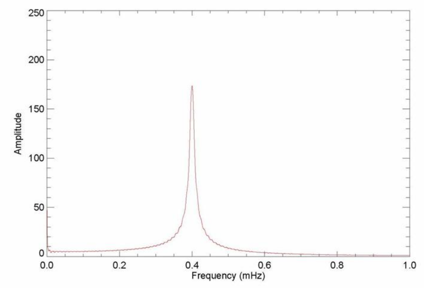
The first non-coherent oscillation we discuss is a damped oscillator where the simple harmonic oscillation is multiplied with an exponential decaying amplitude. The decaying time is T_{life}

The power spectrum of this oscillation is a so-called Lorentzian function. Examples of Lorentzians is shown in the figure.



If we take a simple harmonic oscillator and multiply with an exponential function:

$$e^{-t/T_{life}}$$

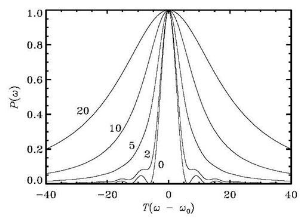


We find that the power spectrum is not a sinc-function but a stated above a Lorentzian.

$$v(t) = a_0 \cdot \cos(\omega_0 t - \delta_0) \cdot \mathrm{e}^{-\eta t}$$

$$\eta = \frac{1}{T_{life}}$$

$$P(\omega) = \frac{1}{4} \frac{a_0^2}{(\omega - \omega_0)^2 + \eta^2}$$



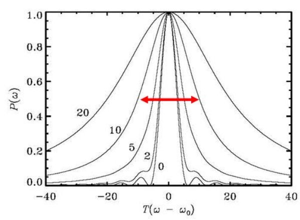
There is a simple relation between the FWHM of the Lorentzian and life time.

$$v(t) = a_0 \cdot \cos(\omega_0 t - \delta_0) \cdot \mathrm{e}^{-\eta t}$$

$$P(\omega) = \frac{1}{4} \frac{a_0^2}{(\omega - \omega_0)^2 + \eta^2}$$

Angular frequencies:

$$FWHM \approx 2\eta$$

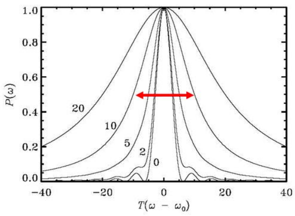


In angular frequencies the FWHM is given by:

$$FWHM = 2\eta$$

$$v(t) = a_0 \cdot \cos(\omega_0 t - \delta_0) \cdot e^{-\eta t}$$

$$P(\omega) = \frac{1}{4} \frac{a_0^2}{(\omega - \omega_0)^2 + \eta^2}$$



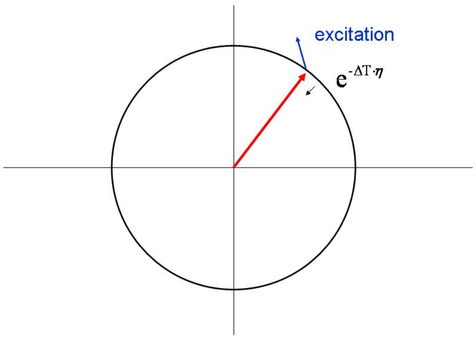
Cyclic frequency:

$$FWHM \approx \frac{\eta}{\pi}$$

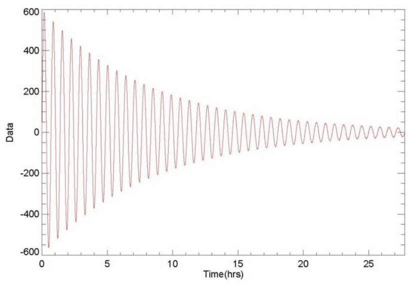
While in cyclic frequencies we find:

$$FWHM = \frac{2\eta}{2\pi} = \frac{\eta}{\pi}$$

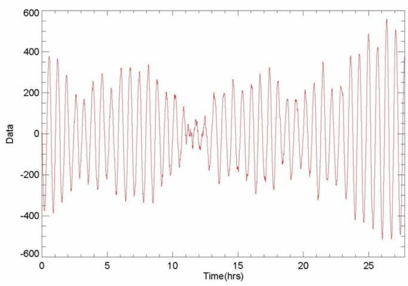
Damping



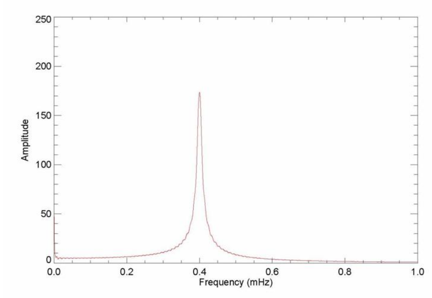
In oscillating stars we will find examples of damped oscillations. However those oscillations will of course also need to be excited. If we look at the “vector model” for the oscillations we may understand the damped oscillation as an exponential decrease in the length of the vector while the excitation is an extension to the vector (that can be out of phase, i.e. having a different direction than the oscillation vector).



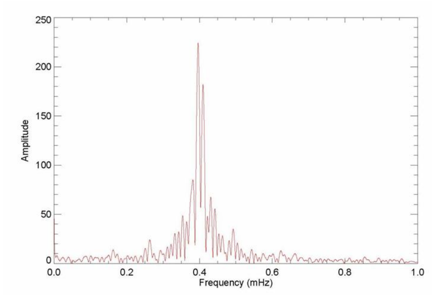
We will therefore often find a damped and re-excited oscillator which will not look like the simple decaying oscillation.



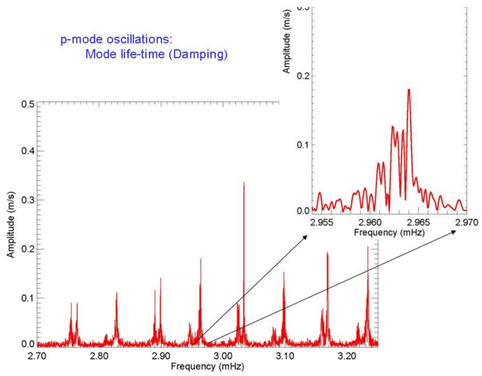
but will be an oscillation that show variations in amplitude (and phase).



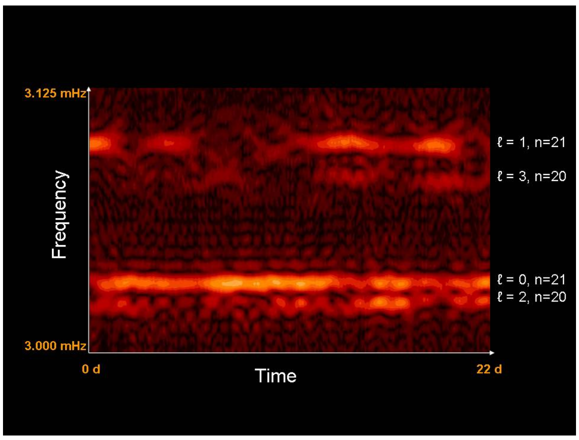
The power spectrum of the simple harmonic damped oscillator (The Lorentzian) will…



… still have a Lorentzian shape but due to the phase variations and amplitude variations (not only decaying) we will find a forest of peaks under a Lorenzian envelope.

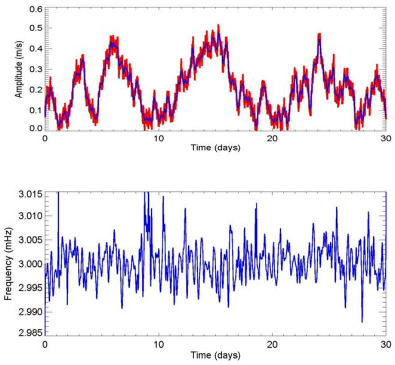


An excellent example of a damped and re-excited oscillator is the solar p-mode oscillations. The figure shows the power spectrum of the solar radial velocity time series and a zoom of one of those oscillations. The shape of the peak is clearly Lorentzian.



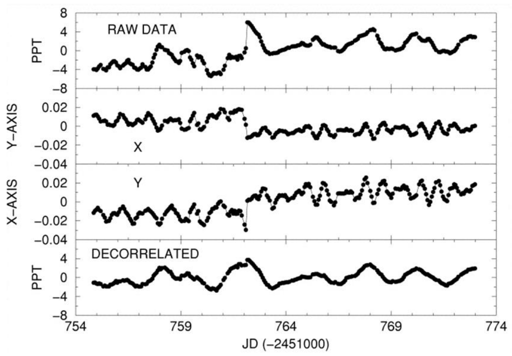
One may show the damping and re-excitation as an image where the colour is related to energy.

The present figure shows the frequency and power as a function of time (for a period of 22 days). The lifetime of the oscillation modes is 3-4 days.



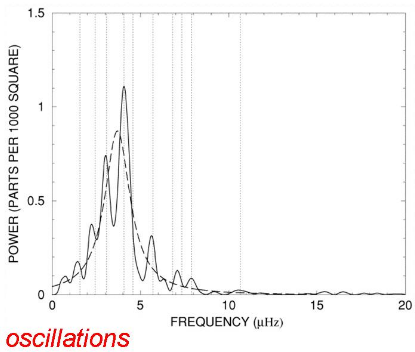
This property is also shown in the present plot where the frequency and amplitude is show as a function of time for a period of one month (simulated p-mode spectrum).

Arcturus: WIRE photometry



If one is analyzing the power spectrum for a time series it may be difficult to know if you are seeing a series of simultaneous oscillations within a narrow frequency interval or we are seeing a damped and re-excited oscillator.

The present data for the red giant star Arcturus shows an example of this ambiguity.



p-mode oscillations
or a single mode with a short lifetime (2 days)?

Are we seeing a single mode with a short mode lifetime or several closely spaced p-modes?

Weighted power spectrum

$$P(\nu) = \alpha(\nu)^2 + \beta(\nu)^2$$

$$\alpha(\nu) = \frac{s \cdot cc - c \cdot sc}{ss \cdot cc - sc^2}$$

$$\beta(\nu) = \frac{c \cdot ss - s \cdot sc}{ss \cdot cc - sc^2}$$

$$s = \sum w(t) \cdot f(t) \cdot \sin(\nu \cdot t)$$

$$c = \sum w(t) \cdot f(t) \cdot \cos(\nu \cdot t)$$

$$ss = \sum w(t) \cdot \sin^2(\nu \cdot t)$$

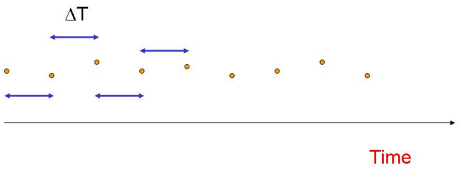
$$cc = \sum w(t) \cdot \cos^2(\nu \cdot t)$$

$$sc = \sum w(t) \cdot \sin(\nu \cdot t) \cdot \cos(\nu \cdot t)$$

The next thing we will discuss is the so-called Nyquist frequency which indicate the highest frequency where one can measure an oscillation for a given sampling in time.

This frequency is formally only defined for regular sampled data with no gabs.

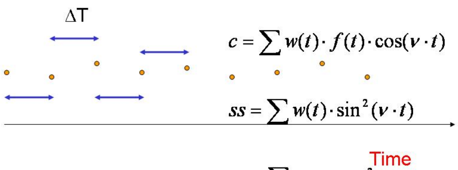
Nyquist frequency



$$f = \frac{1}{\Delta \mathrm{T}}$$

If we take a regular sampled time series with ΔT being the time between two data points we may define the so-called sampling frequency as f=1/ΔT.

Nyquist frequency



$$s = \sum w(t) \cdot f(t) \cdot \sin(\nu \cdot t)$$

$$c = \sum w(t) \cdot f(t) \cdot \cos(\nu \cdot t)$$

$$ss = \sum w(t) \cdot \sin^2(\nu \cdot t)$$

$$cc = \sum w(t) \cdot \cos^2(\nu \cdot t)$$

$$sc = \sum w(t) \cdot \sin(\nu \cdot t) \cdot \cos(\nu \cdot t)$$

$$f = \frac{1}{\Delta \mathrm{T}}$$

If we look at the power spectrum calculation we see that sin(v·t) and cos(v·t) will end up giving exactly the same values fore many frequencies.

$$ss = \sum w(t) \cdot \sin^2(\nu \cdot t)$$

$$\sin(\nu \cdot t) = -\sin(-\nu \cdot t)$$

$$\sin(\nu \cdot t) = -\sin(\nu \cdot t + \pi) = -\sin((\nu + \pi/t) \cdot t)$$

$$t_n = n \cdot \Delta \mathrm{T} \qquad \nu_{\text{Nyquist}} = \frac{\pi}{\Delta \mathrm{T}}$$

Let us e.g. look at “ss”

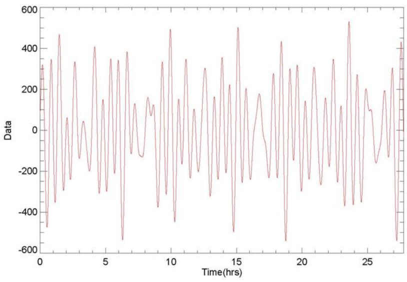
We find that sin(v·t) = - sin(-v·t).. and shifts of π and 2π will also give exactly the same values.

The result is that the power spectrum will repeat it self at intervals of half the sampling frequency.

$$\nu_{\text{Nyquist}} = \frac{\pi}{\Delta \mathrm{T}}$$ Angular frequency

$$f_{\text{Nyquist}} = \frac{1}{2 \cdot \Delta \mathrm{T}}$$ Cyclic frequency

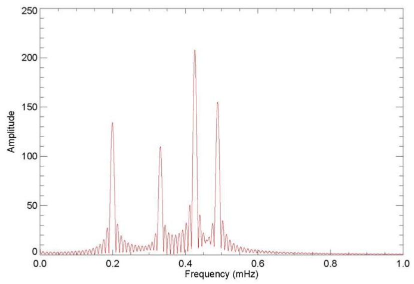
This frequency is the Nyquist frequency.



ΔT=50 sec

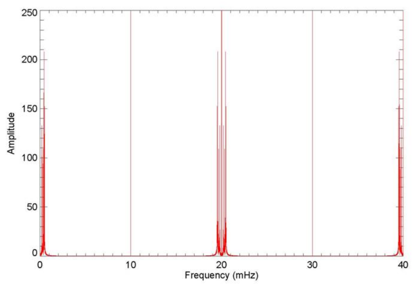
$$f_{\text{Nyquist}} = \frac{1}{2 \cdot 50\,\text{sec}} = 0{,}01\,\text{Hz} = 10\,\text{mHz}$$

Let us consider an example. The present time series contain four oscillation modes and the sampling is 50 sec which will give a sampling frequency of 0.02 Hz = 20 mHz and a Nyquist frequency of 10 mHz.



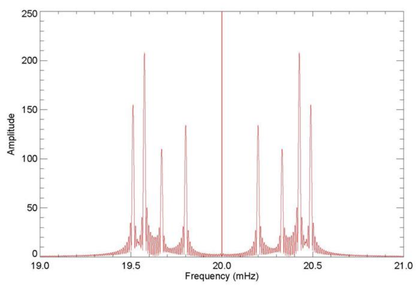
$$f_{\text{Nyquist}} = \frac{1}{2 \cdot 50\,\text{sec}} = 0{,}01\,\text{Hz} = 10\,\text{mHz}$$

The oscillation frequencies are much lower and the four frequencies are not affected by the Nyquist frequency.



$$f_{\text{Nyquist}} = \frac{1}{2 \cdot 50\,\text{sec}} = 0{,}01\,\text{Hz} = 10\,\text{mHz}$$

If we however show the power spectrum for frequencies between zero and 40 mHz (four times the Nyquist frequency) we will find power from all those frequencies both near zero, on both sides of 20 mHz and just below 40 mHz.

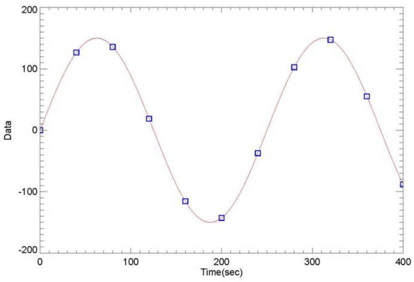


$$\text{Amp}(f) = \text{Amp}(2 \cdot f_{\text{Nyquist}} - f)$$

$$\text{Amp}(f) = \text{Amp}(2n \cdot f_{\text{Nyquist}} \pm f)$$

The same is true if we check the negative frequencies and in general we find:

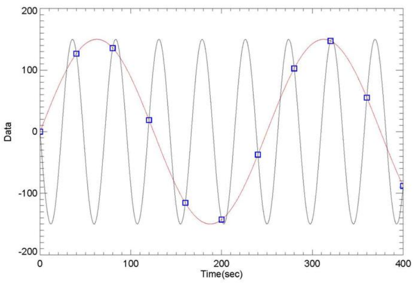
$$\text{Amp}(f) = \text{Amp}(2n \cdot f_{\text{Nyquist}} \pm f)$$



$P = 250\,\text{sec} \qquad \Delta T = 40\,\text{sec}$

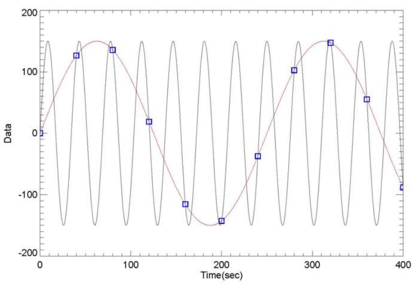
The reason that we find those many extra frequencies can be seen quite clear if we as an example take a simple harmonic oscillator with P=250 sec and a sampling of e.g. 40 sec.

The frequency of the oscillation is 4 mHz and the sampling frequency is 25 mHz.



$$P = 47{,}62\,\text{sec} \qquad \frac{1}{P} = \frac{1}{40\,\text{sec}} - \frac{1}{250\,\text{sec}}$$

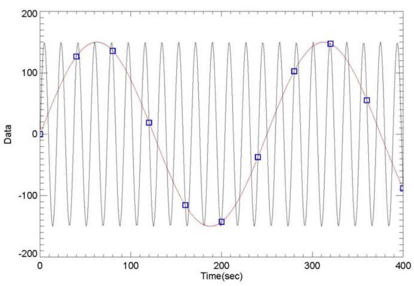
However a frequency of 21 mHz (25 mHz – 4 mHz) will also provide a perfect solution (21 mHz corresponds to a period of 47,62 sec).



$$P = 34{,}48\,\text{sec} \qquad \frac{1}{P} = \frac{1}{40\,\text{sec}} + \frac{1}{250\,\text{sec}}$$

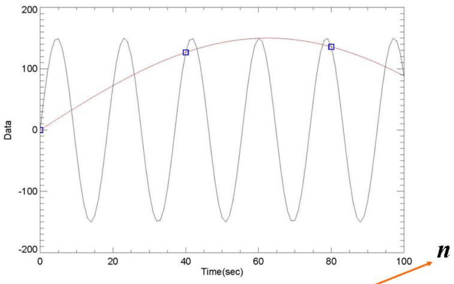
.. but there are many other solutions.

25 mHz + 4 mHz = 29 mHz is another solution (P = 34,48 sec)



$P = 18{,}52\,\text{sec}$

.. and P =18,52 sec corresponding to a frequency of 54 mHz (2 · 25 mHz + 4 mHz).



$$P = 18{,}52\,\mathrm{sec} \qquad \frac{1}{P} = \frac{2}{40\,\mathrm{sec}} + \frac{1}{250\,\mathrm{sec}}$$

In general we find that frequencies that fulfil:

1/P = f = 4 mHz + n · 25 mHz

will all produce the same amplitude when we fit to the data.

$$\nu_{\mathrm{Nyquist}} = \frac{\pi}{\Delta T}$$ Angular frequency

$$f_{\mathrm{Nyquist}} = \frac{1}{2 \cdot \Delta T}$$ Cyclic frequency

.. and this demonstrates how the Nyquist and sampling frequency affect the solutions and results in structures in the power spectrum.

CLEAN

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Pulsation of the δ Scuti star θ^2 Tau: new multisite photometry and modelling of instability

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ABSTRACT
The results of a multisite photometric campaign of θ^2 Tau are reported. This binary system

The final thing I will discus in the present lecture is a process called CLEAN. CLEAN is a process where we use the power spectrum to estimate amplitudes, frequencies and phases for the signal present in a time series and then we correct the time series for the estimated oscillations and create a CLEANed time series without oscillations.

Pulsation of θ^2 Tau

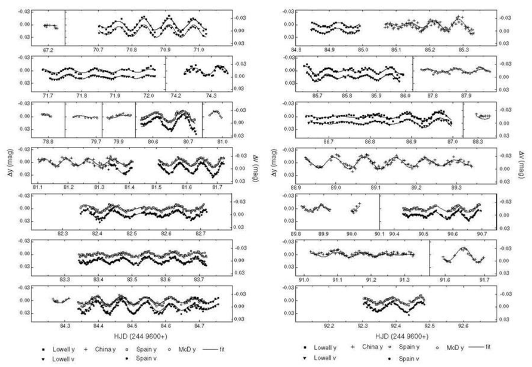


Figure 1. Light curves showing the 1994 measurements of θ^2 Tau. The curves drawn represent the 13-frequency fit found in a later section.

This process can be very powerful and it is used a lot in astronomical time series analysis. One example is the analysis of the time series for the delta scuti star θ^2 Tau. (paper by Breger et al.).

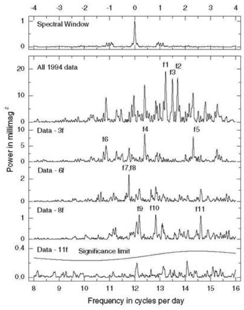


Figure 2. Power spectra of the 1994 photometry of θ^2 Tau. The top panel shows the spectral window, while the other panels present the power spectra before and after prewhitening a given number of frequencies. See text for a discussion of significance levels.

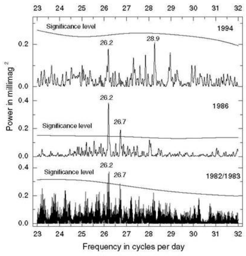


Figure 3. Power spectra in the high-frequency region. The frequency at 26.2 cycle d^{-1} is present in all data sets, while 26.7 cycle d^{-1} is present in both the 1986 and 1982/3 data sets.

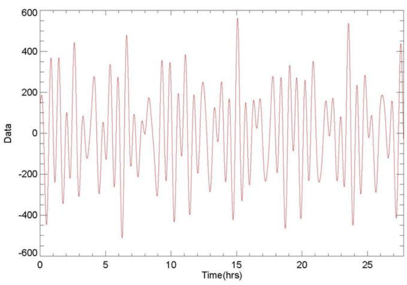
This star shows a number of simultaneously excited oscillation modes. The sampling is not continues so the power spectrum contain a large number of peaks arising from all the excited modes and the extra peaks from the window-function.

CLEAN will remove peaks one by one and in the end the CLEANing will result in a time series with no signal (only noise is left). At the same time we have measured the frequencies, phases and amplitudes for the oscillations modes.

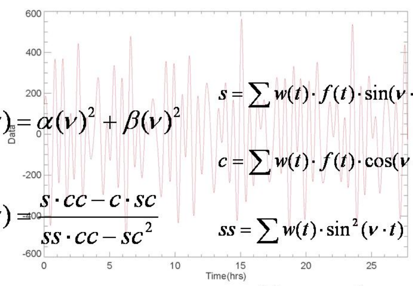
Table 2. The photometric frequency spectrum of θ^2 Tau during 1994. Colons indicate an approximate value.

	Frequency		1994 amplitudes	
	(cycle d^{-1})	(μHz)	v filter (mmag)	y filter (mmag)
			± 0.1	± 0.1
f_1	13.230	153.1	4.8	3.7
f_2	13.694	158.5	5.2	4.1
f_3	13.484	156.1	3.2	2.3
f_4	12.397	143.5	3.7	3.1
f_5	14.316	165.7	3.7	2.8
f_6	10.865	125.8	2.3	1.5
f_7	11.770	136.2	2.3:	2.4:
f_8	11.730	135.8	1.2:	0.9:
f_9	12.177	140.9	1.7	1.0
f_{10}	12.832	148.5	1.4	1.4
f_{11}	14.615	169.2	1.9	1.2

Breger et al. give the frequencies extracted….



CLEAN is in fact a simple process and based on the tools we have discussed and the software you have developed we may be able to construct a simple iterative CLEAN process that will follow the procedure shown for the delta Scuti star above.



$$P(\nu) = \alpha(\nu)^2 + \beta(\nu)^2$$

$$\alpha(\nu) = \frac{s \cdot cc - c \cdot sc}{ss \cdot cc - sc^2}$$

$$\beta(\nu) = \frac{c \cdot ss - s \cdot sc}{ss \cdot cc - sc^2}$$

$$s = \sum w(t) \cdot f(t) \cdot \sin(\nu \cdot t)$$

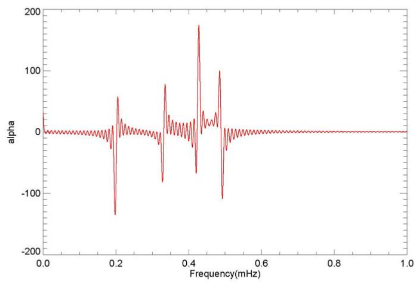
$$c = \sum w(t) \cdot f(t) \cdot \cos(\nu \cdot t)$$

$$ss = \sum w(t) \cdot \sin^2(\nu \cdot t)$$

$$cc = \sum w(t) \cdot \cos^2(\nu \cdot t)$$

$$sc = \sum w(t) \cdot \sin(\nu \cdot t) \cdot \cos(\nu \cdot t)$$

The basic thing to notice is that we through α and β have access to information on phase and amplitude at a specific frequency.

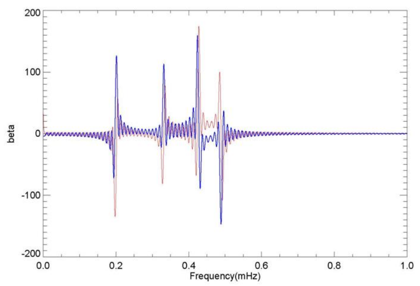


$$\alpha(\nu) = \frac{s \cdot cc - c \cdot sc}{ss \cdot cc - sc^2}, \quad f = \frac{\nu}{2\pi}$$

sine

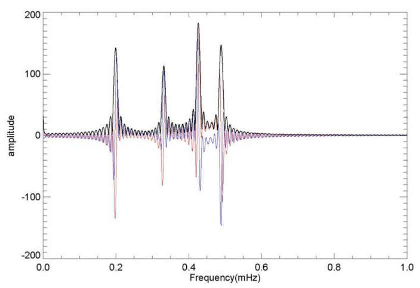
The CLEAN process will therefore first require that we calculate the two coefficients α and β for sine and cosine

This is shown in the two figures.



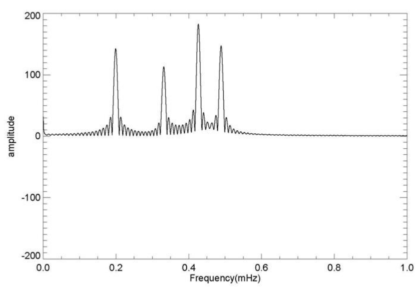
$$\beta(\nu) = \frac{c \cdot ss - s \cdot sc}{ss \cdot cc - sc^2}$$

cosine

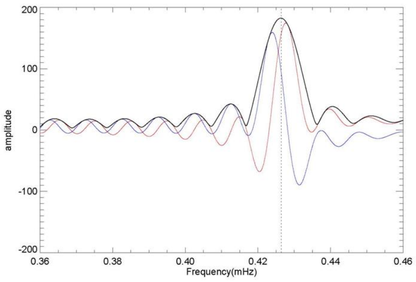


$$\mathrm{Amp}^2(f) = P(\nu) = \alpha(\nu)^2 + \beta(\nu)^2$$

We then calculate the power spectrum from α and β



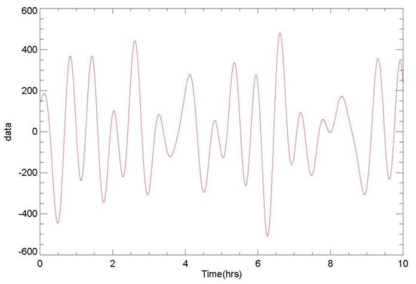
.. and based on this we see four peaks which corresponds to the four oscillation modes in the time series.



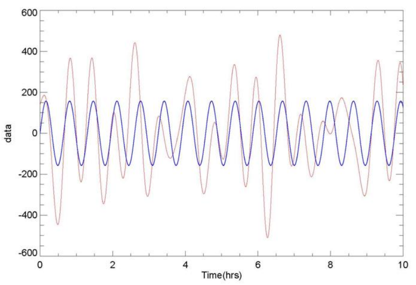
f_0

If we zoom in on the main peak we see in this figure both α and β as a function of frequency and the amplitude spectrum (amplitude as a function of frequency).

f_0 is the frequency at peak power.

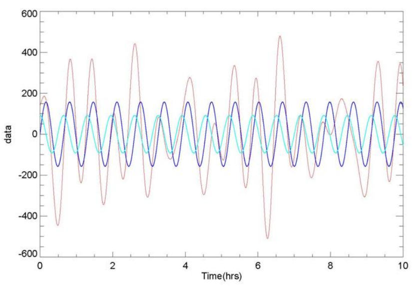


If we look in the time domain we see the combined signal…



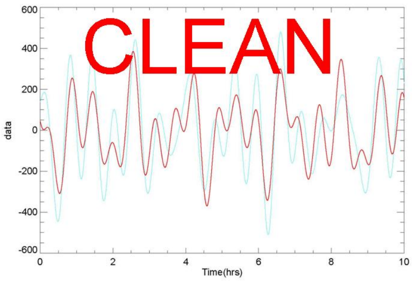
$$\alpha(f_0) = \frac{s \cdot cc - c \cdot sc}{ss \cdot cc - sc^2}$$

.. and $\alpha(f_0)$ multiplied with a sine having a frequency of f_0.



$$\beta(f_0) = \frac{c \cdot ss - s \cdot sc}{ss \cdot cc - sc^2}$$

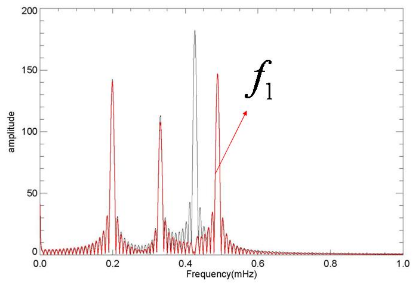
…same for $\beta(f_0)$ and cosine.



$$data(t) - \alpha(f_0)\cdot\sin(2\pi\cdot f_0\cdot t) - \beta(f_0)\cdot\cos(2\pi\cdot f_0\cdot t)$$

If we now remove this signal from the time series we may start to CLEAN the series for the detected oscillations. The first step of the cleaning is therefore

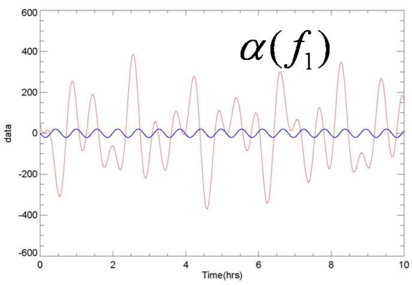
$$data(t) - \alpha(f_0)\cdot\sin(2\pi\cdot f_0\cdot t) - \beta(f_0)\cdot\cos(2\pi\cdot f_0\cdot t)$$



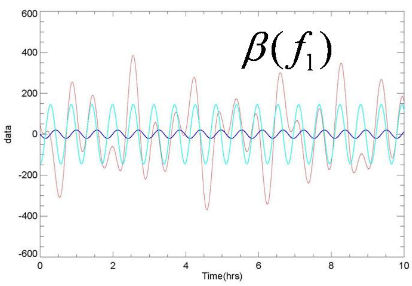
$$data(t) - \alpha(f_0)\cdot\sin(2\pi\cdot f_0\cdot t) - \beta(f_0)\cdot\cos(2\pi\cdot f_0\cdot t)$$

If we calculate the time series after removing the signal we can see that the power from this peak disappears completely.

The next step is then removing the second highest peak (we name the frequency for this peak f_1) and we can calculate the values for α and β for this frequency.

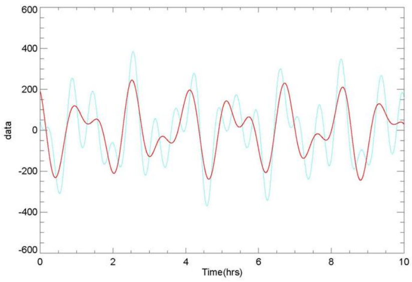


In the time series the oscillation for f_1 can be shown for $\alpha(f_1)$ and $\beta(f_1)$.



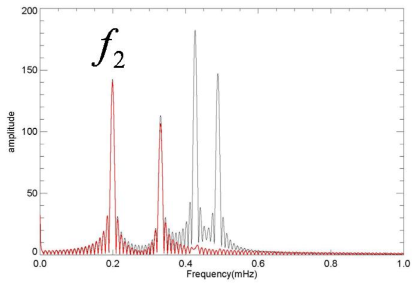
We do exactly the same as was done for f_0 but now we do in on the time series where the f_0 mode has already been removed.

After remove both modes we get another new time series.



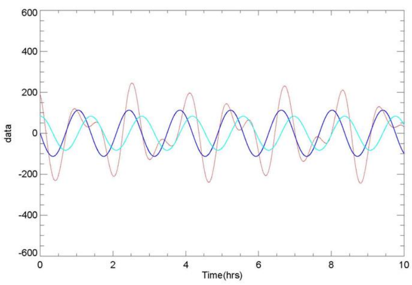
$$data(t) - \alpha(f_0)\cdot\sin(2\pi\cdot f_0\cdot t) - \beta(f_0)\cdot\cos(2\pi\cdot f_0\cdot t)$$
$$- \alpha(f_1)\cdot\sin(2\pi\cdot f_1\cdot t) - \beta(f_1)\cdot\cos(2\pi\cdot f_1\cdot t)$$

In the amplitude spectrum one can clearly see the effect of the CLEANing.

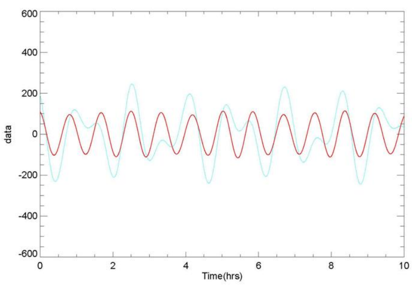


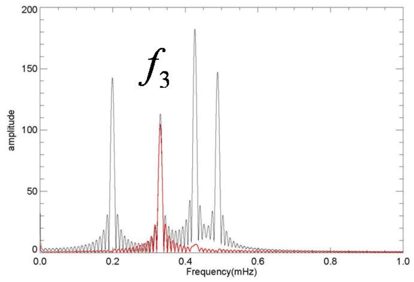
$$data(t) - \alpha(f_0)\cdot\sin(2\pi\cdot f_0\cdot t) - \beta(f_0)\cdot\cos(2\pi\cdot f_0\cdot t)$$
$$- \alpha(f_1)\cdot\sin(2\pi\cdot f_1\cdot t) - \beta(f_1)\cdot\cos(2\pi\cdot f_1\cdot t)$$

We now continue with the third frequency (f_2)…

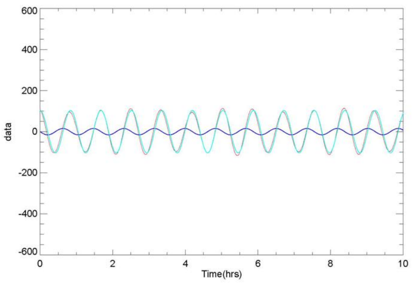


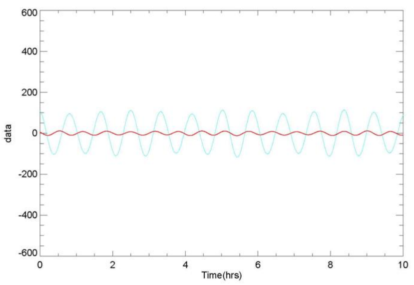
.. by calculating α and β…

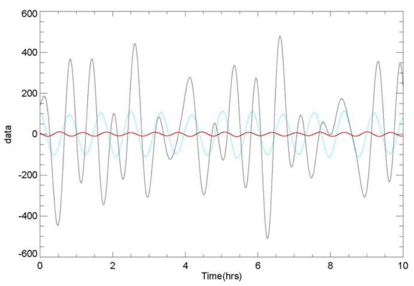




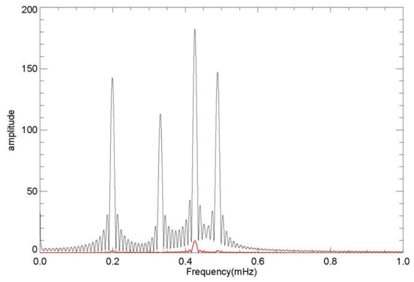
We repeat this process for the fourth frequency and we are left with a time series with almost no signal left.



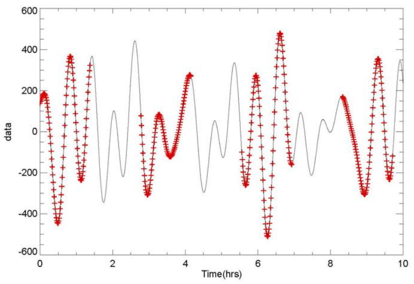




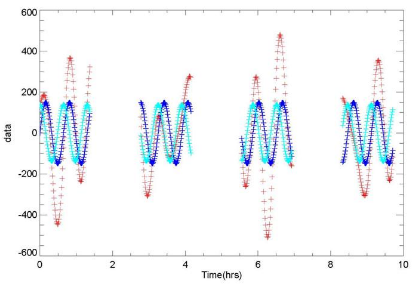
..after removing the four modes with highest amplitude. The figure shows the original series and the CLEANed series.



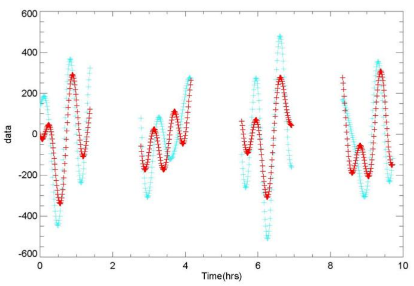
The final CLEANed amplitude spectrum only contains a small residual signal arising from small errors in measuring the frequencies, amplitudes and phases for the four modes.



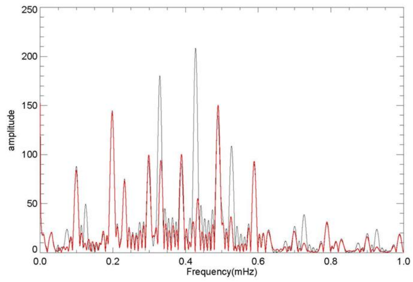
CLEAN will not only work on uninterrupted time series data. The following example shows how well it performs on data with gabs.



As before we localize the top peak in the power spectrum and calculate α and β for this frequency. Based on those coefficients we subtract the main oscillation from the time series.

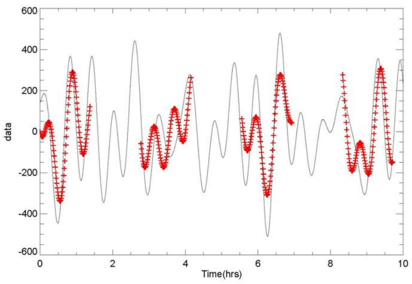


This is seen here…



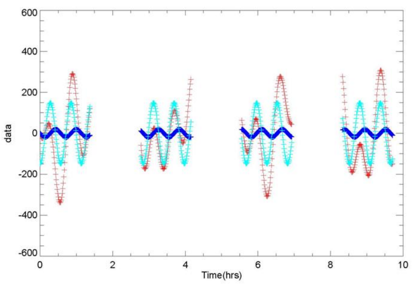
The amplitude spectrum after removing the main peak. The figure shows the amplitude for the CLEANed series:

$$data(t) - \alpha(f_0)\cdot\sin(2\pi\cdot f_0\cdot t) - \beta(f_0)\cdot\cos(2\pi\cdot f_0\cdot t)$$

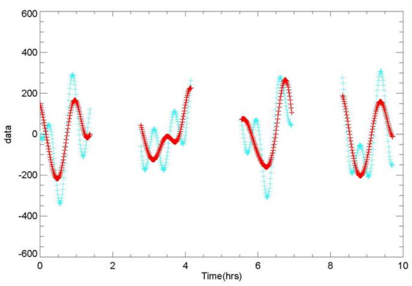


We then continue on the CLEANed series.

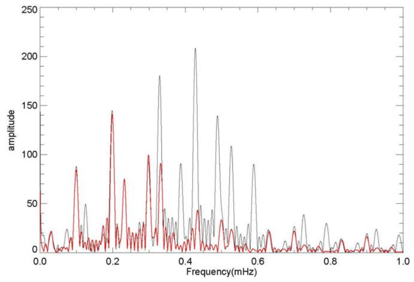
We find the second highest peak and calculate α and β for the corresponding frequency.



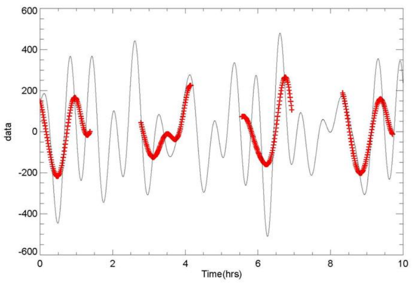
Those fits are seen here…



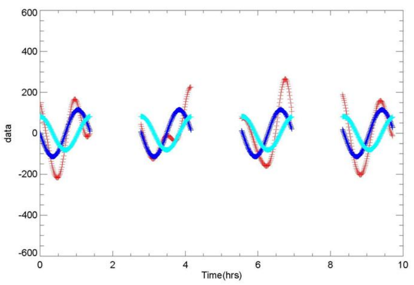
.. and after removing the second oscillation.

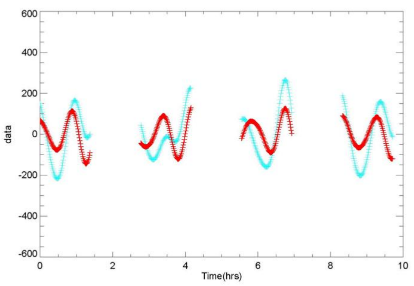


The resulting amplitude spectrum are being CLEANed and one can see that not only do we remove the peaks corresponding to the frequencies we measure.. but we also remove all the peaks from the window.

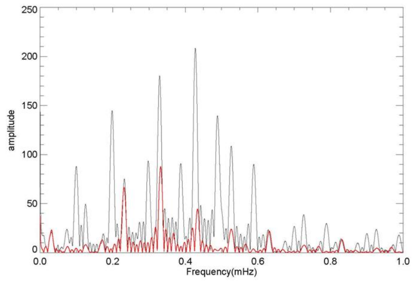


We now repeat the process for the third peak..

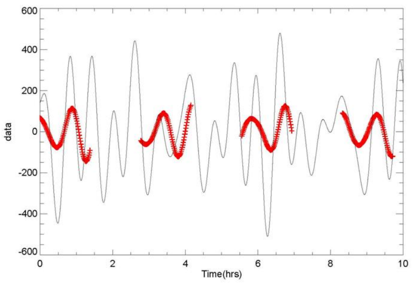




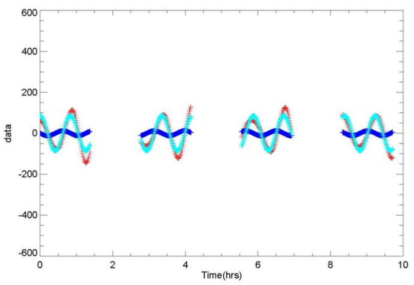
.. and after removing this peak we get as before a new series.

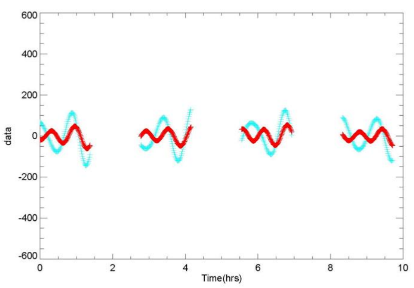


.. with most of the power from the three strongest peaks being removed.

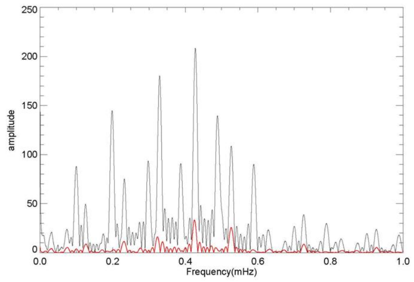


Continuing the process for peak number four..

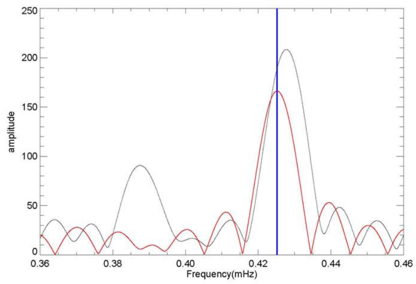




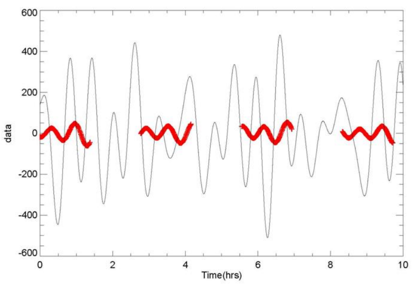
.. we get the final series where all four strong modes has been removed.



As is clear from the amplitude spectrum we have however not removed all power.

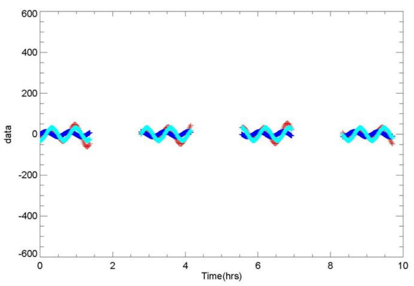


Zooming in on the spectrum shows that there is a small residual peak near the frequency for the main mode (f_0).

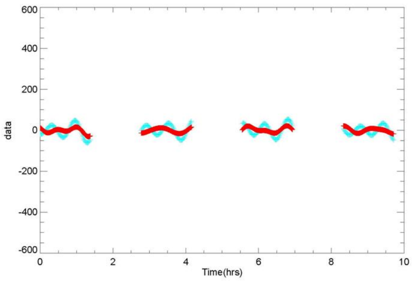


We know that this is an error (since we only included four modes in the simulation), but CLEAN can of course also remove this peak.

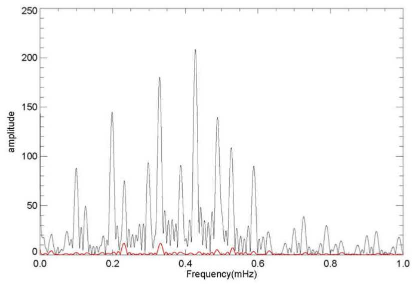
CLEAN will not tell us if the peaks are real oscillations or just residual errors.



Performing the CLEANing on the fifth peak…



… result in a time series where the power is close to zero (although there are still a bit of power left).



The amplitude spectrum of the residual time series shows that only tiny peaks are left in the spectrum. We have localized the power and described it as arising from five harmonic oscillations.