

Chapter 2

Problem 2.1

A point charge q is brought to a position a distance d away from an infinite plane conductor held at zero potential. Using the method of images, find:

- (a) the surface-charge density induced on the plane, and plot it;
- (b) the force between the plane and the charge by using Coulomb's law for the force between the charge and its image;
- (c) the total force acting on the plane by integrating $\sigma^2/2\epsilon_0$ over the whole plane;
- (d) the work necessary to remove the charge q from its position to infinity;
- (e) the potential energy between the charge q and its image [compare the answer to part d and discuss].
- (f) Find the answer to part d in electron volts for an electron originally one angstrom from the surface.

Part a

Figure 2.1 shows the arrangement of charges corresponding to a unit charge q placed a distance d from a grounded plane conductor. Using the definition of coordinates from the figure, q is placed at $\mathbf{z} = d\hat{\mathbf{z}}$ and $-q$ at $\mathbf{z}' = -d\hat{\mathbf{z}}$. The potential for $z \geq 0$ is then given by the potential of the two point charges

$$\begin{aligned}\Phi(\mathbf{x}) &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{|\mathbf{x} - d\mathbf{z}|} - \frac{1}{|\mathbf{x} + d\mathbf{z}|} \right) \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{[x^2 + y^2 + (z - d)^2]^{1/2}} - \frac{1}{[x^2 + y^2 + (z + d)^2]^{1/2}} \right)\end{aligned}$$

The expression for the induced surface charge density is basically given by the normal derivative evaluated at the surface

$$\left. \frac{\partial\Phi}{\partial z} \right|_{z=0} = \frac{q}{4\pi\epsilon_0} \left(\frac{(-\frac{1}{2}) \cdot 2(z - d)}{[x^2 + y^2 + (z - d)^2]^{3/2}} - \frac{(-\frac{1}{2}) \cdot 2(z + d)}{[x^2 + y^2 + (z + d)^2]^{3/2}} \right) \Big|_{z=0}$$

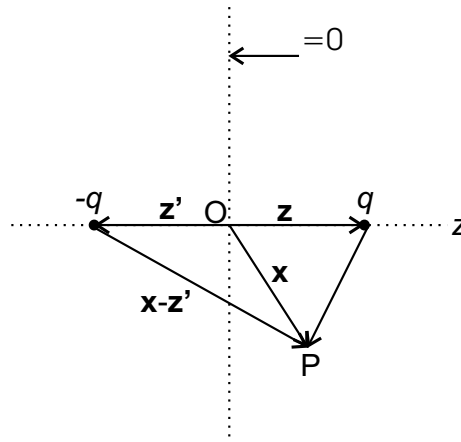


Figure 2.1: Problem 2.1. The geometry of the situation.

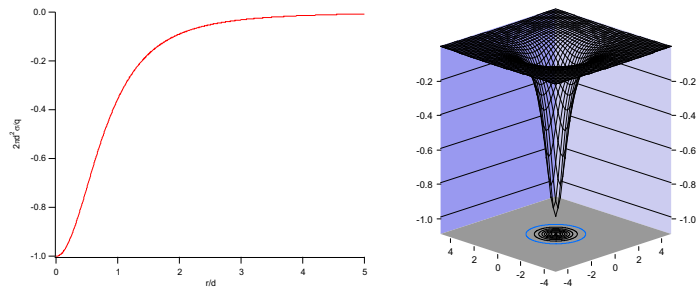


Figure 2.2: Problem 2.1. 2D and 3D plots of the induced surface charge density.

so the induced surface charge density σ is

$$\sigma = -\epsilon_0 \left. \frac{\partial \Phi}{\partial z} \right|_{z=0} = -\frac{q}{2\pi} \frac{d}{[x^2 + y^2 + d^2]^{3/2}} = -\frac{q}{2\pi d^2} \left(1 + \left(\frac{r}{d} \right)^2 \right)^{-3/2}$$

with $r^2 = x^2 + y^2$ the radial distance from the origin in the xy -plane. Figure 2.2 shows the desired plot.

Part b

The force between the plane and the point charge is the attractive force between the two point charges placed at a distance $2d$ and from Coulomb's law this is simply

$$F = \frac{q^2}{4 \times 4\pi\epsilon_0 d^2}$$

Part c

The force per unit area on the plane is $\sigma^2/2\epsilon_0$, as derived in Section 1.11. The total force is

$$\begin{aligned} F &= \int_S \frac{\sigma^2}{2\epsilon_0} da \\ &= \frac{q^2}{8\pi^2\epsilon_0 d^4} \int_0^\infty 2\pi r dr \left(1 + \frac{r^2}{d^2}\right)^{-3} \\ &= \frac{q^2}{4 \times 4\pi\epsilon_0 d^2} \end{aligned}$$

as in part b.

Part d

The work W_1 needed to move q to infinity is (1.18)

$$\begin{aligned} W_1 &= \int_d^\infty F(z) dz \\ &= \int_d^\infty \frac{q^2}{4 \times 4\pi\epsilon_0 z^2} dz \\ &= \frac{q^2}{4 \times 4\pi\epsilon_0 d} \end{aligned}$$

Part e

The potential energy W_2 between q and its image is

$$\begin{aligned} W_2 &= q\Phi(z=d) \\ &= \frac{q^2}{4\pi\epsilon_0} \left(-\frac{1}{2d}\right) \\ &= -\frac{q^2}{2 \times 4\pi\epsilon_0 d} \\ &= -2W_1 \end{aligned}$$

The magnitudes in parts d and e are not equal because in d the image charge moved with q . Put in another way, when q is moved to infinity, we do work *only* on q . It is true that induced charge is moving in over the conductor, but this costs nothing, since the whole conductor is at zero potential. By contrast, if we simultaneously move in *two* point charges (with no conductor), we do work on *both* of them, and the total is twice as great.

Part f

With $d=1 \text{ \AA}$ and $q=-e$ the work needed to move q to infinity becomes

$$W = \frac{e^2}{16\pi\epsilon_0 d} = 3.6 \text{ eV}$$

This is a typical value for a work function of a metal. As Jackson remarks at the end of Sec. 2.3, a work function of a metal is in large part due to attraction from an image force when an electron is removed from the surface.

Problem 2.2

Using the method of images, discuss the problem of a point charge q *inside* a hollow, grounded, conducting sphere of inner radius a . Find

- (a) the potential inside the sphere
- (b) the induced surface-charge density
- (c) the magnitude and direction of the force acting on q .
- (d) Is there any change in the solution if the sphere is kept at a fixed potential V ? If the sphere has a total charge Q on its inner and outer surfaces?

Solution

This situation is very similar to the ones in Secs. 2.2–2.4 so we can take over most of the results from these sections. Jackson is even kind enough to tell us the correct results in the last paragraph of Sec. 2.2. With q inside the sphere, the image charge is luckily outside the region of interest. The situation is similar to that of Jackson's Fig. 2.2, with q and q' exchanged.

Part a

The potential is given by

$$\Phi(\mathbf{x}) = \frac{q/4\pi\epsilon_0}{|\mathbf{x} - \mathbf{y}|} + \frac{q'/4\pi\epsilon_0}{|\mathbf{x} - \mathbf{y}'|}$$

It is only possible to have $\Phi(x = a) = 0$ if

$$q' = -\frac{a}{y}q, \quad y' = \frac{a^2}{y}$$

and thus the potential becomes

$$\Phi(\mathbf{x}) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{(x^2 + y^2 - 2xy\cos\gamma)^{1/2}} - \frac{1}{(a^2 + (xy/a)^2 - 2xy\cos\gamma)^{1/2}} \right)$$

with γ the angle between \mathbf{x} and \mathbf{y} . In this form, it is obvious that $\Phi = 0$ when $x = a$.

Part b

The induced surface charge density can be calculated from the normal derivative of Φ at the surface. The normal derivative out of the conductor is now radially inward:

$$\sigma = -\epsilon_0 \left. \frac{\partial\Phi}{\partial n} \right|_{x=a} = +\epsilon_0 \left. \frac{\partial\Phi}{\partial x} \right|_{x=a}$$

This is exactly the same derivative as the one leading to (2.5); the only difference is a sign change and now $a > y$ so we rearrange to a more suggestive form

$$\sigma = -\frac{q}{4\pi a^2} \frac{1 - \frac{y^2}{a^2}}{\left(1 + \frac{y^2}{a^2} - 2\frac{y}{a}\cos\gamma\right)^{3/2}}$$

Note that now the total induced charge is equal to $-q$, independent of y , as can easily be verified.

Part c

The force acting on q is most easily computed by writing up the force between q and its image. The distance between q and q' is $y' - y = y(a^2/y^2 - 1)$. q is attracted towards the conductor with a force of magnitude

$$|\mathbf{F}| = \frac{1}{4\pi\epsilon_0} qq' \frac{1}{y^2} \left(\frac{a^2}{y^2} - 1 \right)^{-2} = \frac{q^2}{4\pi\epsilon_0 a^2} \left(\frac{a}{y} \right)^3 \left(\frac{a^2}{y^2} - 1 \right)^{-2}$$

Part d

If the sphere is kept at a fixed potential V :

- The potential inside is raised by a constant V
- No change to the inner surface charge density, but a charge $4\pi\epsilon_0 Va$ is uniformly distributed on the outer surface
- No change to the force

If the sphere has a total charge Q :

- The potential inside is raised by a constant equal to the potential of the sphere
- No change to the inner surface charge density, but a charge $Q + q$ is uniformly distributed on the outer surface
- No change to the force

Problem 2.3

A straight-line charge with constant linear charge density λ is located perpendicular to the x - y plane in the first quadrant at (x_0, y_0) . The intersecting planes $x = 0, y \geq 0$ and $y = 0, x \geq 0$ are conducting boundary surfaces held at zero potential. Consider the potential, fields, and surface charges in the first quadrant.

- (a) The well-known potential for an isolated line charge at (x_0, y_0) is $\Phi(x, y) = (\lambda/4\pi\epsilon_0) \ln(R^2/r^2)$, where $r^2 = (x - x_0)^2 + (y - y_0)^2$ and R is a constant. Determine the expression for the potential of the line charge in the presence of the intersecting planes. Verify explicitly that the potential and the tangential electric field vanish on the boundary surface.
- (b) Determine the surface charge density σ on the plane $y = 0, x \geq 0$. Plot σ/λ versus x for $(x_0 = 2, y_0 = 1)$, $(x_0 = 1, y_0 = 1)$, and $(x_0 = 1, y_0 = 2)$.
- (c) Show that the total charge (per unit length in z) on the plane $y = 0, x \geq 0$ is

$$Q_x = -\frac{2}{\pi} \lambda \tan^{-1} \left(\frac{x_0}{y_0} \right)$$

What is the total charge on the plane $x = 0$?

- (d) Show that far from the origin [$\rho \gg \rho_0$, where $\rho = \sqrt{x^2 + y^2}$ and $\rho_0 = \sqrt{x_0^2 + y_0^2}$] the leading term in the potential is

$$\Phi \rightarrow \Phi_{asym} = \frac{4\lambda}{\pi\epsilon_0} \frac{(x_0 y_0)(xy)}{\rho^4}$$

Interpret.

Solution

The setup of conductors and line charges is that of Fig. 2.3.

Part a

The potential in the first quadrant is the sum of potentials from the four line charges, e.g. the contribution from the line charge in the second quadrant is

$$\Phi_2(x, y) = -\frac{\lambda}{4\pi\epsilon_0} \ln \frac{R^2}{(x + x_0)^2 + (y - y_0)^2}$$

and the total potential is:

$$\begin{aligned} \Phi(x, y) &= \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 \\ &= \frac{\lambda}{4\pi\epsilon_0} \ln \left\{ \frac{(x - x_0)^2 + (y + y_0)^2}{(x - x_0)^2 + (y - y_0)^2} \cdot \frac{(x + x_0)^2 + (y - y_0)^2}{(x + x_0)^2 + (y + y_0)^2} \right\} \end{aligned}$$

In this form, it is easy to see that $\Phi(0, y) = \Phi(x, 0) = 0$. To verify that the tangential electric field vanishes at the conductor surfaces, we need (for the horizontal plane)

$$E_{\parallel}(y = 0) = - \left. \frac{\partial \Phi}{\partial x} \right|_{y=0}$$

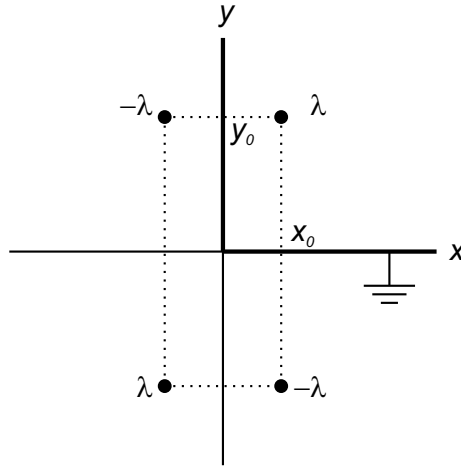


Figure 2.3: Problem 2.3. The setup of conductors and line charges.

This derivative is done more easily if the logarithm is split into four terms. I just quote the result:

$$\frac{\partial \Phi}{\partial x} = \frac{\lambda}{4\pi\epsilon_0} \left\{ -\frac{2(x-x_0)}{(x-x_0)^2+(y-y_0)^2} + \frac{2(x+x_0)}{(x+x_0)^2+(y-y_0)^2} - \frac{2(x+x_0)}{(x+x_0)^2+(y+y_0)^2} + \frac{2(x-x_0)}{(x-x_0)^2+(y+y_0)^2} \right\}$$

Inserting $y = 0$ it is easy to see that terms 1,4 and 2,3 cancel. We have thus verified that the tangential component of the electric field vanishes.

Part b

To get the induced surface charge density, we now need the normal derivative evaluated at the surface ($\partial\Phi/\partial y|_{y=0}$ for the horizontal conductor). Again, this is more easily done with four terms and proceeds similarly:

$$\frac{\partial \Phi}{\partial y} = -\frac{2\lambda}{4\pi\epsilon_0} \left\{ \frac{y-y_0}{(x-x_0)^2+(y-y_0)^2} - \frac{y-y_0}{(x+x_0)^2+(y-y_0)^2} + \frac{y+y_0}{(x+x_0)^2+(y+y_0)^2} - \frac{y+y_0}{(x-x_0)^2+(y+y_0)^2} \right\}$$

The induced surface charge density for the horizontal conductor is then:

$$\sigma(x) = -\frac{\lambda}{\pi} \left\{ \frac{y_0}{(x-x_0)^2+y_0^2} - \frac{y_0}{(x+x_0)^2+y_0^2} \right\}$$

Figure 2.4 shows the desired plot of the induced surface charge density. When the line charge is placed symmetrically ($x_0 = 1$, $y_0 = 1$), of course the distribution of charge on the two conductors is the same. With an asymmetric placement ($x_0 = 1$, $y_0 = 2$) of the line charge, the charge distribution on the other plane is the distribution for ($x_0 = 2$, $y_0 = 1$).

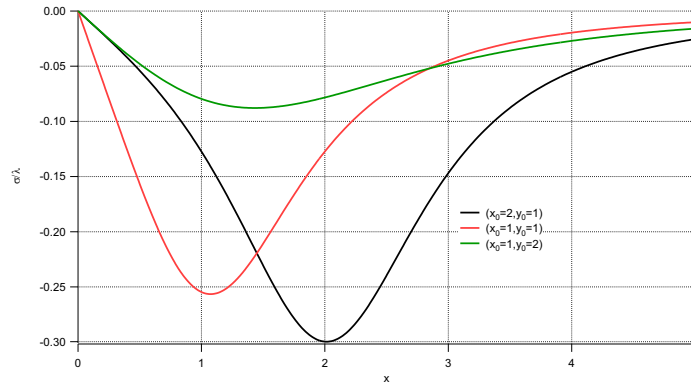


Figure 2.4: Problem 2.3. Plots of the surface charge density on the horizontal plane for selected sets of (x_0, y_0)

Part c

The total charge on the horizontal plane $y = 0$, $x \geq 0$ is obtained by integrating σdx :

$$\begin{aligned}
 Q_x &= \int_0^\infty \sigma dx \\
 &= -\frac{y_0 \lambda}{\pi} \left\{ \int_{-x_0}^\infty \frac{dt}{t^2 + y_0^2} - \int_{x_0}^\infty \frac{dt}{t^2 + y_0^2} \right\} \\
 &= -\frac{\lambda}{\pi} \tan^{-1} \left(\frac{t}{y_0} \right) \Big|_{-x_0}^{x_0} \\
 &= -\frac{2}{\pi} \lambda \tan^{-1} \left(\frac{x_0}{y_0} \right)
 \end{aligned}$$

The total charge on the other plane is, as noted in part b, the same expression with x_0 and y_0 exchanged:

$$Q_y = -\frac{2}{\pi} \lambda \tan^{-1} \left(\frac{y_0}{x_0} \right) = -\frac{2}{\pi} \lambda \cot^{-1} \left(\frac{x_0}{y_0} \right)$$

Since $\tan^{-1} x + \cot^{-1} x = \pi/2$ we see that the total induced surface charge per unit length in z on the conductor surface is always $-\lambda$, as it should be in order to prevent the electric field from the line charge from penetrating the conductors.

Part d

We shall find the leading term for $\rho \gg \rho_0$ in several steps. First consider the expressions under the logarithm, e.g.:

$$(x - x_0)^2 + (y + y_0)^2 = x^2 + x_0^2 - 2xx_0 + y^2 + y_0^2 + 2yy_0 \simeq x^2 - 2xx_0 + y^2 + 2yy_0$$

Inserting this in the expression for $\Phi(x, y)$ from part a yields

$$\begin{aligned}
\Phi(x, y) &\simeq \frac{\lambda}{4\pi\epsilon_0} \ln \left\{ \frac{\rho^4 - 4(xx_0 - yy_0)^2}{\rho^4 - 4(xx_0 + yy_0)^2} \right\} \\
&= \frac{\lambda}{4\pi\epsilon_0} \left[\ln \left\{ 1 - 4 \left(\frac{xx_0 - yy_0}{\rho^2} \right)^2 \right\} - \ln \left\{ 1 - 4 \left(\frac{xx_0 + yy_0}{\rho^2} \right)^2 \right\} \right] \\
&\simeq \frac{\lambda}{4\pi\epsilon_0} \left[-\frac{4}{\rho^4} (xx_0 - yy_0)^2 + \frac{4}{\rho^4} (xx_0 + yy_0)^2 \right] \\
&= \frac{4\lambda}{\pi\epsilon_0} \frac{(x_0 y_0)(xy)}{\rho^4}
\end{aligned}$$

This is the far field from a quadrupole.

Problem 2.4

A point charge is placed a distance $d > R$ from the center of an equally charged, isolated, conducting sphere of radius R .

- (a) Inside of what distance from the surface of the sphere is the point charge attracted rather than repelled by the charged sphere?
- (b) what is the limiting value of the force of attraction when the point charge is located a distance $a (= d - R)$ from the surface of the sphere, if $a \ll R$?
- (c) What are the results for parts a and b if the charge on the sphere is twice (half) as large as the point charge, but still the same sign?

[Answers: (a) $d/R - 1 = 0.6180$, (b) $F = -q^2/(16\pi\epsilon_0 a^2)$, i.e., image force, (c) for $Q = 2q$, $d/R - 1 = 0.4276$; for $Q = q/2$, $d/R - 1 = 0.8823$. The answer for part b is the same.]

Solution

Now we return to the situation of Sec. 2.3. Refer to Jackson's Fig. 2.5 (showing the force on the point charge) to get an overview of the situation. NB: the asymptotic (Coulomb-like) dependence has been divided out in the figure.

Part a

All we need to do here is find the root $d/R > 1$ of Eq. 2.9:

$$|\mathbf{F}| = \frac{1}{4\pi\epsilon_0} \frac{1}{d^2} \left[Q - \frac{qR^3(2d - R^2)}{d(d^2 - R^2)^2} \right] = 0$$

and the force is attractive for values of d/R less than the root. Defining $x \equiv d/R$ this reduces to

$$\frac{Q}{q}x^5 - 2\frac{Q}{q}x^3 - 2x^2 + \frac{Q}{q}x + 1 = 0$$

For $Q/q = 1$ and with the restraint that $x > 1$ my pocket calculator found $x = 1.618034$, i.e. for positions closer than $d/R - 1 = 0.6180$ to the surface the charge q is attracted towards the sphere. This also fits well with Jackson's Figure 2.5.

Part b

When the charge is a small distance $a = d - R$ from the surface of the sphere, we can perform a Taylor expansion of the force expression ($x = d/R = 1 + a/R$):

$$\begin{aligned} |\mathbf{F}| &= \frac{1}{4\pi\epsilon_0} \frac{1}{d^2} \left[Q - \frac{qR^3(2d - R^2)}{d(d^2 - R^2)^2} \right] \\ &= \frac{q^2}{4\pi\epsilon_0} \frac{1}{R^2} \left[1 - \frac{1}{x^5} \frac{2x^2 - 1}{(1 - \frac{1}{x^2})^2} \right] \\ &= -\frac{q^2}{4\pi\epsilon_0} \frac{1}{(2a)^2} \end{aligned}$$

i.e. the image force! This limiting value is independent of the radius of the sphere R and the charge on the sphere Q .

Part c

For $Q/q = 2$, we find from the equation in part a $d/R - 1 = 0.4276$ while for $Q/q = 1/2$ we find $d/R - 1 = 0.8823$. As mentioned in part b, the limiting value for the attractive force when q is very close to the sphere is independent of Q so the answers for the force are the same as before.

Problem 2.5

- (a) Show that the work done to remove the charge q from a distance $r > a$ to infinity against the force, Eq. (2.6), of a grounded conducting sphere is

$$W = \frac{q^2 a}{8\pi\epsilon_0(r^2 - a^2)}$$

Relate this result to the electrostatic potential, Eq. (2.3), and the energy discussion of Section 1.11.

- (b) Repeat the calculation of the work done to remove the charge q against the force, Eq. (2.9), of an isolated charged conducting sphere. Show that the work done is

$$W = \frac{1}{4\pi\epsilon_0} \left[\frac{q^2 a}{2(r^2 - a^2)} - \frac{q^2 a}{2r^2} - \frac{qQ}{r} \right]$$

Relate the work to the electrostatic potential, Eq. (2.8), and the energy discussion of Section 1.11.

Part a

The force attracting q towards the grounded sphere, i.e. opposite to the direction of the position vector of q , is given by Eq. (2.6)

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \left(\frac{a}{y}\right)^3 \left(1 - \frac{a^2}{y^2}\right)^{-2}$$

Then the work required to move q to infinity is

$$\begin{aligned} W &= + \int_r^\infty F dy \\ &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \int_r^\infty dy \left(\frac{a}{y}\right)^3 \left(1 - \frac{a^2}{y^2}\right)^{-2} \end{aligned}$$

This integral is obviously performed by substituting $x = 1 - a^2/y^2$ and is otherwise elementary. We get

$$W = \frac{q^2 a}{8\pi\epsilon_0(r^2 - a^2)}$$

We can compare this result with the energy required to disassemble the charges by using Eq. (1.51) for the potential energy of a distribution of discrete charges (which is the negative value of the energy required to disassemble the charges)

$$W' = \frac{1}{8\pi\epsilon_0} \sum_i \sum_j \frac{q_i q_j}{|\mathbf{x}_i - \mathbf{x}_j|}$$

It is understood that $i = j$ terms (infinite “self-energy” terms) are omitted in the double sum. This gives two (equal) terms as follows

$$-W' = \frac{q}{4\pi\epsilon_0} \left(\frac{aq^2}{r} \frac{1}{r(1 - a^2/r^2)} \right) = \frac{q^2 a}{4\pi\epsilon_0(r^2 - a^2)} = 2W$$

This is the same effect found in Problem 2.1. The reason for the unequal 1:2 magnitudes is again that in the former case, work is only done on one charge (the “real” one) and the image charge is changing position and size continually. In the latter case, we do work on both (constant magnitude) charges.

Part b

Now the force is (2.9)

$$F = \frac{1}{4\pi\epsilon_0} \frac{q}{y^2} \left[Q - \frac{qa^3(2-y^2-a^2)}{y(y^2-a^2)^2} \right]$$

One could attack this expression head-on but it is easier to deconvolute the force expression into its two constituents from the image charge $q' = -aq/y$ and the charge $(Q - q')$ sitting at the center of the sphere. Then the work is the expression from part a plus

$$W_{Q-q'} = -\frac{q}{4\pi\epsilon_0} \int_r^\infty \left(\frac{Q}{r^2} + \frac{a}{r^3}q \right) dr$$

This is very simple and we get

$$W = \frac{1}{4\pi\epsilon_0} \left[\frac{q^2a}{2(r^2-a^2)} - \frac{q^2a}{2r^2} - \frac{qQ}{r} \right]$$

The first term is the one from part a while the two latter terms is the work done against the attraction from the charge $(Q - q')$ at the center of the sphere. Comparing again to the work needed to disassemble the discrete (constant magnitude) charges through Eq. (1.51) we get

$$-W' = \frac{1}{4\pi\epsilon_0} \left[\frac{q^2a}{r^2-a^2} - \frac{q(Q + \frac{a}{r}q)}{r} \right] = \frac{1}{4\pi\epsilon_0} \left[\frac{q^2a}{r^2-a^2} - \frac{q^2a}{r^2} - \frac{qQ}{r} \right]$$

For the same reasons as in part a, the two former terms are double the corresponding ones in part a, but the last term is the same because Q on the sphere is fixed in both cases.

Problem 2.6

The electrostatic problem of a point charge outside an isolated, charged conducting sphere is equivalent to that of three charges, the original and two others, one located at the center of the sphere and another (“the image charge”) inside the now imaginary sphere, on the line joining the center and the original charge.

If the point charge and sphere are replaced by two conducting spheres of radii r_a and r_b , carrying total charges Q_a and Q_b , respectively, with centers separated by a distance $d > r_a + r_b$, there is an equivalence with an infinite set of charges within each sphere, one at the center and a set of images along the line joining the centers. The charges and their locations can be determined iteratively, starting with a charge $q_a(1)$ at the center of the first sphere and $q_b(1)$ correspondingly for the second sphere. The charge $q_b(1)$ has its image $q_a(2)$ within the first sphere and vice versa. Then the image charge within the first sphere induces another image within the second sphere, and so on. The sum of all the charges within each sphere must be scaled to be equal to Q_a and Q_b .

The electrostatic potential outside the spheres, the force between the spheres, etc. can be found by summing the contributions from all the charges.

- (a) Show that the charges and their positions are determined iteratively by the relations,

$$\begin{aligned} q_a(j) &= -r_a q_b(j-1)/d_b(j-1), & x_a(j) &= r_a^2/d_b(j-1), & d_a(j) &= d - x_a(j) \\ q_b(j) &= -r_b q_a(j-1)/d_a(j-1), & x_b(j) &= r_b^2/d_a(j-1), & d_b(j) &= d - x_b(j) \end{aligned}$$

for $j = 2, 3, 4, \dots$, with $d_a(1) = d_b(1) = d$, and $x_a(1) = x_b(1) = 0$.

- (b) Find the image charges and their locations as well as the potentials on the spheres and force between them by means of a suitable computer program. [In computing the potential on each sphere, evaluate it in different places: e.g., in the equatorial plane and at the pole opposite the other sphere. This permits a check on the equipotential of the conductor and on the accuracy of computation.]
- (c) As an example, show that for two equally charged spheres of the same radius R , the force between them when almost in contact is 0.6189 times the value that would be obtained if all the charge on each sphere were concentrated at its center. Show numerically and by explicit summation of the series that the capacitance of two identical conducting spheres in contact is $C/4\pi\epsilon_0 R = 1.3863 \dots [= \ln 4]$