Chapter 1

Introduction

There are two reasons for studying stellar pulsations: to understand why, and how, certain types of stars pulsate; and to use the pulsations to learn about the more general properties of these, and hence perhaps other, types of stars.

Stars whose luminosity varies periodically have been known for centuries. However, only within the last hundred years has it been definitely established that in many cases these variations are due to *intrinsic* pulsations of the stars themselves. For obvious reasons studies of pulsating stars initially concentrated on stars with large amplitudes, such as the Cepheids and the long period variables. The variations of these stars could be understood in terms of pulsations in the fundamental radial mode, where the star expands and contracts, while preserving spherical symmetry. It was realized very early (Shapley 1914) that the period of such motion is approximately given by the dynamical time scale of the star:

$$t_{\rm dyn} \simeq \left(\frac{R^3}{GM} \right)^{1/2} \simeq (G \bar{\rho})^{-1/2} \; , \tag{1.1}$$

where R is the radius of the star, M is its mass, $\bar{\rho}$ is its mean density, and G is the gravitational constant. Thus observation of the period immediately gives an estimate of one intrinsic property of the star, *viz.* its mean density.

It is a characteristic property of the Cepheids that they lie in a narrow, almost vertical strip in the HR diagram, the so-called *instability strip.* As a result, there is a direct relation between the luminosities of these stars and their radii; assuming also a mass-luminosity relation one obtains a relation between the luminosities and the periods, provided that the latter scale as $t_{\rm dyn}$. This argument motivates the existence of a *period-luminosity relation* for the Cepheids: thus the periods, which are easy to determine observationally, may be used to infer the intrinsic luminosities; since the apparent luminosities can be measured, one can determine the distance to the stars. This provides one of the most important distance indicators in astrophysics.

The main emphasis in the early studies was on understanding the causes of the pulsations, particularly the concentration of pulsating stars in the instability strip. As in many other branches of astrophysics major contributions to the understanding of stellar pulsation were made by Eddington (*e.g.* Eddington 1926). However, the identification of the actual cause of the pulsations, and of the reason for the instability strip, was first arrived at independently by Zhevakin (1953) and by Cox & Whitney (1958).

In parallel with these developments, it has come to be realized that some, and probably very many, stars pulsate in more complicated manners than the Cepheids. In many instances more than one mode of oscillation is excited simultaneously in a star; these modes may include both radial overtones, in addition to the fundamental, and *nonradial* modes, where the motion does not preserve spherical symmetry. (It is interesting that Emden [1907], who laid the foundation for the study of polytropic stellar models, also considered a rudimentary description of such nonradial oscillations.) This development is extremely important for attempts to use pulsations to learn about the properties of stars: each observed period is in principle (and often in practice) an independent measure of the structure of the star, and hence the amount of information about the star grows with the number of modes that can be detected. A very simple example are the *double mode Cepheids*, which have been studied extensively by, among others, J. Otzen Petersen, Copenhagen (*e.g.* Petersen 1973, 1974, 1978). These are apparently normal Cepheids which pulsate simultaneously in two modes, in most cases identified as the fundamental and the first overtone of radial pulsation. While measurement of a single mode, as discussed above, provides a measure of the mean density of the star, two periods roughly speaking allow determination of its mass and radius. It is striking that, as discussed by Petersen, even this limited information about the stars led to a conflict with the results of stellar evolution theory which has only been resolved very recently with the computation of new, improved opacity tables.

In other stars, the number of modes is larger. An extreme case is the Sun, where currently several thousand individual modes have been identified. It is expected that with more careful observation, frequencies for as many as 10^6 modes can be determined accurately. Even given likely advances in observations of other pulsating stars, this would mean that more than half the total number of known oscillation frequencies for *all* stars would belong to the Sun. This vast amount of information about the solar interior forms the basis for *helioseismology*, the science of learning about the Sun from the observed frequencies. This has already led to a considerable amount of information about the structure and rotation of the solar interior; much more is expected from observations, including some from space, now being prepared.

The observed solar oscillations mostly have periods in the vicinity of five minutes, considerably shorter than the fundamental radial period for the Sun, which is approximately 1 hour. Both the solar five-minute oscillations and the fundamental radial oscillation are acoustic modes, or *p modes*, driven predominantly by pressure fluctuations; but whereas the fundamental radial mode has no nodes, the five-minute modes are of high radial order, with $20 - 30$ nodes in the radial direction.

The observational basis for helioseismology, and the applications of the theory developed in these notes, are described in a number of reviews. General background information was provided by, for example, Deubner & Gough (1984), Leibacher *et al.* (1985), Christensen-Dalsgaard, Gough & Toomre (1985), Libbrecht (1988), Gough & Toomre (1991) and Christensen-Dalsgaard & Berthomieu (1991). Examples of more specialized applications of helioseismology to the study of the solar interior were given by Christensen-Dalsgaard (1988a, 1996a).

Since we believe the Sun to be a normal star, similarly rich spectra of oscillations would be expected in other similar stars. An immediate problem in observations of stars, however, is that they have no, or very limited, spatial resolution. Most of the observed solar modes have relatively short horizontal wavelength on the solar surface, and hence would not be detected in stellar observations. A second problem in trying to detect the expected solar-like oscillations in other stars is their very small amplitudes. On the Sun the *maximum* velocity amplitude in a single mode is about $15\,\mathrm{cm\,s^{-1}}$, whereas the luminosity amplitudes are of the order of 1 micromagnitude or less. Clearly extreme care is required in observing such oscillations in other stars, where the total light-level is low. In fact, despite several attempts and some tentative results, no definite detection of oscillations in a solar-like star has been made. Nevertheless, to obtain information, although less detailed than available for the Sun, for other stars would be extremely valuable; hence a great deal of effort is being spent on developing new instrumentation with the required sensitivity.

Although oscillations in solar-like stars have not been definitely detected, other types of stars display rich spectra of oscillations. A particularly interesting case are the white dwarfs; pulsations are observed in several groups of white dwarfs, at different effective temperatures. Here the periods are considerably *longer* than the period of the fundamental radial oscillation, indicating that a radically different type of pulsation is responsible for the variations. In fact it now seems certain that the oscillations are driven by buoyancy, as are internal gravity waves; such modes are called *g modes.* An excellent review of the properties of pulsating white dwarfs was given by Winget (1988). Another group of stars of considerable interest are the δ Scuti stars, which fall in the instability strip near the main sequence.

The present notes are mainly concerned with the basic theory of stellar pulsation, particularly with regards to the oscillation periods and their use to probe stellar interiors. However, as a background to the theoretical developments, Chapter 2 gives a brief introduction to the problems encountered in analyses of observations of pulsating stars, and summarizes the existing data on the Sun, as well as on δ Scuti stars and white dwarfs. A main theme in the theoretical analysis is the interplay between numerical calculations and simpler analytical considerations. It is a characteristic feature of many of the observed modes of oscillation that their overall properties can be understood quite simply in terms of asymptotic theory, which therefore gives an excellent insight into the relation between the structure of a star, say, and its oscillation frequencies. Asymptotic results also form the basis for some of the techniques for *inverse analysis* used to infer properties of the solar interior from observed oscillation frequencies. However, to make full use of the observations accurate numerical techniques are evidently required. This demand for accuracy motivates including a short chapter on some of the numerical techniques that are used to compute frequencies of stellar models. Departures from spherical symmetry, in particular rotation, induces fine structure in the frequencies. This provides a way of probing the internal rotation of stars, including the Sun, in substantial detail. A chapter on inverse analyses discusses the techniques that are used to analyse the observed solar frequencies and gives brief summaries of some of the results. The notes end with an outline of some aspects of the theory of the excitation of stellar pulsations, and how they may be used to understand the location of the Cepheid instability strip.