The Physics of Stellar Collapse and Core-Collapse Supernovae

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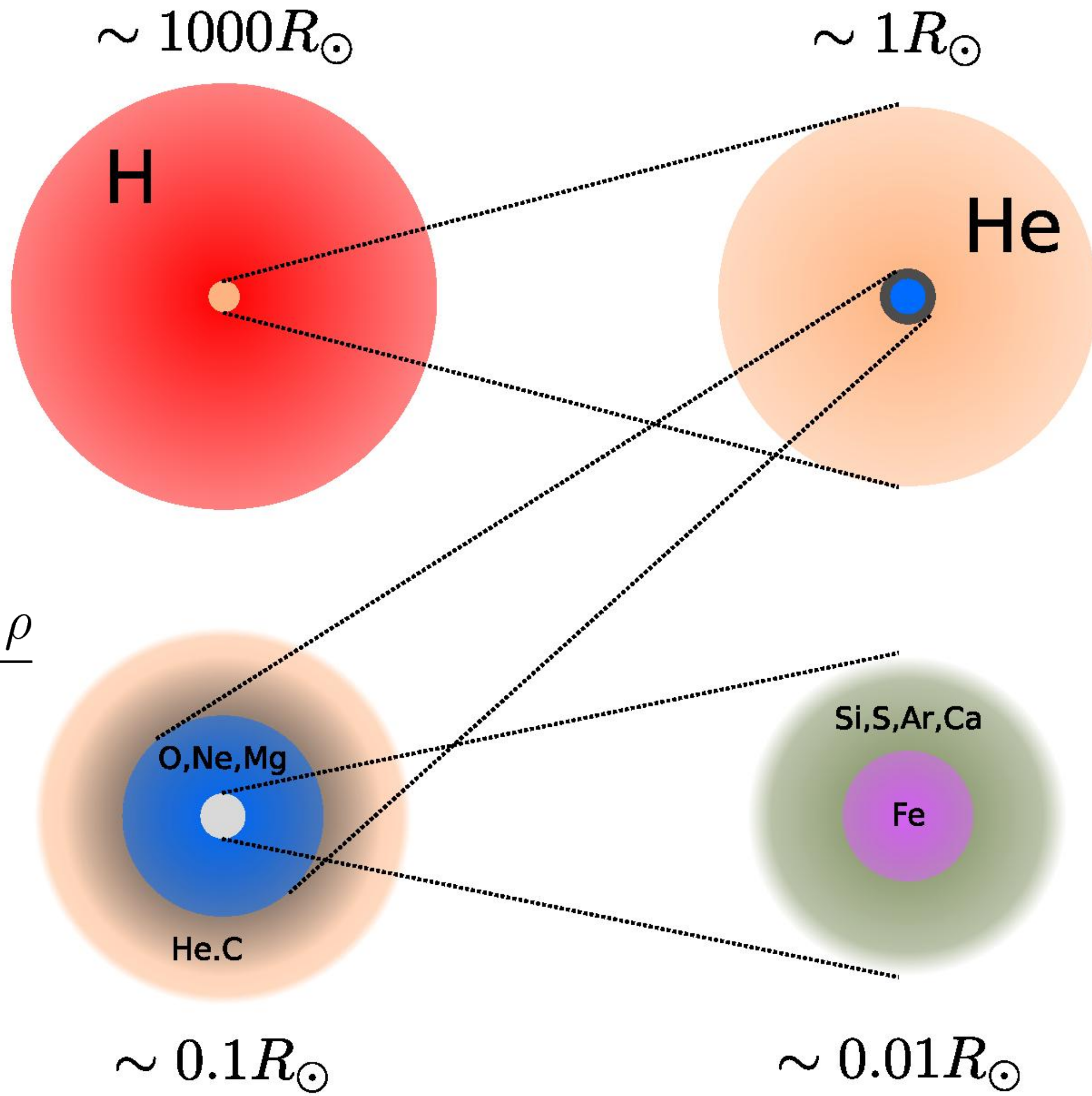
Outline

- Onset of Collapse, Hydrodynamics, and Homologous Collapse.
- Collapse Microphysics and Neutrino Trapping.
- Core Bounce and Shock Formation.
- The Supernova Problem and its Energetics.
- Supernova simulations:
  1D, 2D, and 3D and their ingredients.
- Candidate supernova mechanisms.

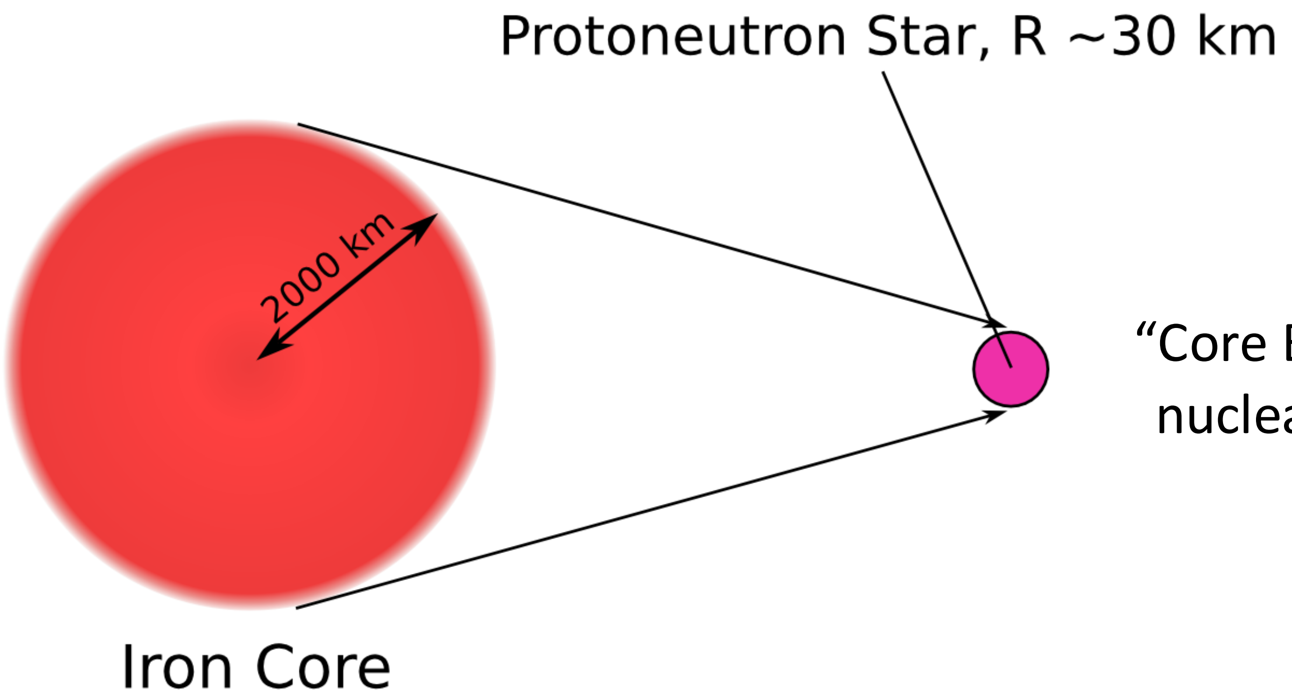
Situation at the Onset of Collapse

Hydrostatic Equilibrium

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$



The General Picture

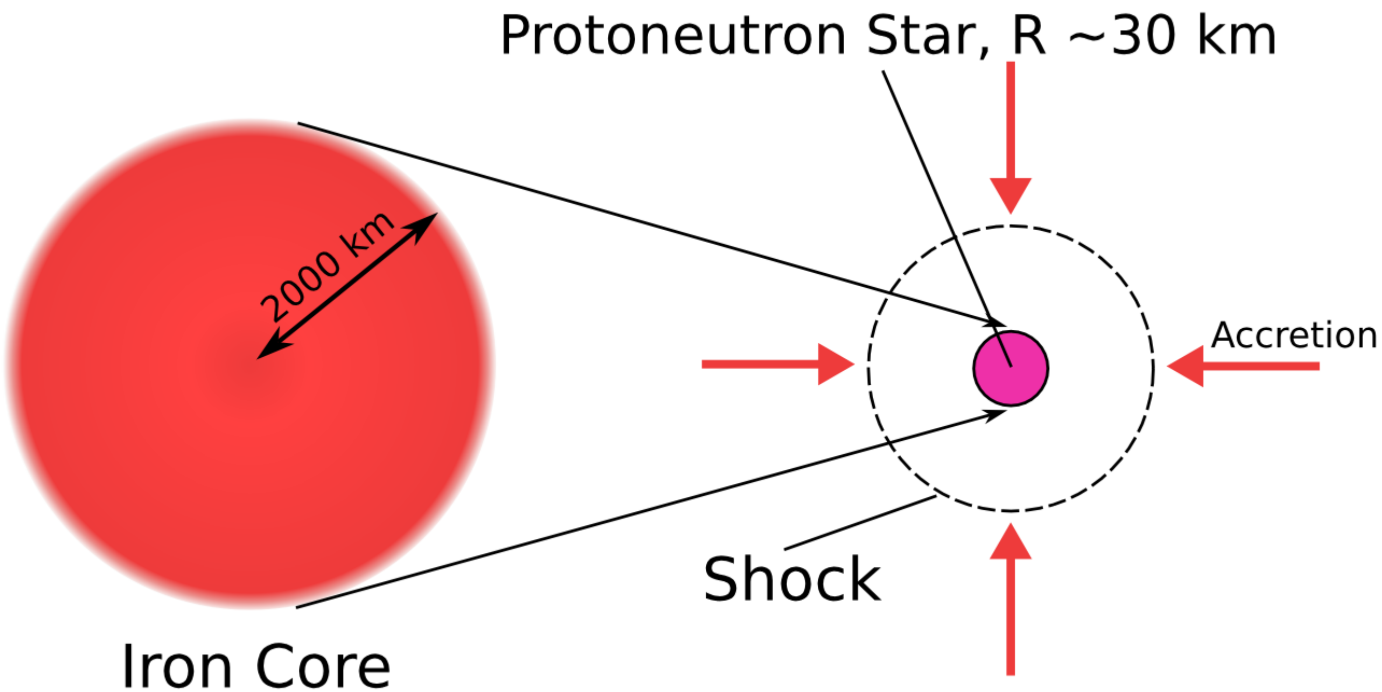


"Core Bounce" at nuclear density.

$M \approx 1.3 - 2.2\ M_{SUN}$

$M = M_{CH,0}$ + corrections (thermal, GR, etc.)

The General Picture



$M \approx 1.3 - 2.2\ M_{SUN}$

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Blackboard/Lecture Notes

Studying Stellar Collapse: Essential Ingredients

Fully coupled!

- Hydrodynamics
- Gravity: Newton / General Relativity
- Microphysics: Nuclear & Neutrino Physics
- Transport Theory: Neutrino Transport

Hydrodynamics

Conservation Laws: Mass, Momentum, Energy

(Ideal fluid approximation -> No viscosity, radiation)

- Mass Conservation -> Equation of Continuity

$$\frac{\partial}{\partial t}\int \rho dV = -\int_{\partial V} \rho \vec{v}\cdot\vec{n}dS$$

Using Gauss's theorem, we can rewrite this to:

$$\frac{\partial}{\partial t}\int \rho dV = -\int \nabla(\rho\vec{v})dV$$

Since this must hold for any volume V, the **continuity equation** follows:

$$\frac{\partial}{\partial t}\rho + \nabla(\rho\vec{v}) = 0$$

- Same approach: momentum and energy equations

The Equations of Newtonian Hydrodynamics

(Eulerian Formulation -> “laboratory frame”)

Continuity

$$\frac{\partial}{\partial t}\rho + \nabla(\rho\vec{v}) = 0$$

Momentum

$$\frac{\partial}{\partial t}(\rho\vec{v}) + \nabla(\rho\vec{v}\vec{v}) + \nabla P = -\rho\nabla\Phi$$

Energy

$$\frac{\partial}{\partial t}(\rho e) + \nabla(\vec{v}(\rho e + P)) = -\rho\vec{v}\,\nabla\Phi$$

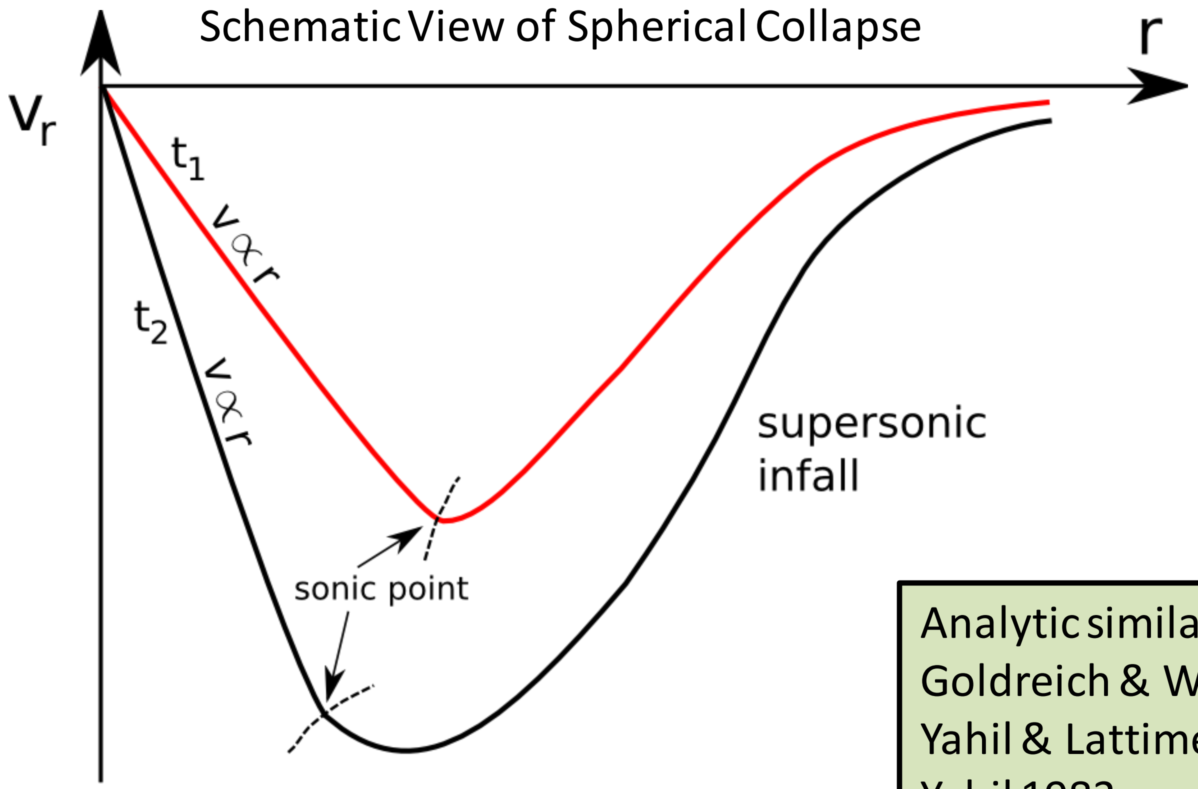
Newtonian Gravity

$$e = \epsilon + \frac{1}{2}v^2$$

- Alternative: Lagrangian formulation (“comoving frame”). Transformation:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}\,\nabla$$

Self-Similarity in Stellar Collapse



Analytic similarity solutions:
Goldreich & Weber 1980
Yahil & Lattimer 1982
Yahil 1983

- Separation into **homologously ($v \propto r$) collapsing inner core** and **supersonically collapsing outer core**.

Important Collapse Microphysics

In collapse, pressure support is reduced by

- **Photodissociation** of heavy nuclei: ~125 MeV/reaction

$$\gamma + {}^{56}_{26}\mathrm{Fe} \rightleftharpoons 13\alpha + 4n$$

- **Electron Capture**

$$e^- + (Z, A) \overset{(W)}{\to} \nu_e + (Z-1, A)$$
$$e^- + p \overset{(W)}{\to} \nu_e + n \ .$$

Capture rates: $\frac{\partial}{\partial t} Y_e \propto \mu_e^5 \propto \rho^{5/3}$

- Neutrinos stream off freely at densities below ~10^{12} g/cm^3.
  -> core "deleptonizes" during collapse.
- Net entropy change very small,
  -> collapse proceeds practically adiabatic.

Neutrino Trapping

- Collapse phase: Neutrino opacity dominated by coherent neutrino-nucleus scattering: $\nu + (A,Z) \longleftrightarrow \nu + (A,Z)$

Neutrino mean-free path:

$$\lambda_\nu \approx 10^7\,\mathrm{cm}\left(\frac{10^{12}\,\mathrm{g\,cm^{-3}}}{\rho}\right)\frac{A}{N^2}\left(\frac{10\,\mathrm{MeV}}{\epsilon_\nu^2}\right)$$

- For $\rho \geq 3 \times 10^{12}$ g/cm^3, diffusion time τ_{diff} >> time between collisions τ_{coll} -> **neutrinos become trapped in the collapsing core.**
- **Consequences**:

Deleptonization stopped

$$Y_{\mathrm{lep}} = Y_e + Y_\nu = \mathrm{const.}$$

Detailed simulations:

$$Y_{\mathrm{lep}} \approx 0.32$$

Beta Equilibrium

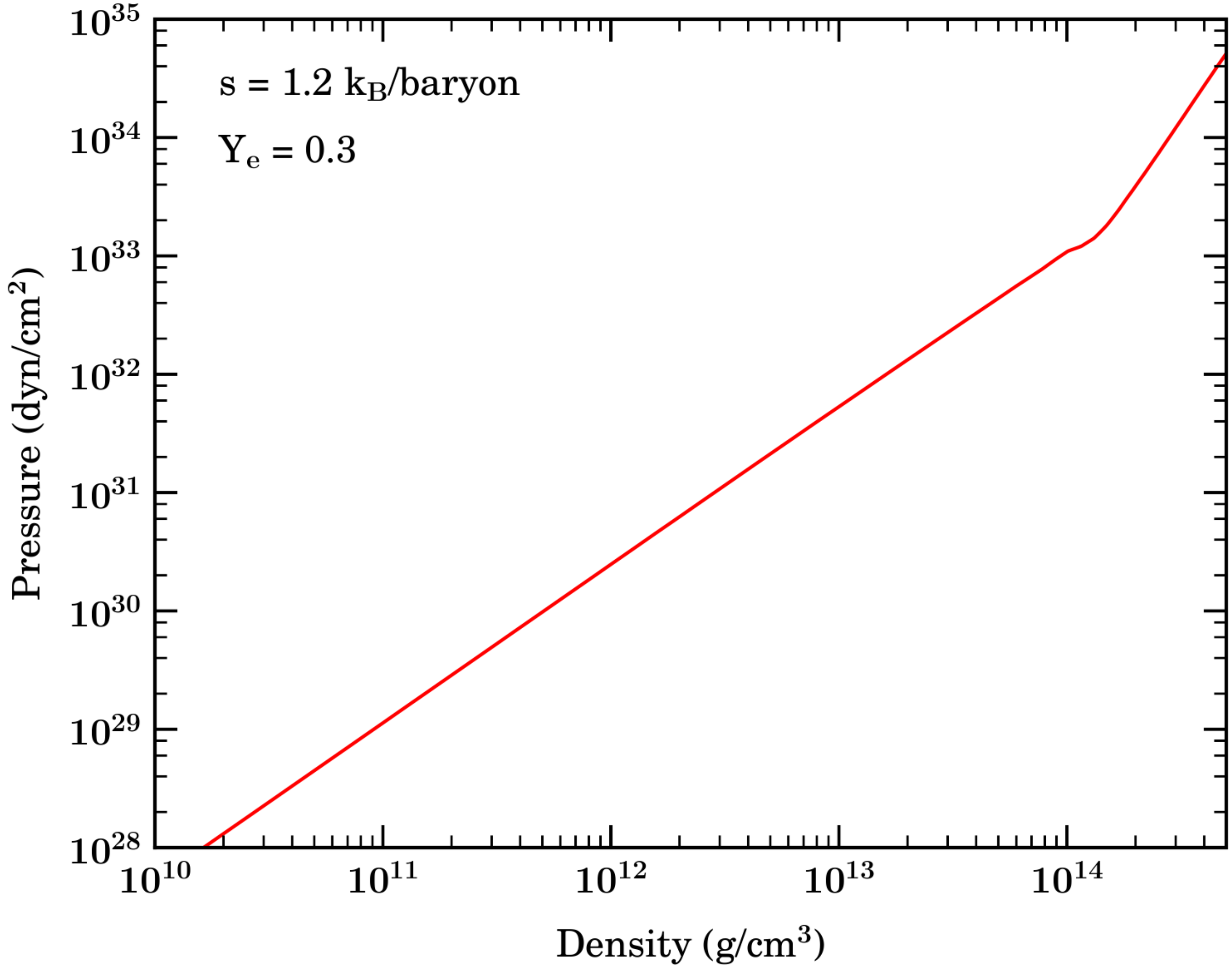
$$e^- + p \longleftrightarrow \nu_e + n$$

$$\mu_e + \mu_p = \mu_\nu + \mu_n$$

The Nuclear Equation of State (EOS)

Nuclear Statistical Equilibrium ($\rho > 10^7$ g/cm^3, $T > 0.5$ MeV)

-> $P = P(\rho, T, Y_e)$



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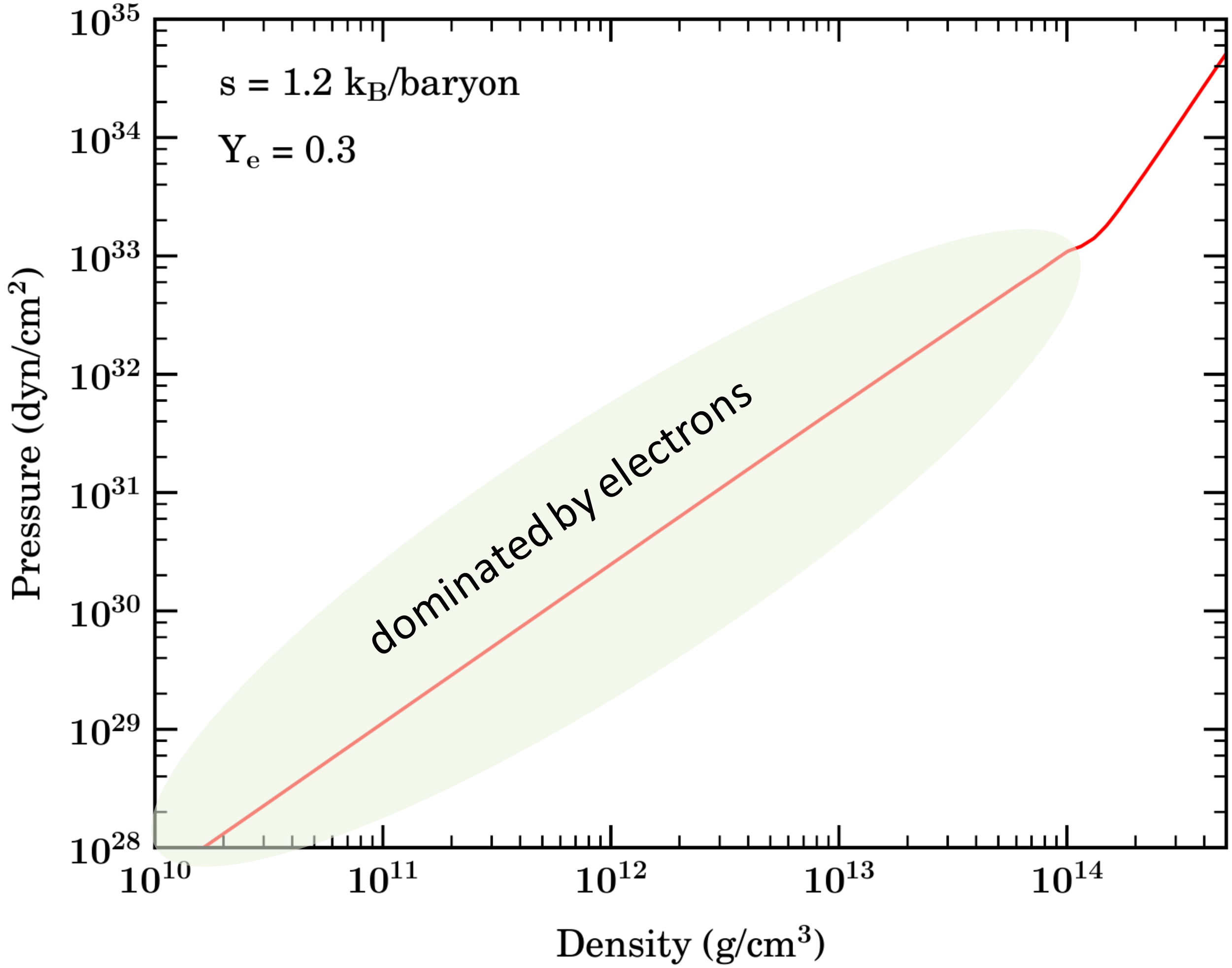

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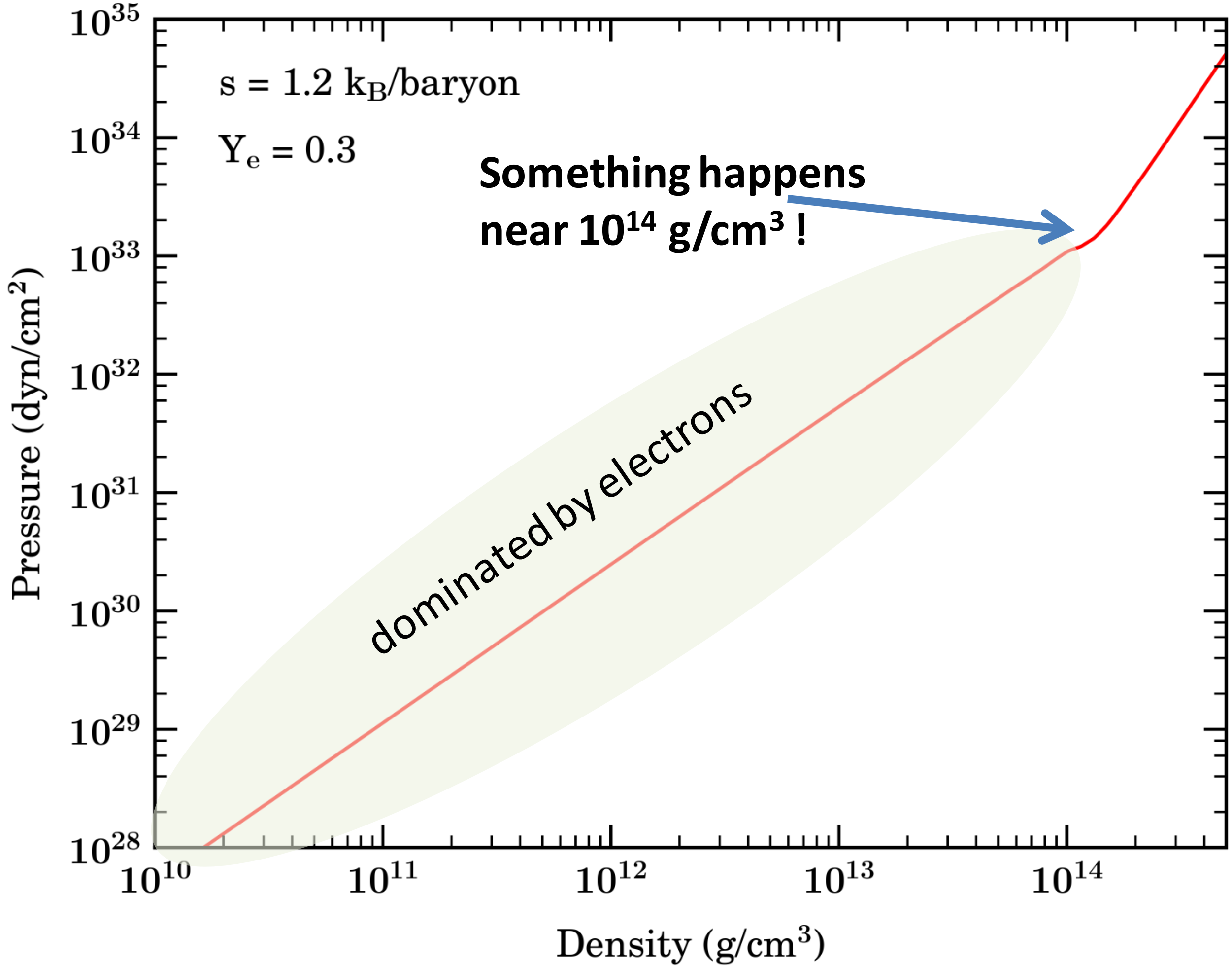


The Nuclear Equation of State (EOS)

Nuclear Physics:

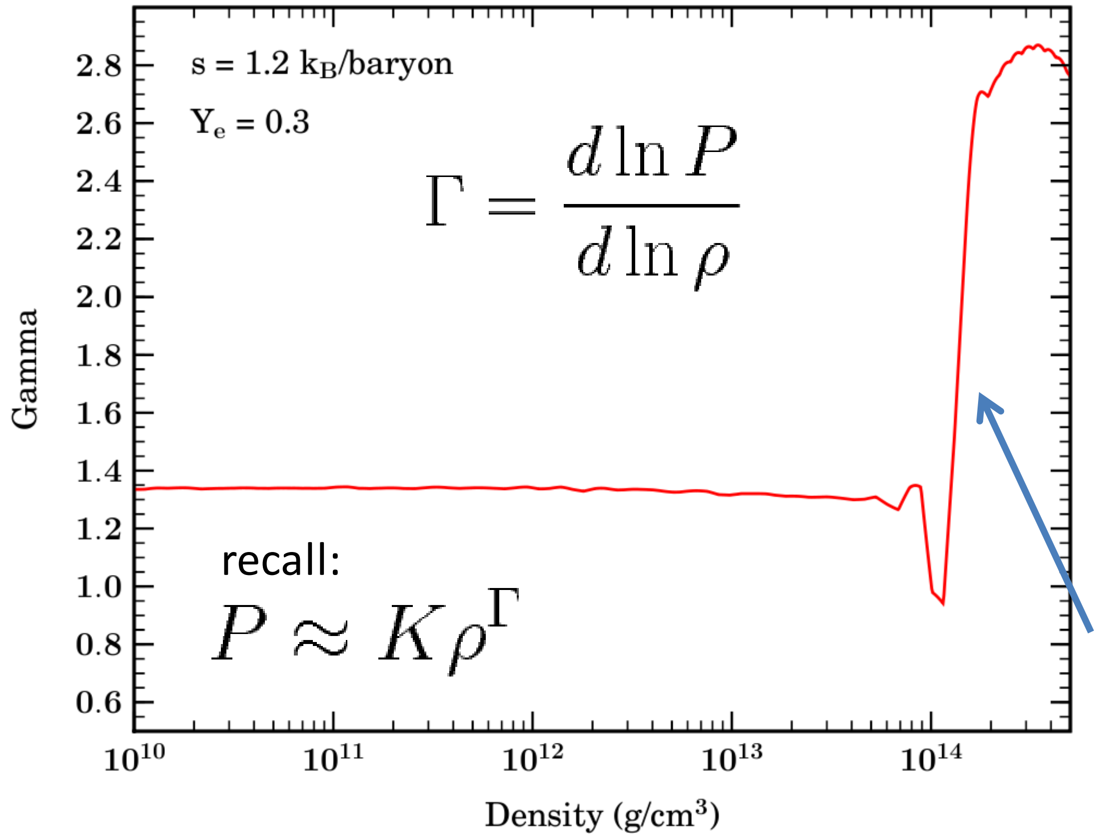
$$R_{\rm nuc} = A^{1/3} r_0$$

$$r_0 = 1.25\,{\rm fm}$$

Nuclear Density:

$$\bar{\rho}_{\rm nuc} = \frac{A\, m_b}{\frac{4}{3}\pi R_{\rm nuc}^3}$$

$$\bar{\rho}_{\rm nuc} \approx 2 \times 10^{14}\,{\rm g/cm^{-3}}$$



"Stiffening" of the EOS

Nuclear EOS: What happens near ρ_{nuc}?

Nuclear Physics:

$$R_{\rm nuc} = A^{1/3} r_0$$

$$r_0 = 1.25\,\mathrm{fm}$$

Nuclear Density:

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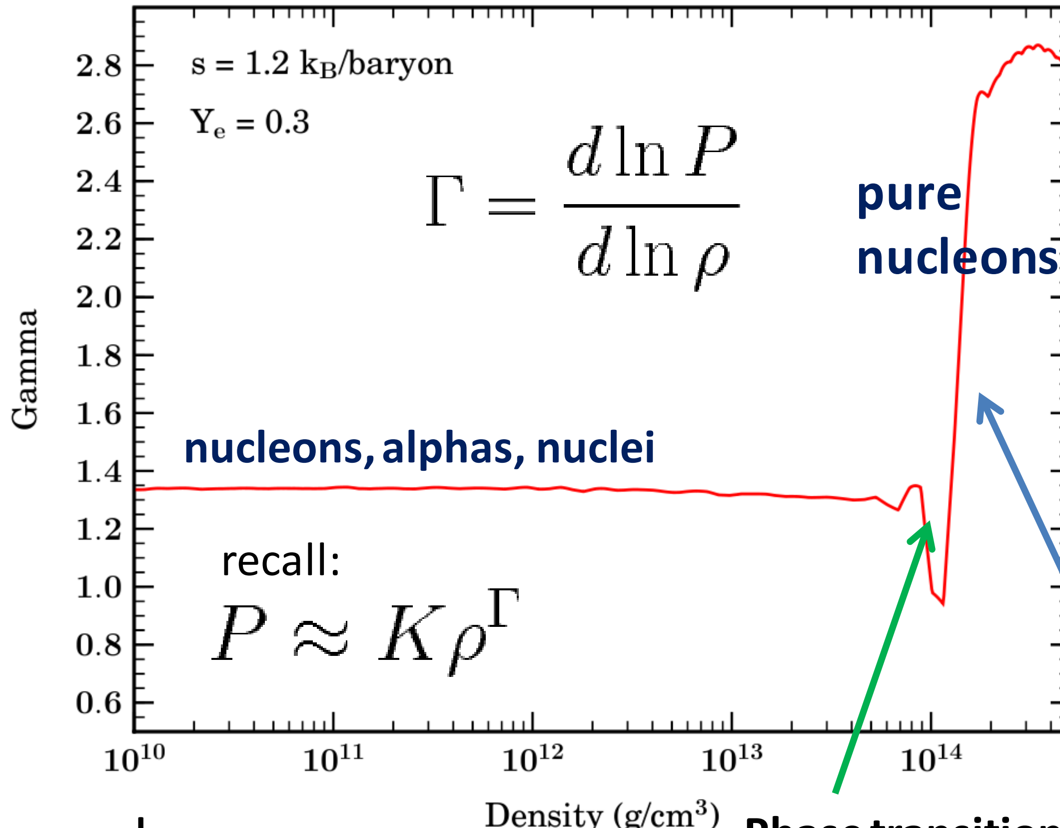


$$\Gamma = \frac{d\ln P}{d\ln \rho}$$

recall:

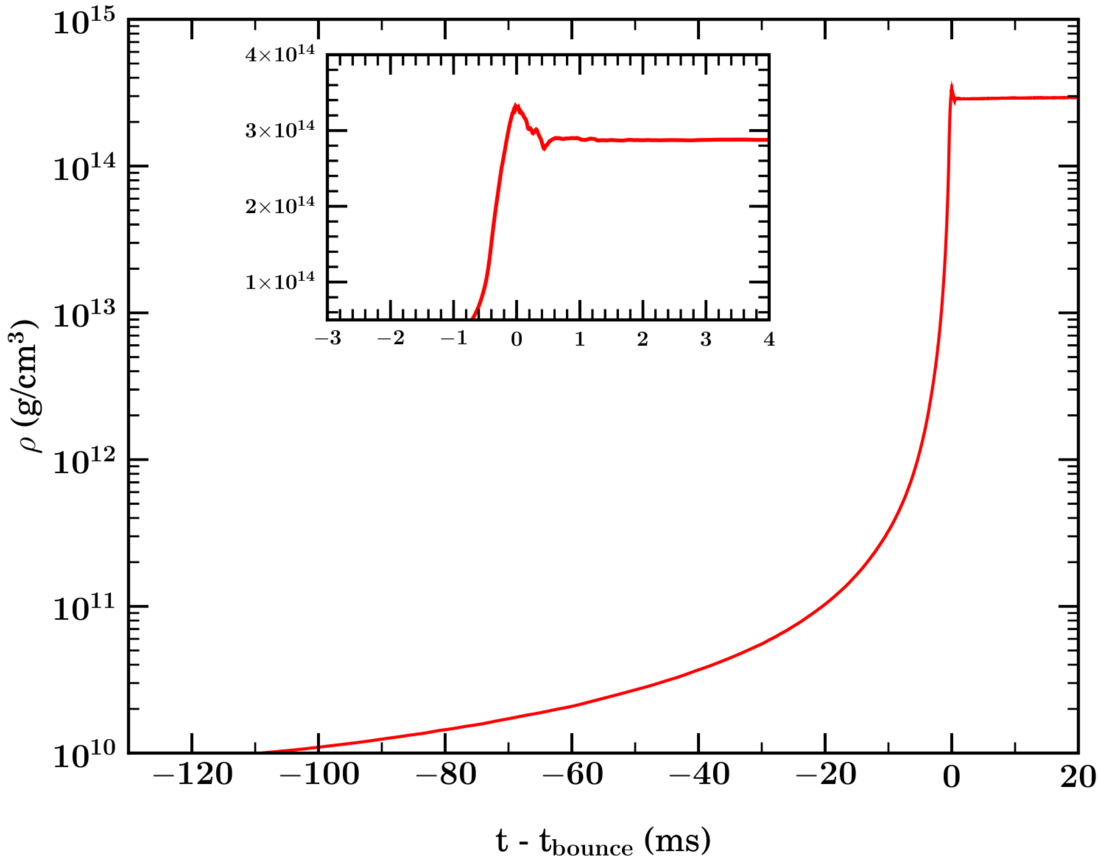
$$P \approx K \rho^{\Gamma}$$

"Stiffening" of the EOS

Phase transition from inhomogeneous to homogeneous nuclear matter

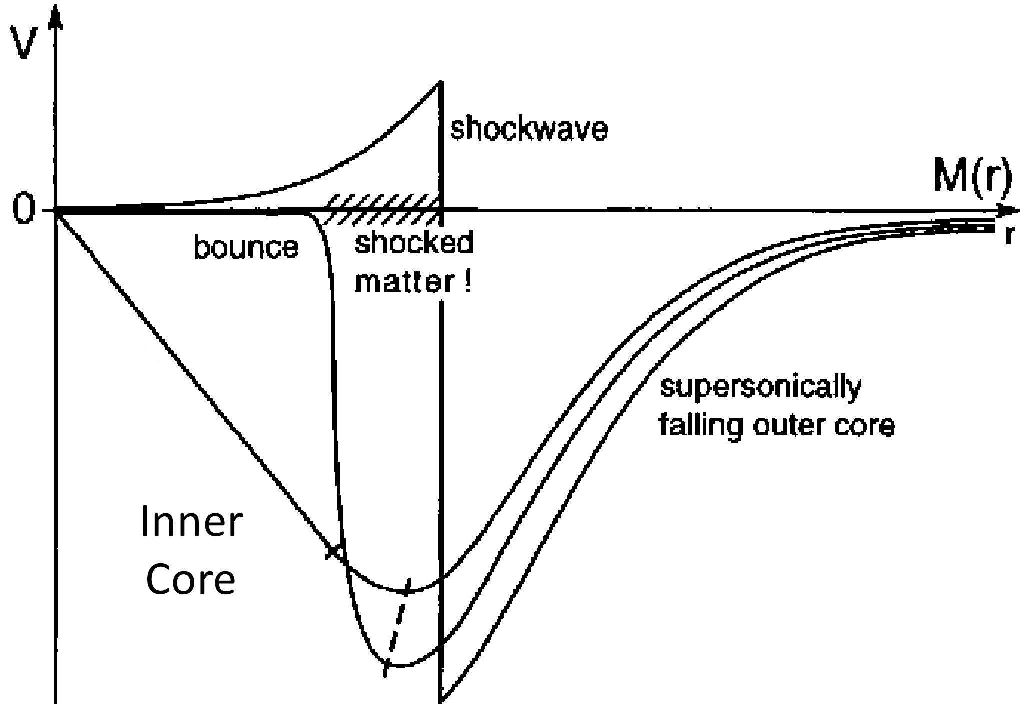
- Above $\approx\rho_{nuc}$ n,p are so close that "repulsive core" of the strong force kicks in and leads to the stiffening of the EOS

Collapse and Bounce



- **Inner Core** reaches ρ_{nuc}, rebounds ("bounces") **into still infalling outer core**.

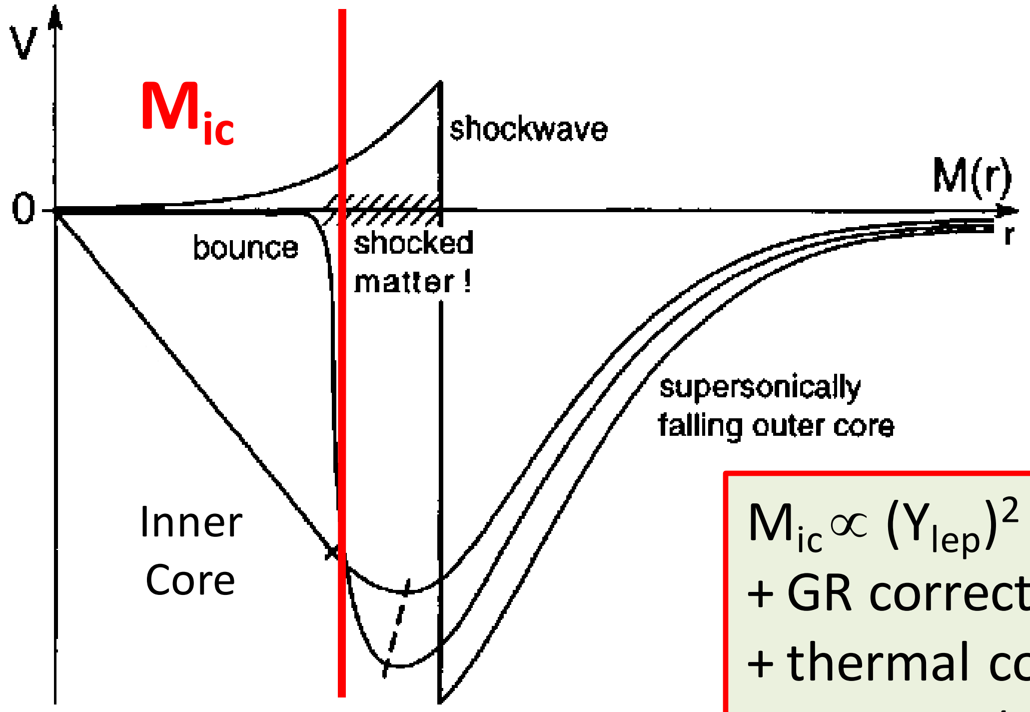
Shock Formation



Credit:
E. Müller
Saas-Fee Lectures 1998

- Stiffening of EOS leads to sound wave that propagates through the inner core and steepens to a shock at the sonic point.

Universality of Core Collapse



Credit:
E. Müller
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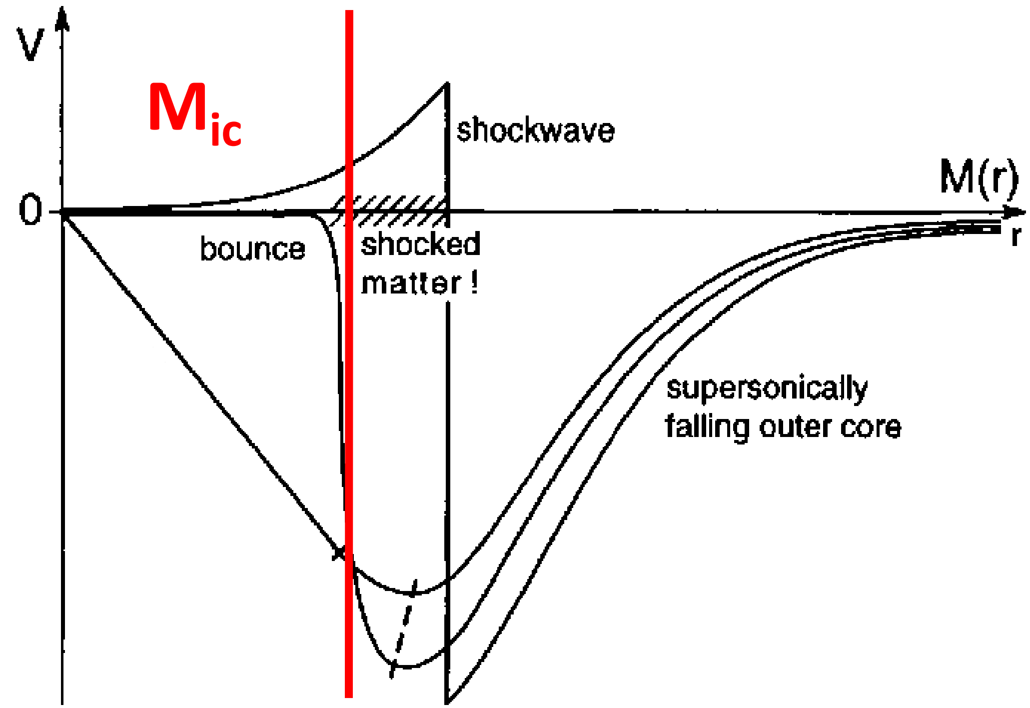
$M_{ic} \propto (Y_{lep})^2$
+ GR correction (-)
+ thermal correction (+)
+ rotation (+)

The Mass M_{ic} of the inner core at bounce is determined by nuclear physics and weak interactions, is ~0.5 M_{SUN}, and is practically independent of progenitor star mass and structure.

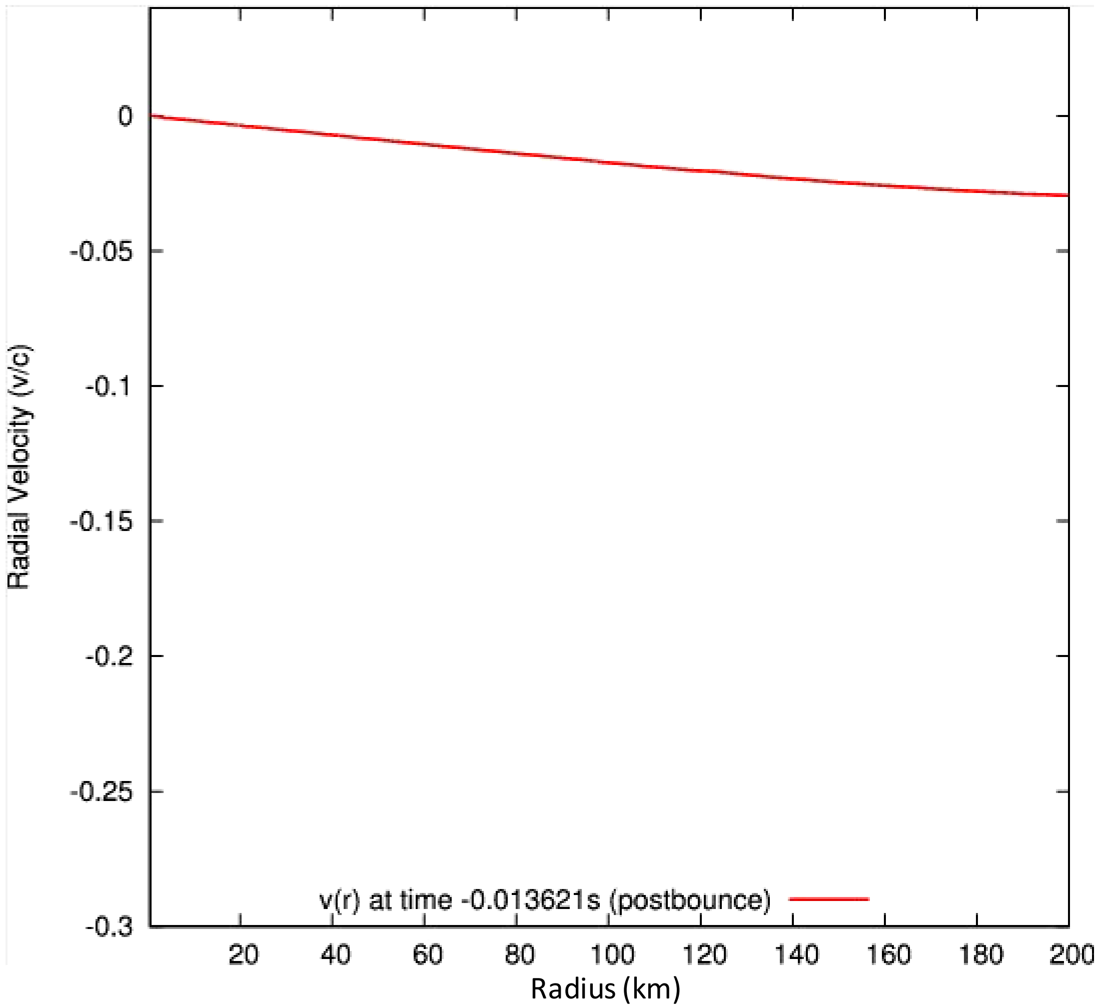
Why worry about M_{ic}?

- M_{ic} is the amount of matter dynamically relevant in bounce.
- M_{ic} sets kinetic energy imparted to the shock.
- M_{ic} (and IC radius) sets the angular momentum that can be dynamically relevant.
- Sets mass cut for material that the shock needs to go through.



- M_{ic} ~0.5 M_{SUN} can easily stabilized by nuclear EOS. No "prompt" Black Hole formation.
- M_{ic} sets the mass that must be accreted (before explosion?) to make a canonical 1.4 M_{SUN} neutron star.

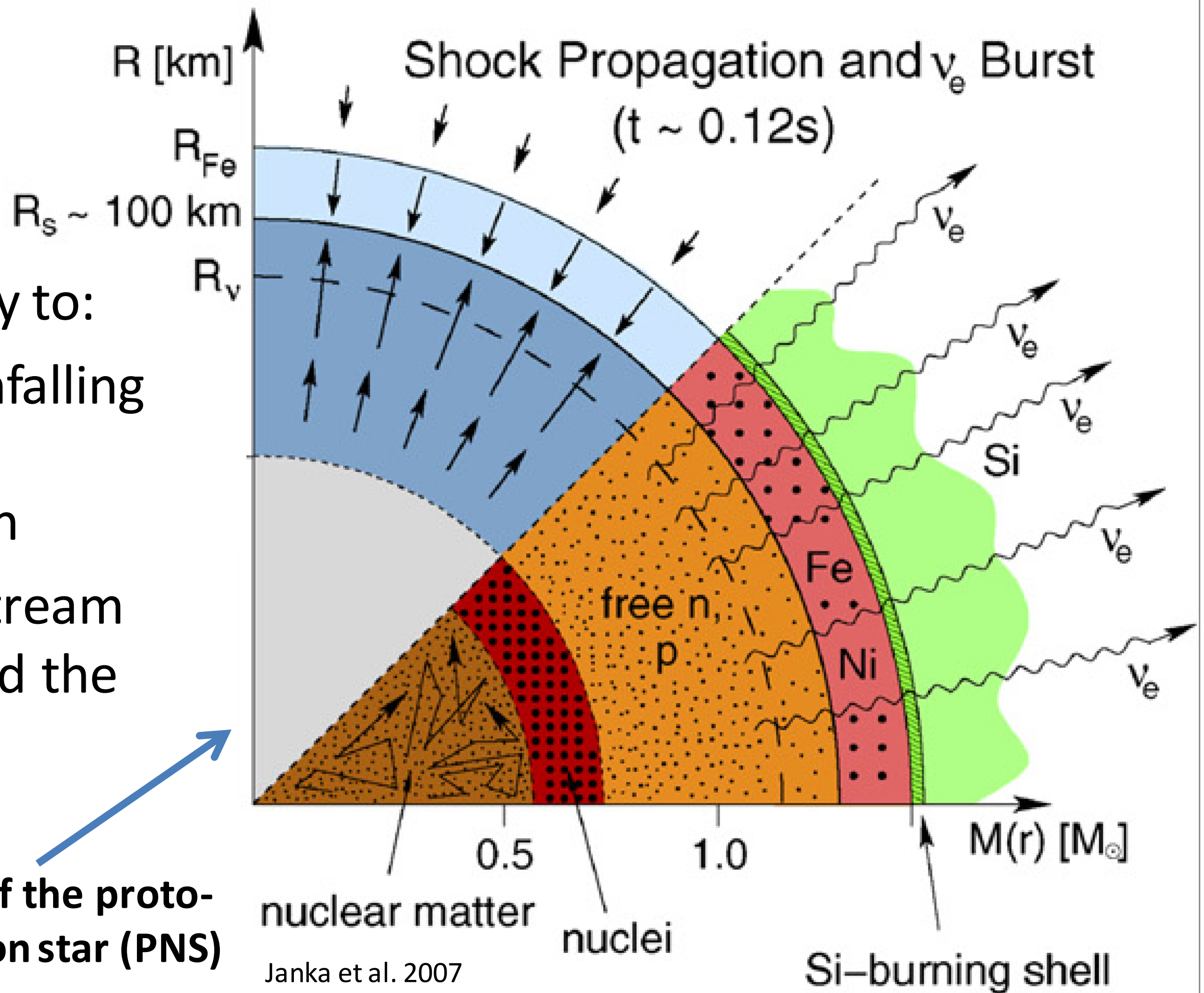
The Supernova Problem



Why Does the Shock Stall

- Shock loses energy to:
  - Dissociation of infalling heavy nuclei: ~8.8 MeV/baryon
  - Neutrinos that stream away from behind the shock.

Inner core -> Core of the proto-neutron star (PNS)



Janka et al. 2007

Neutrino Burst

- Optical depth

$$\tau_\nu(r) = \int_\infty^r \frac{1}{\lambda_\nu} dr'$$

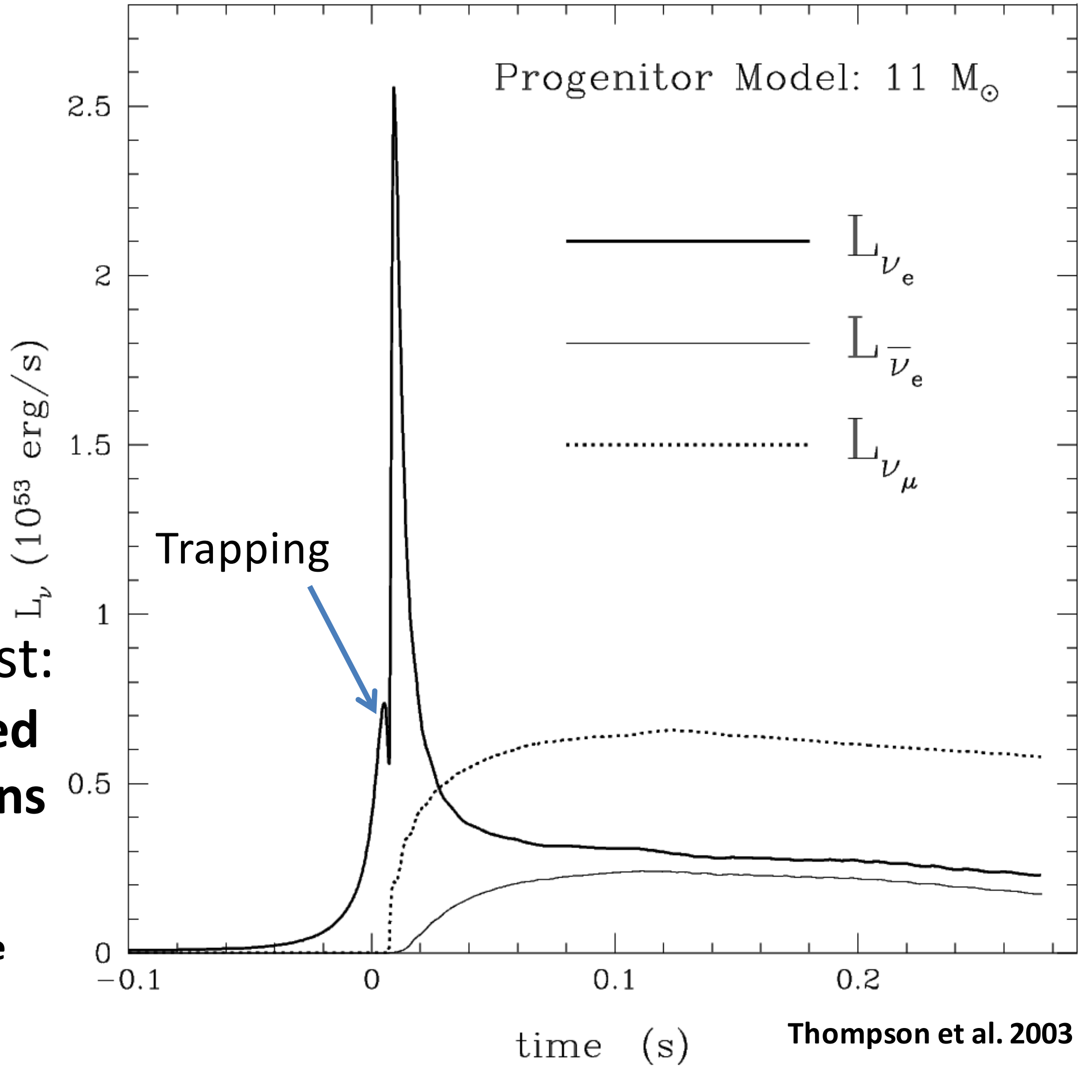
- Neutrinosphere:

$$R_\nu = R\left(\tau_\nu = \frac{2}{3}\right)$$

Depends on $(\epsilon_\nu)^2$

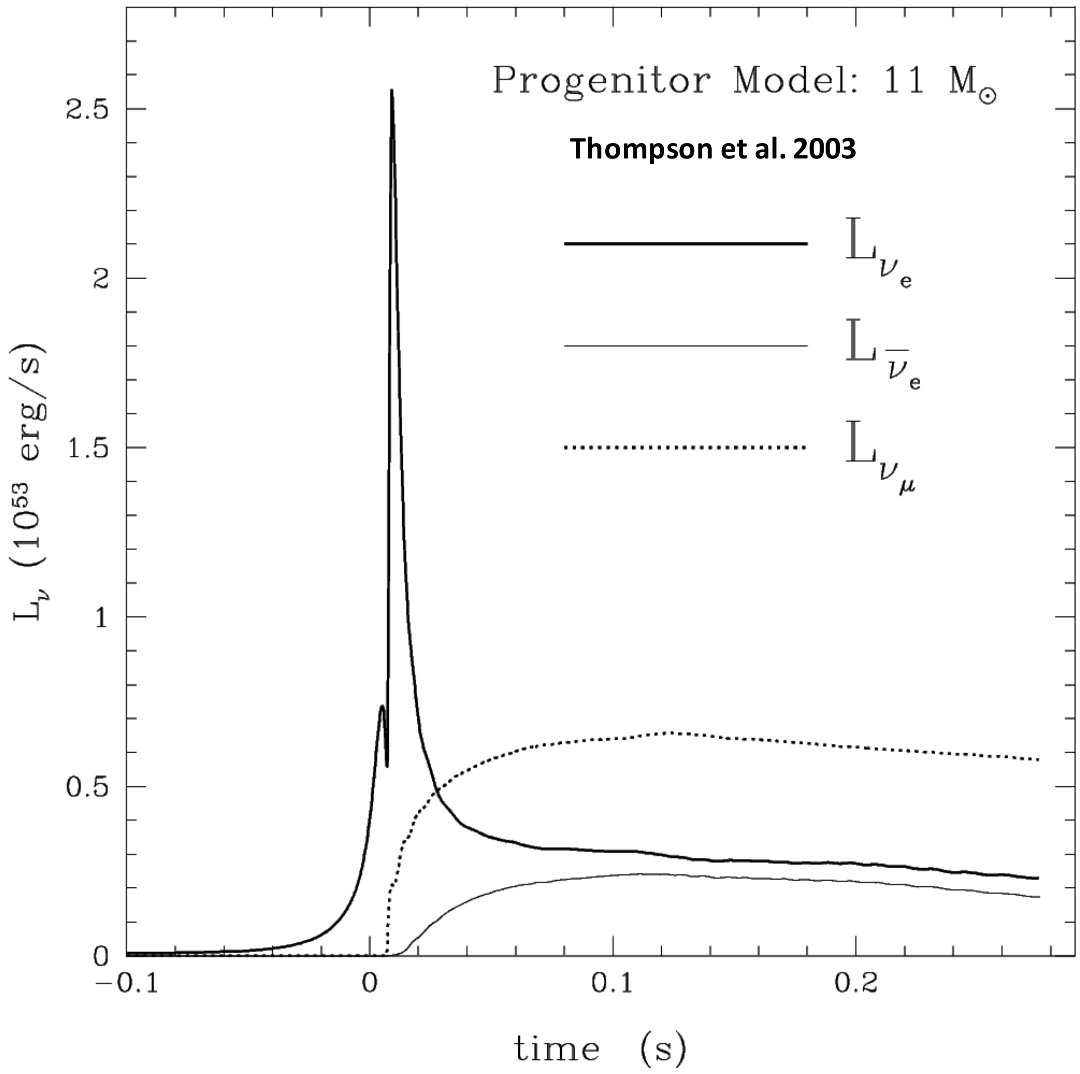
- Postbounce neutrino burst:

Release of neutrinos created by e^- capture on free protons in shocked region when shock 'breaks out' of the ν_e neutrinospheres.



Thompson et al. 2003

Postbounce Neutrino Emission



- Neutrinos and Anti-neutrinos of ALL species:

$$\nu_e,\ \bar{\nu}_e,\ \text{“}\nu_\mu\text{“} = \{\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\}$$

Don't participate in charged-current reactions. Can be treated as 'one'.

- Emission:

$$e^- p \rightleftharpoons \nu_e n$$
$$e^+ n \rightleftharpoons \bar{\nu}_e p$$

$$e^- e^+ \rightleftharpoons \nu_i \bar{\nu}_i$$
$$NN \rightleftharpoons \nu_i \bar{\nu}_i$$
$$\tilde{\gamma} \rightleftharpoons \nu_i \bar{\nu}_i$$

Pair processes: hot & dense environment needed

- Accretion luminosity and diffusive luminosity.

Putting Things Together: Supernova Energetics

- Supernova problem: What revives the shock?
- Precollapse iron core gravitational energy:

$$E_{\mathrm{grav,Fe}(1.5\mathrm{M}_\odot)} \approx 5 \times 10^{51}\,\mathrm{erg} = 5\ \mathrm{B}$$

- Binding energy of a cold 1.5 M_{SUN} NS, R=12.5 km -> **Energy Reservoir**

$$E_{\mathrm{grav,NS}} \approx -\frac{3}{5} G \frac{M^2}{R} \approx -3 \times 10^{53}\,\mathrm{erg} = -3 \times 10^{46}\mathrm{J} = -300\,[\mathrm{B}]\mathrm{ethe}$$

- Initial shock energy: $E_{\mathrm{shock},0} = \frac{1}{2} M v^2 \approx 1.2 \times 10^{51}\,\mathrm{erg} = 1.2\,\mathrm{B}$

(Shock formation at ~0.55 M_{SUN}, v ~ 0.05 c)

- Dissociation:

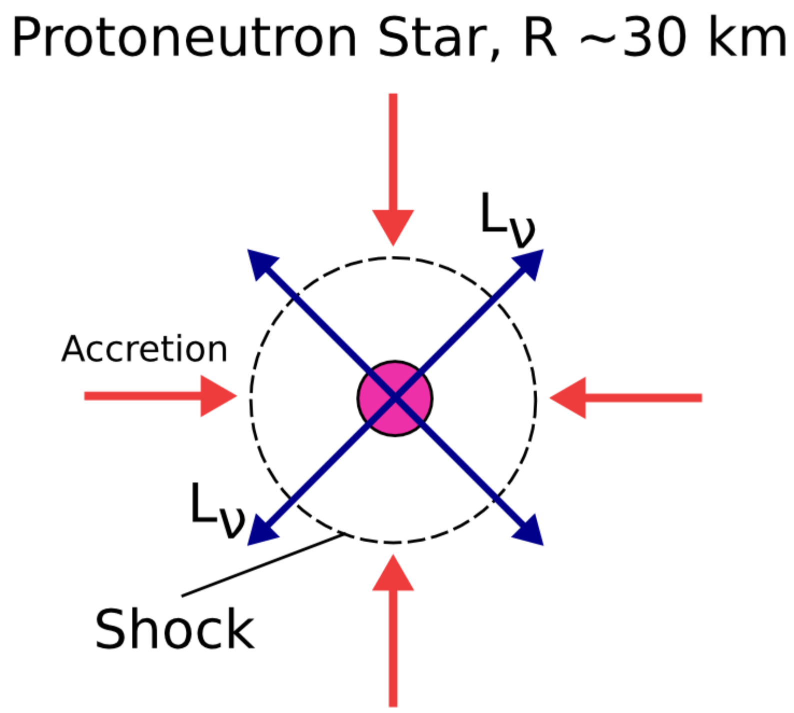
$$E_{\mathrm{diss}} = 17 \left(\frac{M}{M_\odot}\right) \mathrm{B}$$

-> Shock stalls "after" ~0.1 M_{SUN}.

- Neutrinos: initially up to $L_{\nu,\mathrm{total}} \sim 100\,\mathrm{B/s}$
- Binding energy of the mantle (12-M_{SUN} star): $E_{\mathrm{bind},0.6-12\,\mathrm{M}_\odot} = -3.7\,\mathrm{B}$

-> need multiple Bethes to blow up the star!

The General Picture



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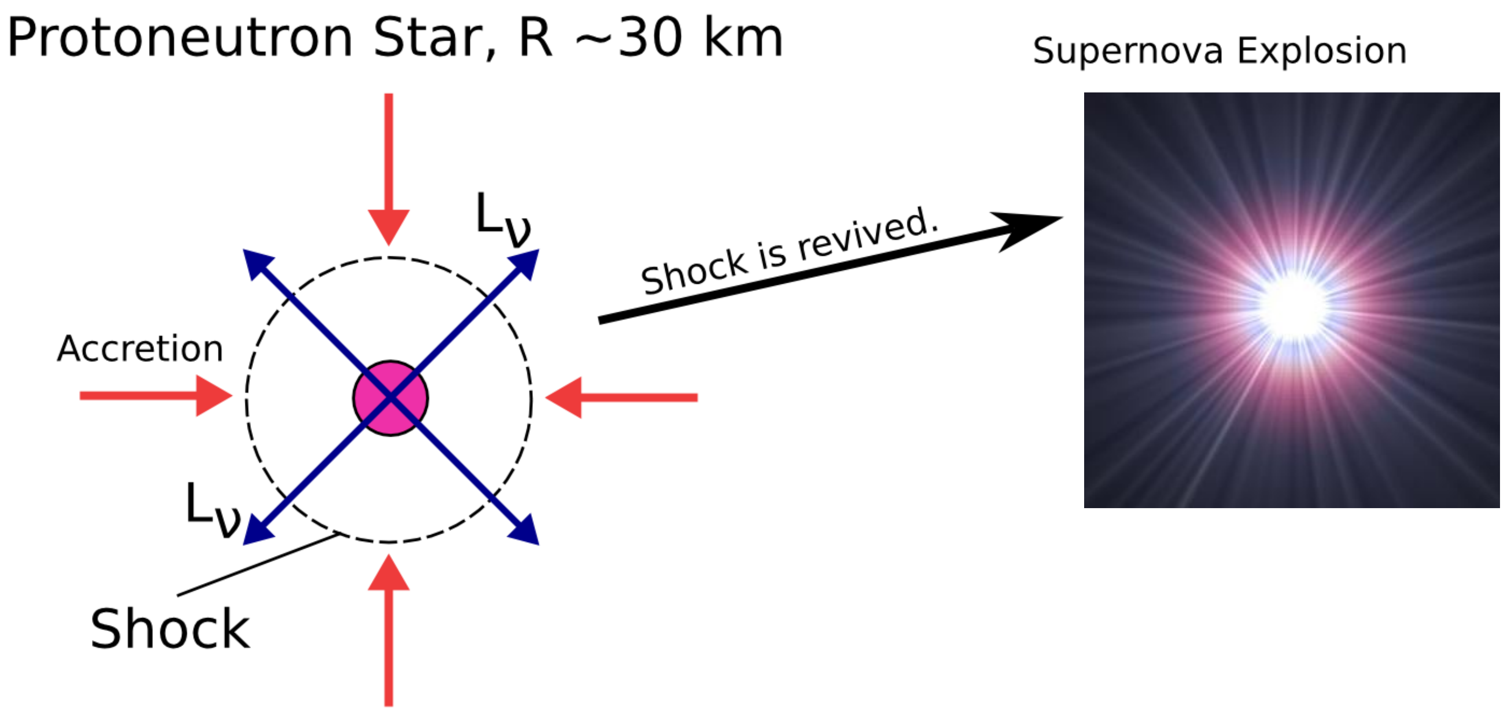

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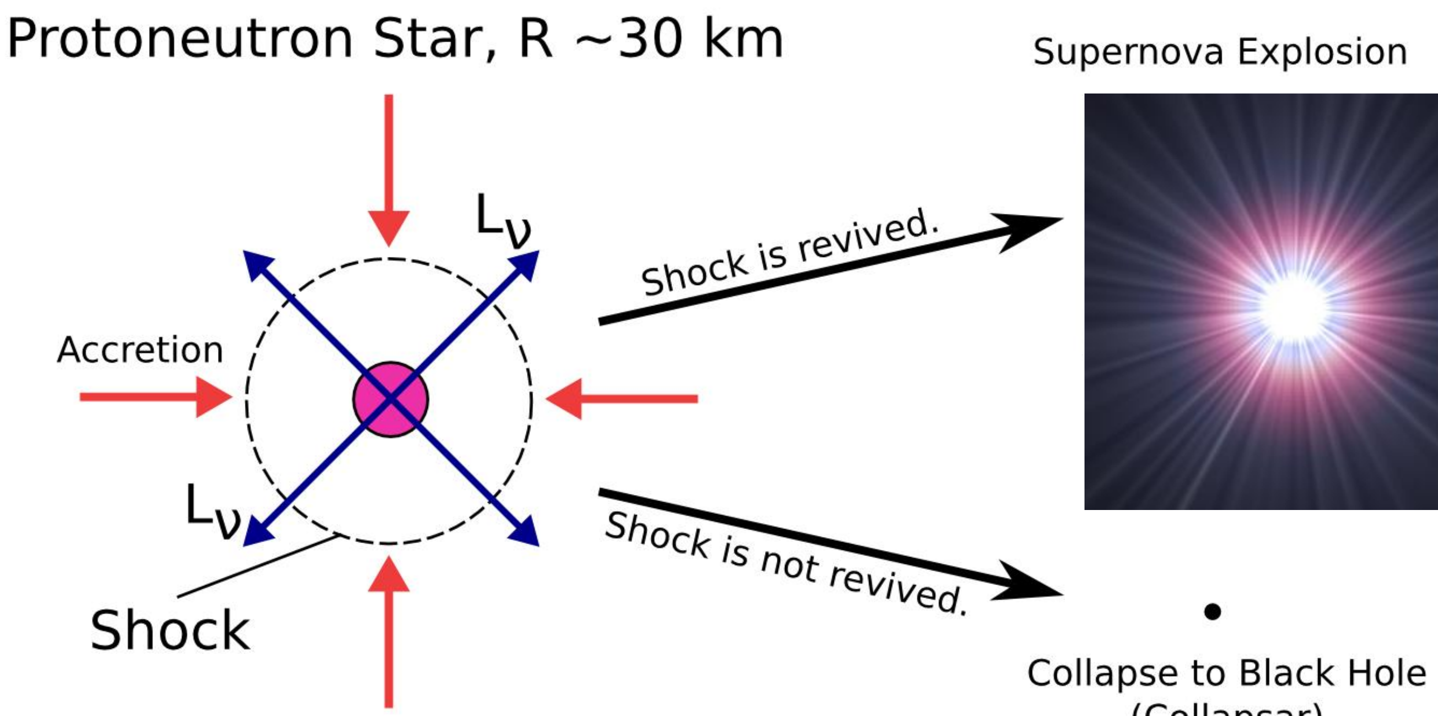
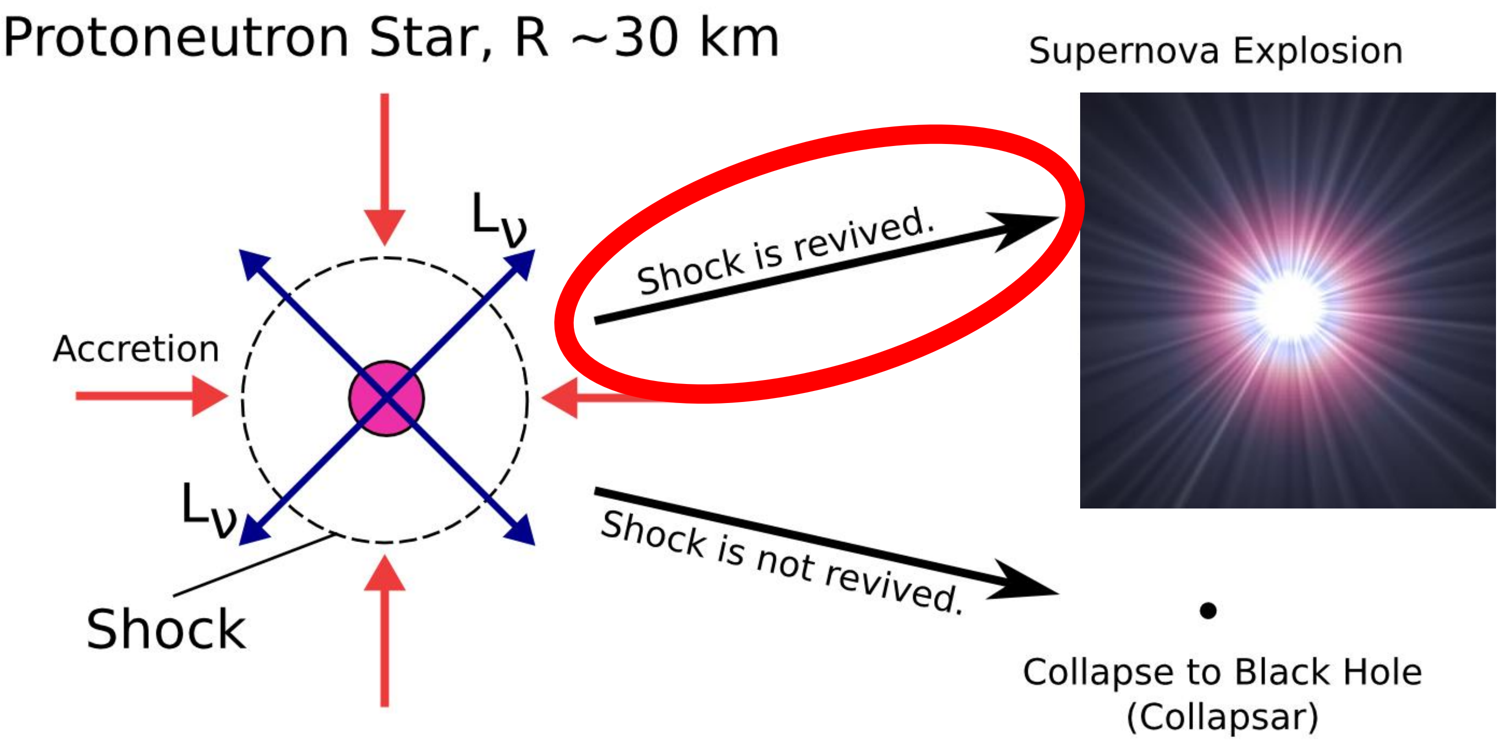


The Supernova Problem



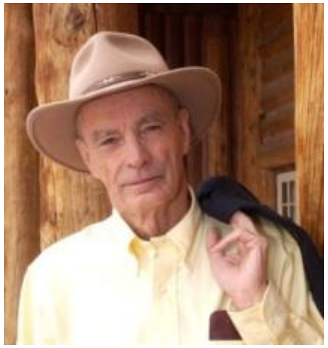
What is the Mechanism of shock revival?

The Essence of any Supernova Mechanism

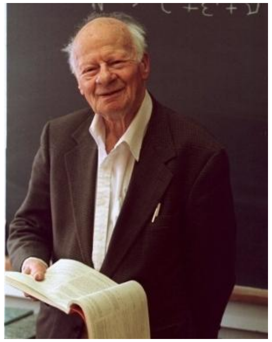
- Collapse to neutron star:
  $\sim$3 x 10^{53} erg = 300 Bethe [B] gravitational energy.
- $\sim 10^{51}$ erg = 1 B kinetic and internal energy of the ejecta. (Extreme cases: 10^{52} erg; "hypernova")
- 99% of the energy is radiated as neutrinos over hundreds of seconds as the protoneutron star (PNS) cools.

Explosion mechanism must tap the gravitational energy reservoir and convert the necessary fraction into energy of the explosion.

Supernova Mechanism: First Simulations


Sterling Colgate


Dave Arnett


Hans


Jim Wilson

Colgate & White 1966

Arnett 1966

Bethe & Wilson 1985

- No supercomputers yet (Cray-I only in 1976!): Limited to spherical symmetry, low resolution, poor neutrino transport.
- Nevertheless: Very important discovery -> **Energy deposition by neutrinos may revive/drive the shock**.

The Neutrino Mechanism

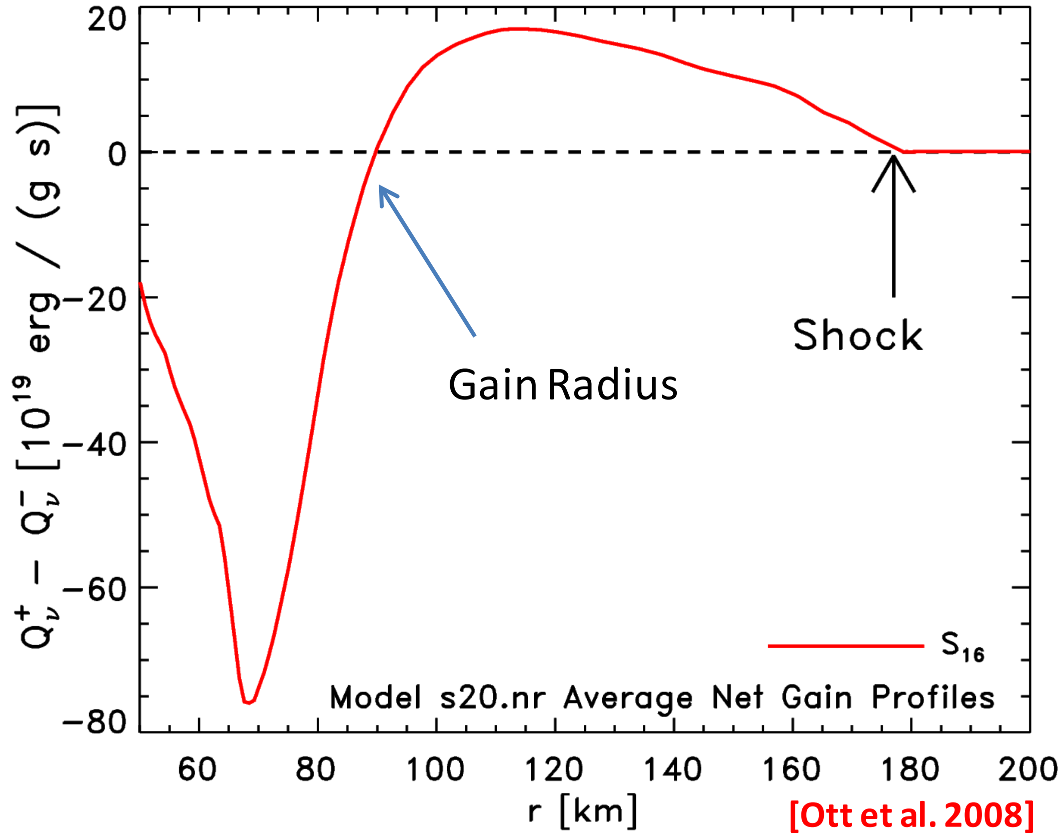
Neutrino cooling: $Q_\nu^- \propto T^6$

Neutrino heating: $Q_\nu^+ \propto L_\nu r^{-2} \langle \epsilon_\nu^2 \rangle$

$\text{Net heating where:}$

$Q_\nu^+ > Q_\nu^-$

- Neutrino-driven mechanism: Based on subtle imbalance between neutrino heating and cooling in postshock region.



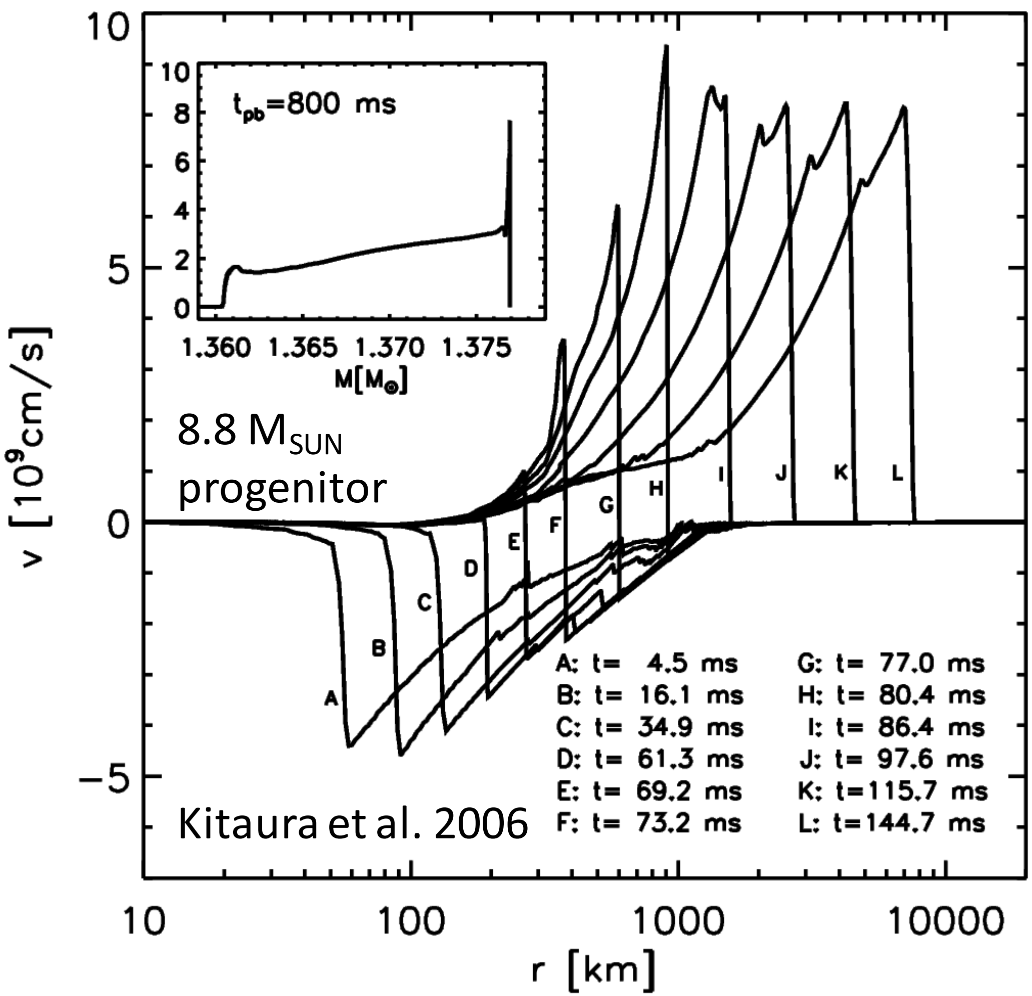
[Ott et al. 2008]

Does it work?

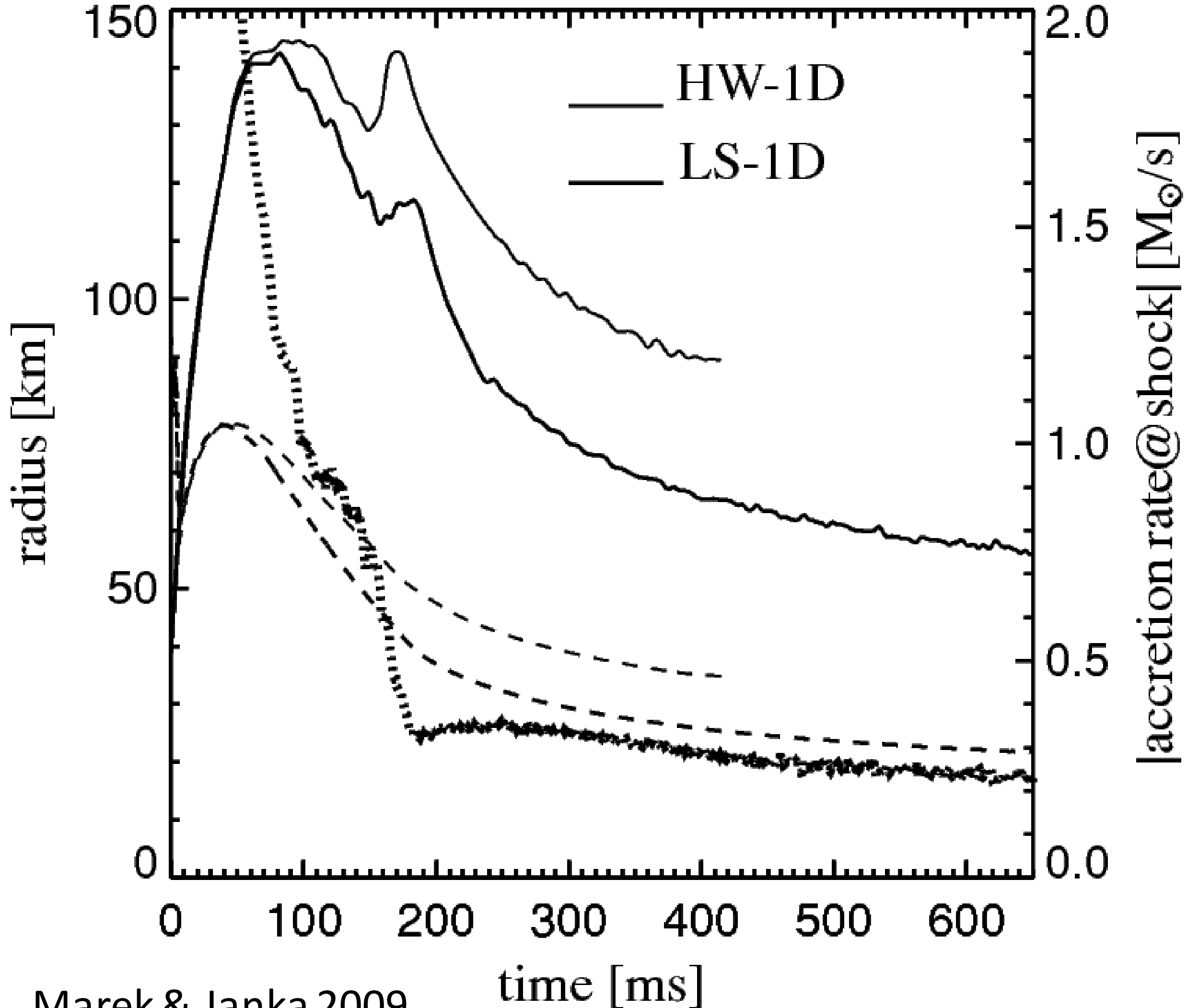
- **Yes!**
  BUT:
  Only for lowest-mass massive stars.

  (Kitaura et al. 2006, Burrows 1988, Burrows, Livne, Dessart 2007)

- **FAILS in spherical symmetry (1D) for garden-variety massive stars (~15 M_{SUN}) in simulations with best neutrino physics and neutrino transport**



Failure of the Neutrino Mechanism in 1D



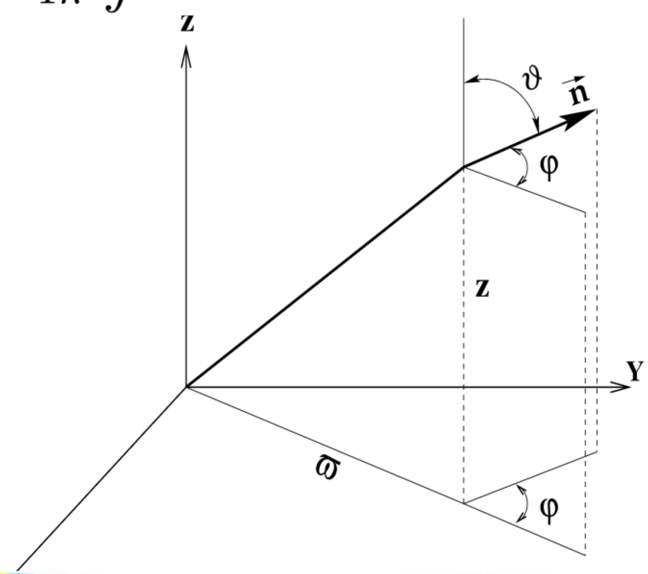
Marek & Janka 2009

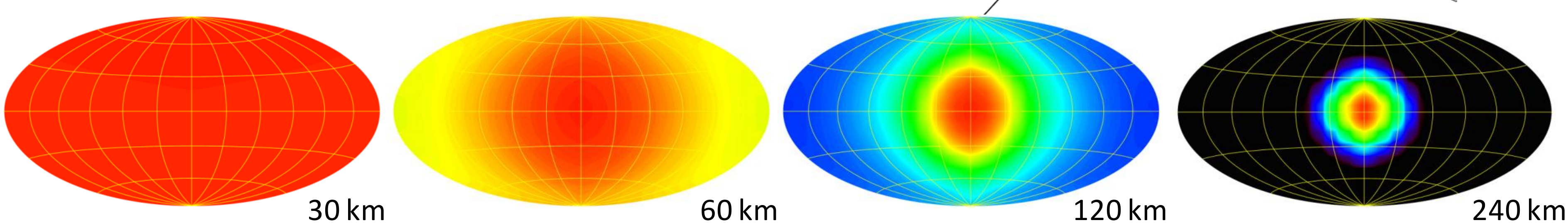
A few Words on Neutrino Transport

$$\frac{1}{c}\frac{\partial I(\vec{r},\vec{n},\epsilon_\nu)}{\partial t} + \vec{n}\,\vec{\nabla} I(\vec{r},\vec{n},\epsilon_\nu) = \Xi[I(\vec{r},\vec{n},\epsilon_\nu),\rho,T,Y_e]$$

$$J = \frac{1}{4\pi}\oint I d\Omega \qquad \vec{H} = \frac{1}{4\pi}\oint \vec{n} I d\Omega \qquad \mathbf{K} = \frac{1}{4\pi}\oint \vec{n}\cdot\vec{n}\, I d\Omega$$

- 6D problem: 3D space, 3D (ϵ, θ, ϕ) momentum space.
- Limiting cases – easy to handle:
  (1) Diffusion (isotropic radiation field)
  (2) Free streaming ("forward-peaked" radiation field)





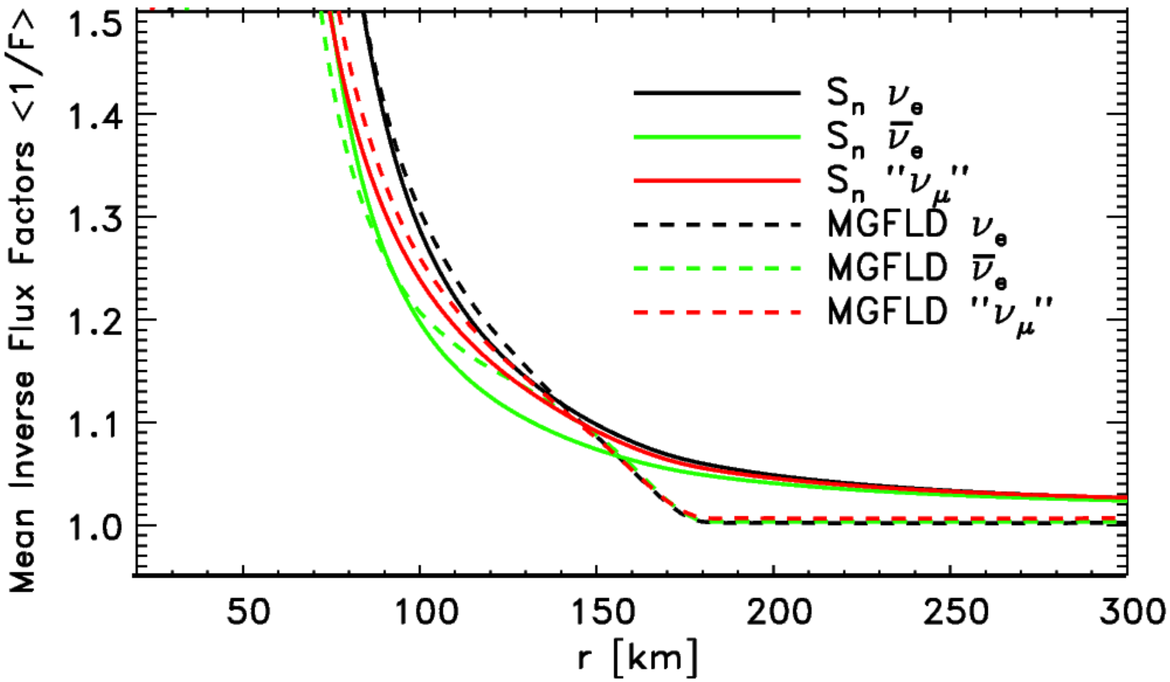
Neutrino Transport in Core-Collapse SNe

- Main complication: Need to track radiation field from complete isotropy to full free streaming over many orders of magnitude of τ.
- Neutrino heating depends on details of the radiation field:

$$Q^+_{\nu_e/\bar{\nu}_e} = 4\pi \int_0^\infty d\epsilon_\nu \, \kappa_{\mathrm{a},\nu_e/\bar{\nu}_e} J_\nu = \frac{1+3g_\mathrm{A}^2}{4} \frac{\sigma_0 N_\mathrm{A} X_{\mathrm{n/p}}}{(m_\mathrm{e}c^2)^2} \boxed{\langle \epsilon_\nu^2 \rangle} \frac{\boxed{L_{\nu_e/\bar{\nu}_e}}}{4\pi r^2} \boxed{\left\langle \frac{1}{\mathrm{F}} \right\rangle}$$

- Inverse Flux factor:

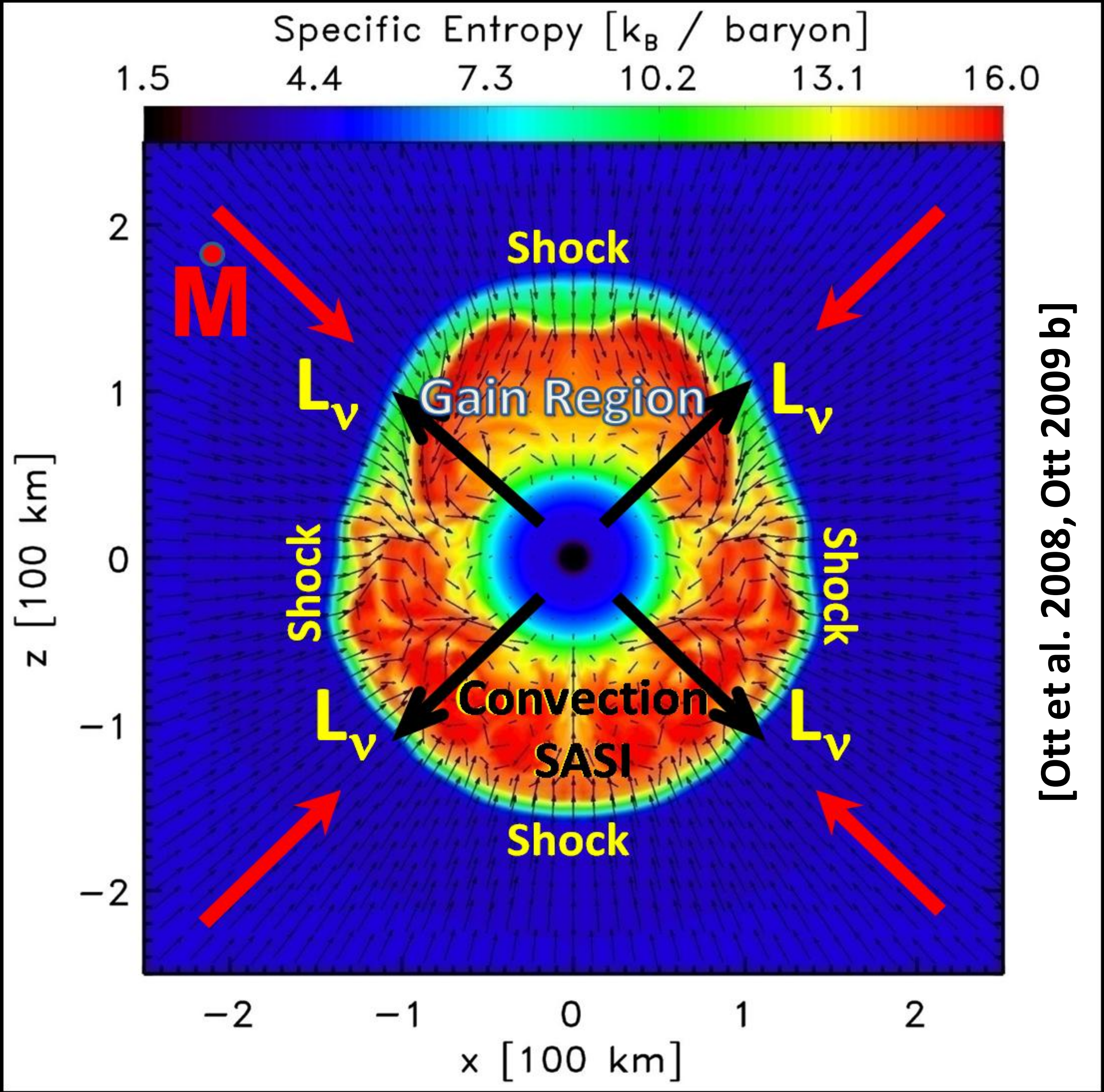
$$\left\langle \frac{1}{\mathbf{F}_{\nu_i}} \right\rangle = \frac{c \int d\varepsilon_\nu E(\varepsilon_\nu, \nu_i)}{\int d\varepsilon_\nu F_r(\varepsilon_\nu, \nu_i)}$$



Anyway... What next?

- Why does the neutrino mechanism fail in 1D?
- Is dimensionality an issue? What is 1D missing?
  - Rotation and magnetohydrodynamics (MHD)
  - Convection/Turbulence
  - Other multi-D processes; e.g., pulsations
- First multi-D radiation-hydrodynamics simulations:
  - early to mid 1990s:
    Herant et al. 1994, Burrows et al. 1995, Janka & Müller 1996.

A Look at the Beast:



[Ott et al. 2008, Ott 2009 b]

Convection

- Ledoux criterion for instability:

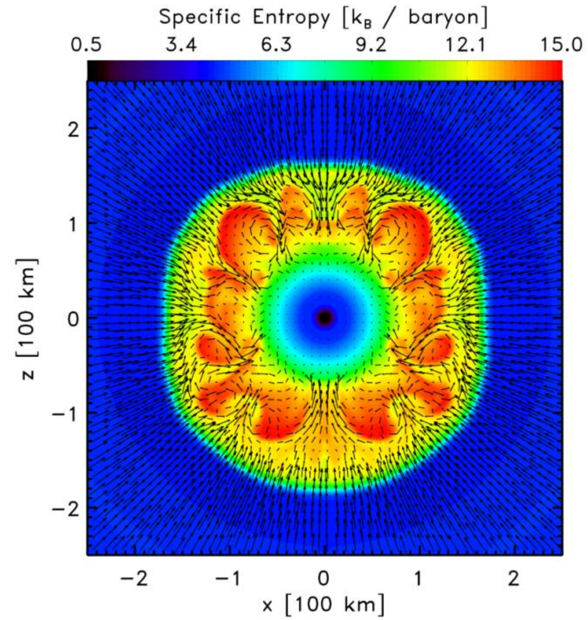
$$C_L \equiv \left(\frac{\partial \rho}{\partial s}\right)\bigg|_{Y,p} \frac{\mathrm{d}s}{\mathrm{d}r} + \left(\frac{\partial \rho}{\partial Y}\right)\bigg|_{s,p} \frac{\mathrm{d}Y}{\mathrm{d}r}$$

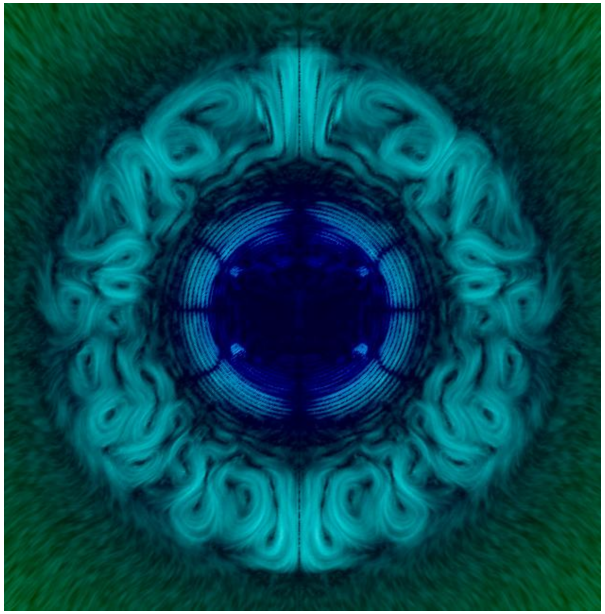
< 0 < 0

Entropy Gradient

Lepton Gradient

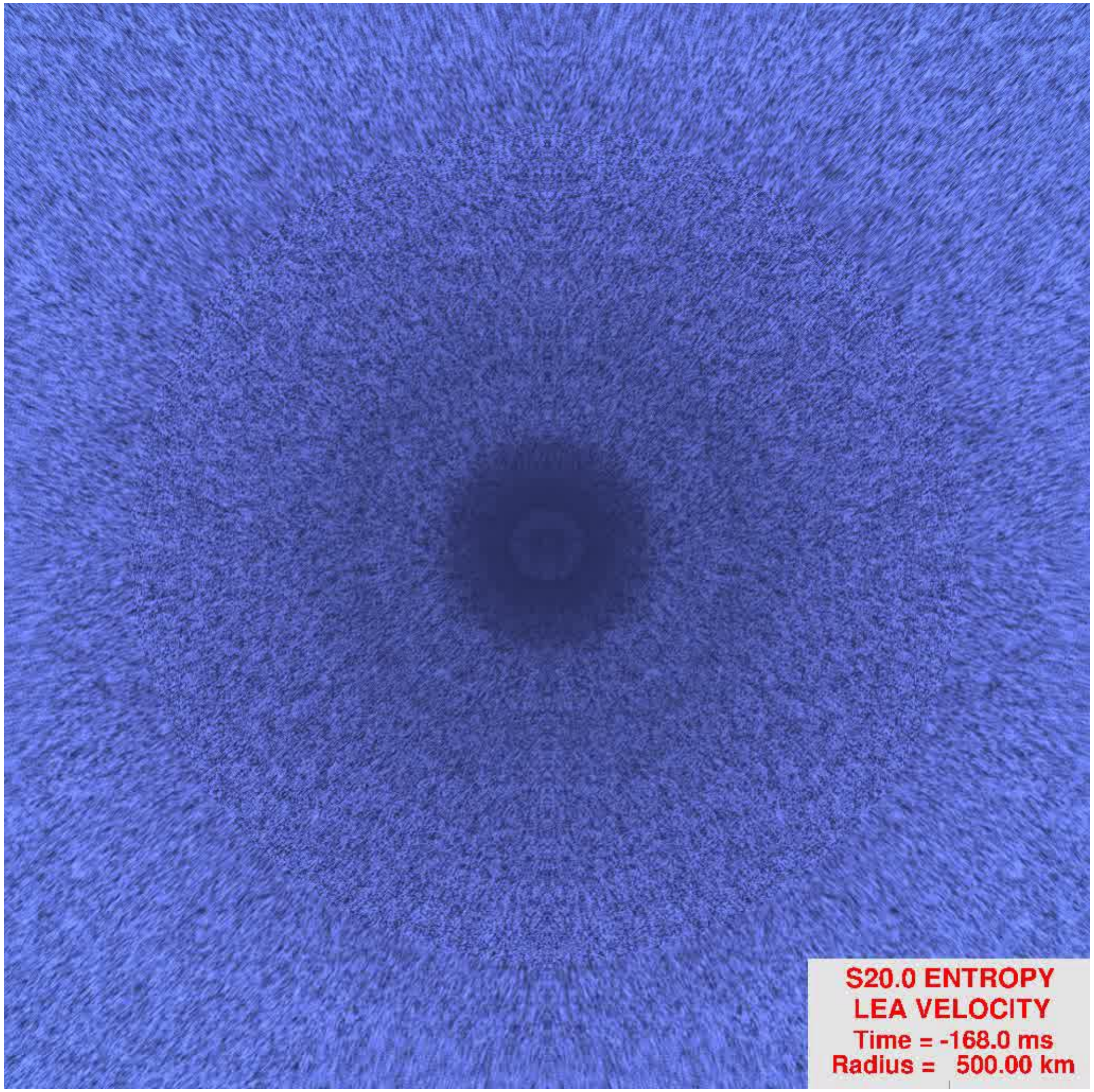
- $C_L > 0$ -> convective instability.
- Postbounce supernova cores:
  - Negative entropy gradient in postshock region -> convection
  - Negative entropy region inside the neutrinosphere in the PNS -> convection
- **Important effect of convection:**
  - "Dwell time" of material in the heating ("gain") region is increased -> leads to more favorable ratio $\tau_{advect} / \tau_{heat}$.





Standing Accretion Shock Instability

[Blondin et al. '03,'06; **Foglizzo et al. '06**, Scheck et al. '06, '07, Burrows et al. '06, '07]

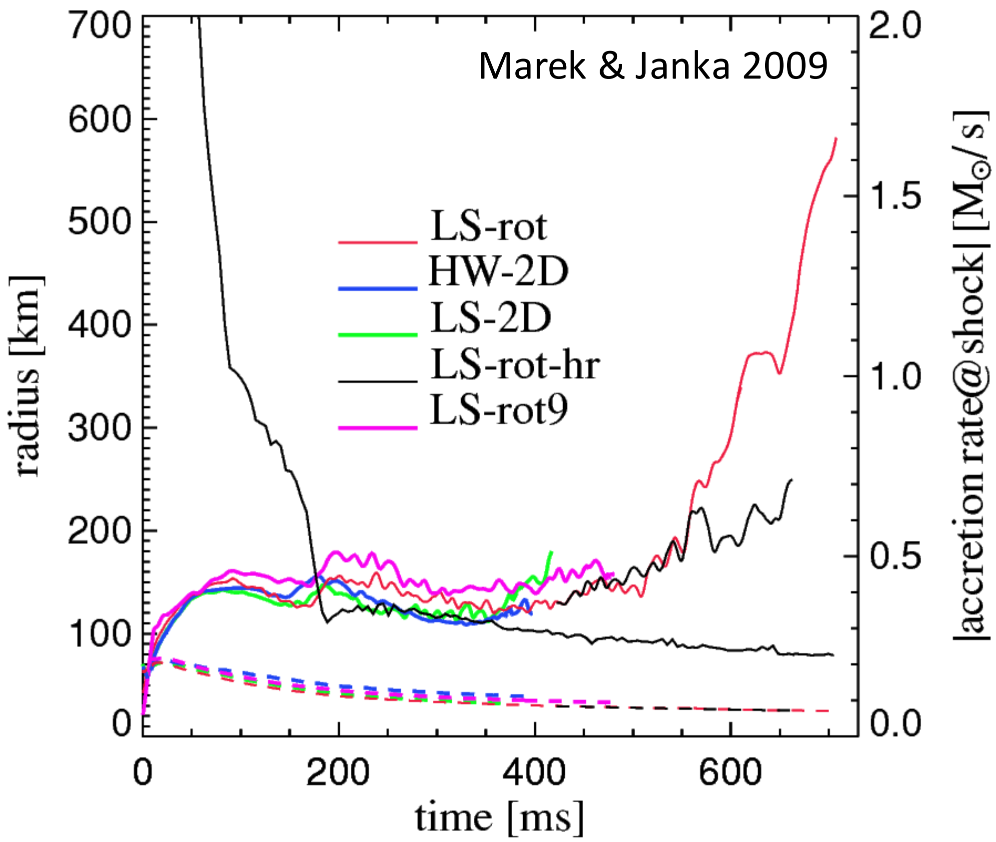


Advective-acoustic cycle drives shock instability.

Seen in simulations by all groups!

Status of the Neutrino Mechanism

- Best simulations are still in 2D.
- Things look better in 2D, some models explode under special circumstances.
- No robust explosions.
- Crucial conditions (?):
  General relativity
  Soft nuclear EOS
- **Robust explosions in 3D?**
  -> ongoing research!



Alternatives to the Neutrino Mechanism

(qualitative discussion!)

Magnetorotational Mechanism

[LeBlanc & Wilson 1970, Bisnovatyi-Kogan et al. 1976, Meier et al. 1976, Symbalisty 1984]

Acoustic Mechanism

[proposed by Burrows et al. 2006, 2007; not (yet?) confirmed by other groups/codes]

Alternatives to the Neutrino Mechanism

(qualitative discussion!)

Magnetorotational Mechanism

[LeBlanc & Wilson 1970, Bisnovatyi-Kogan et al. 1976, Meier et al. 1976, Symbalisty 1984]

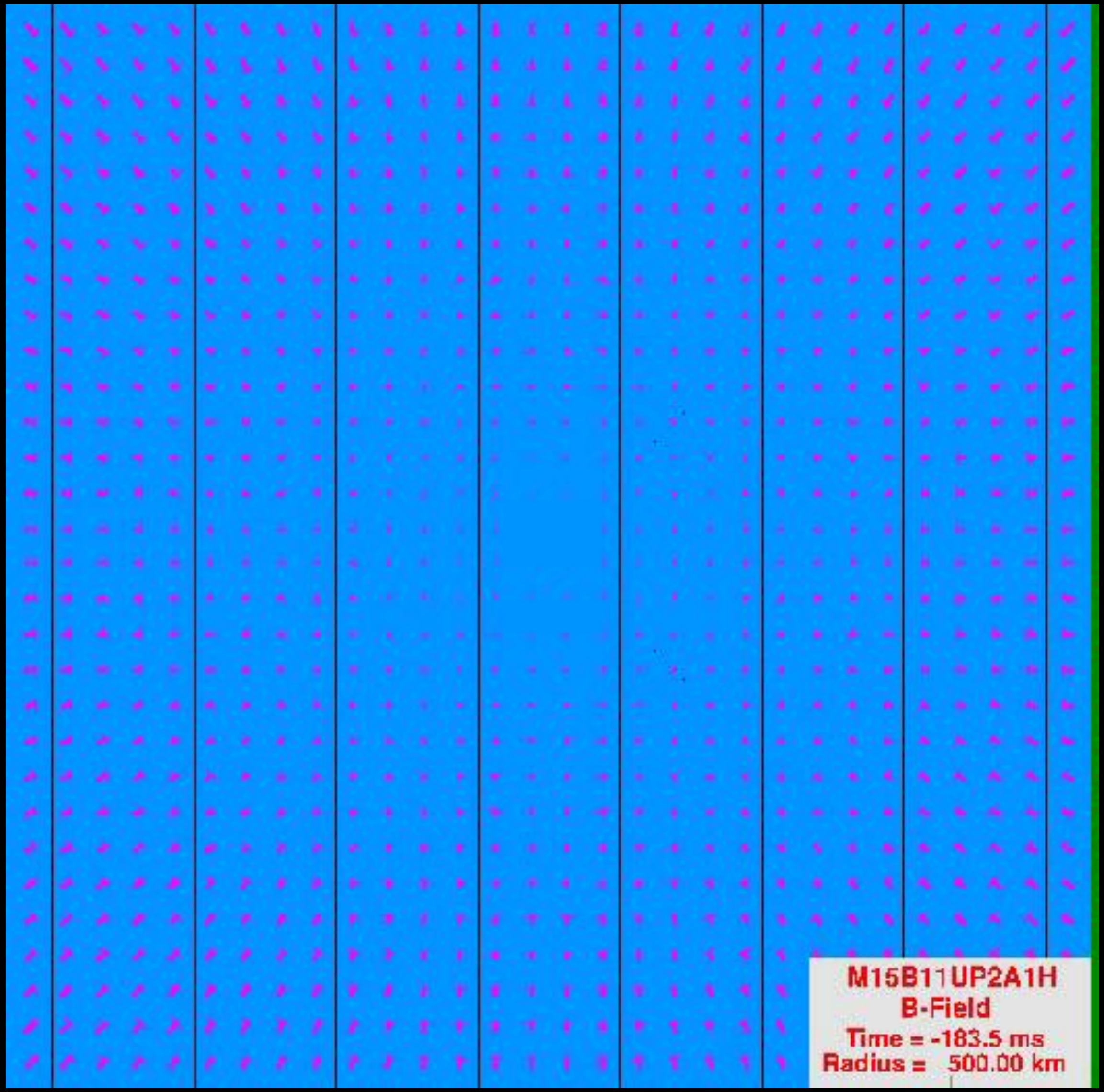
Acoustic Mechanism

[proposed by Burrows et al. 2006, 2007; not (yet?) confirmed by other groups/codes]

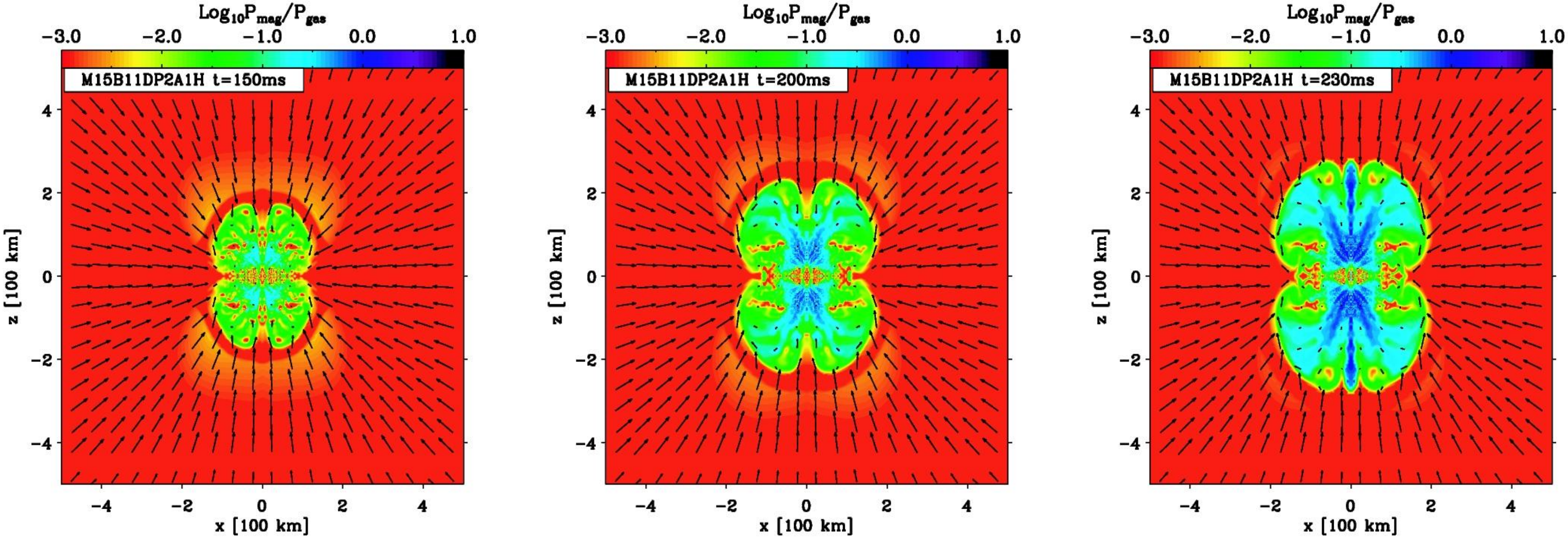
MHD-driven Explosions

[e.g., Burrows et al. 2007, Dessart et al. 2008, Kotake et al. 2004, Yamada & Sawai 2004, Sawai et al. 2008, Takiwaki et al. 2009]

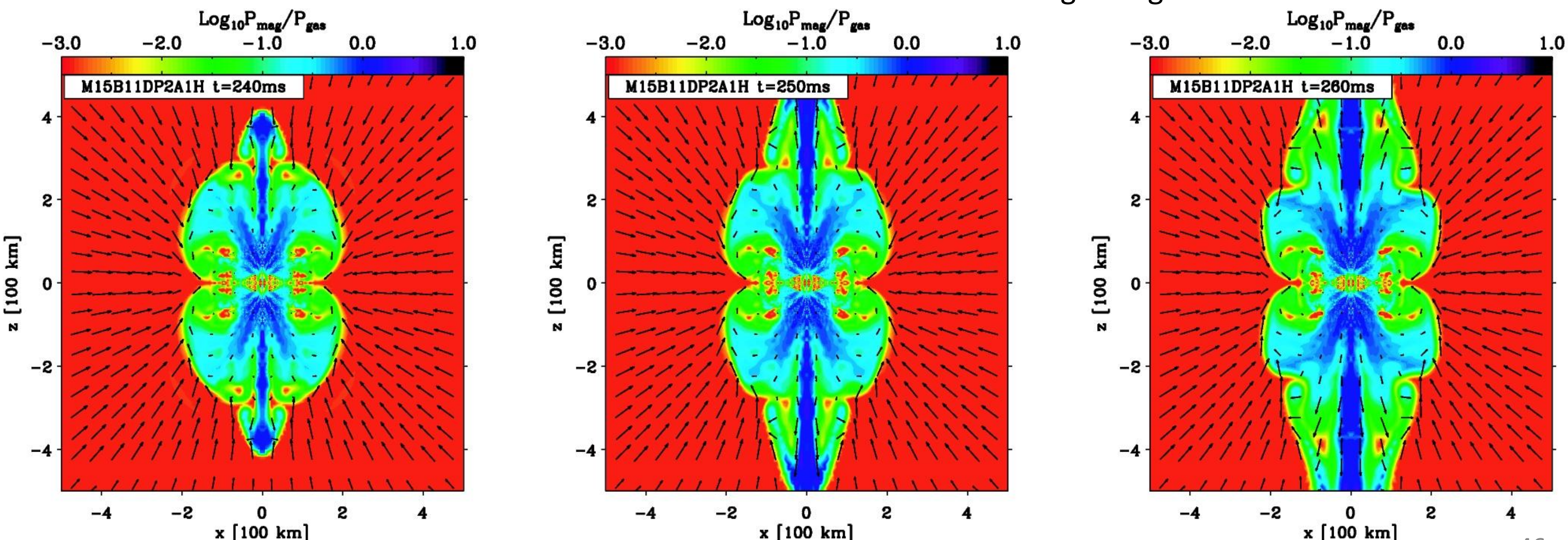
- **Rapid rotation**:
  $P_0 < 4\text{-}6$ s
  -> millisecond PNS
- PNS rotational energy: ~10 B
- Amplification of B fields up to equipartition:
  - compression
  - dynamos
  - magneto-rotational instability (MRI)
- Jet-driven outflows.
- MHD-driven explosion may be GRB precursor.



VULCAN 2D R-MHD code, Livne et al. 2007, Burrows et al. 2007.

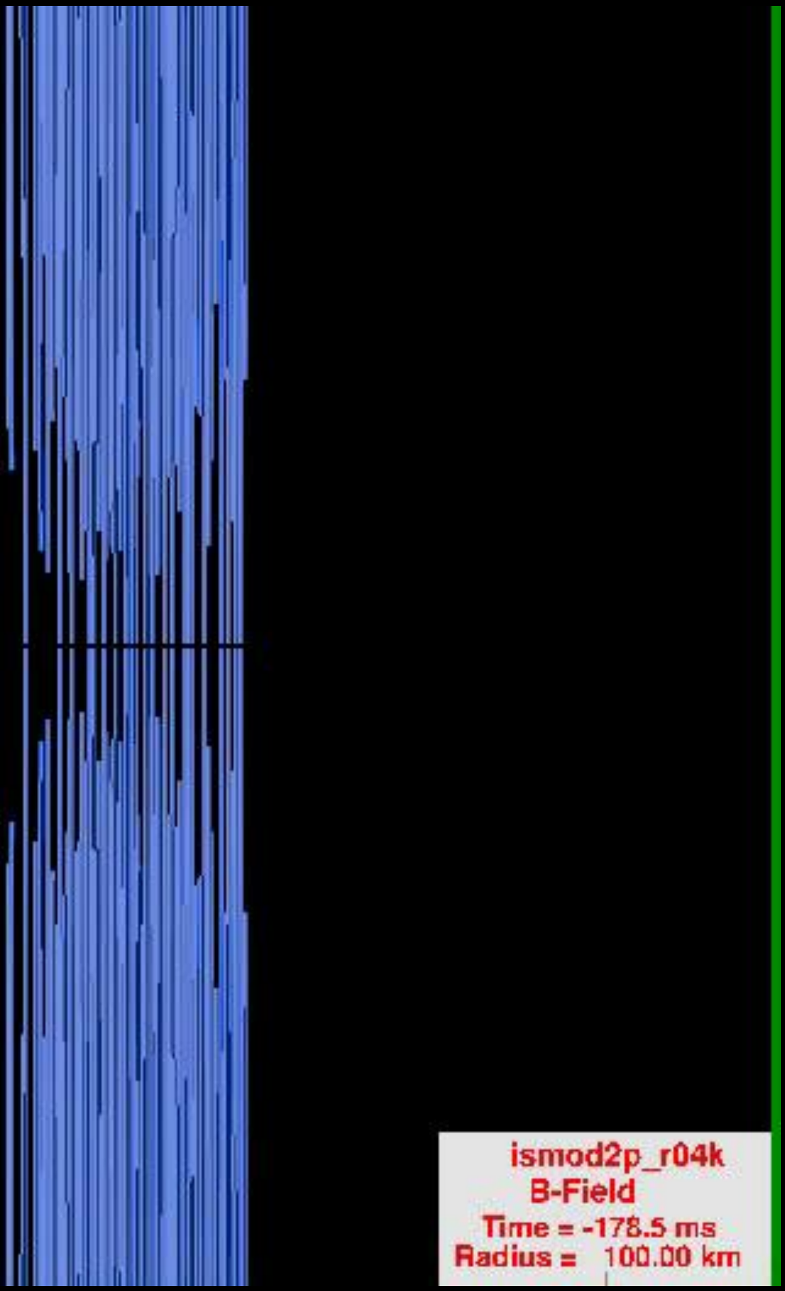


MHD jet/explosion launched when $P_{mag} / P_{gas} \sim 1$



Newtonian
Radiation-MHD
Simulations with
VULCAN/2D

Magnetic field lines in
M15B11UP2A1H of
Burrows, Dessart,
Livne, Ott, Murphy ‘07.



Features/Limitations of the Magnetorotational Mechanism

[Burrows et al. 2007]

- Jet powers up to 10 B/s (10^{52} erg/s).
- **Simultaneous explosion and accretion**.
- **Hypernova** energies (> 10 B) attainable.
- MHD mechanism inefficient for cores with precollapse $P_0 > 4$ s, but stellar evolution + NS birth spin estimates: **$P_0 > 30$ s in most cores**.
  [Heger et al. 2005, Ott et al. 2006]
- MHD explosion — a GRB precursor?
- Limitations: Resolution does not allow to capture Magnetorotational Instability;
  Simulations 2D and Newtonian.

Alternatives to the Neutrino Mechanism

(qualitative discussion!)

Magnetorotational Mechanism

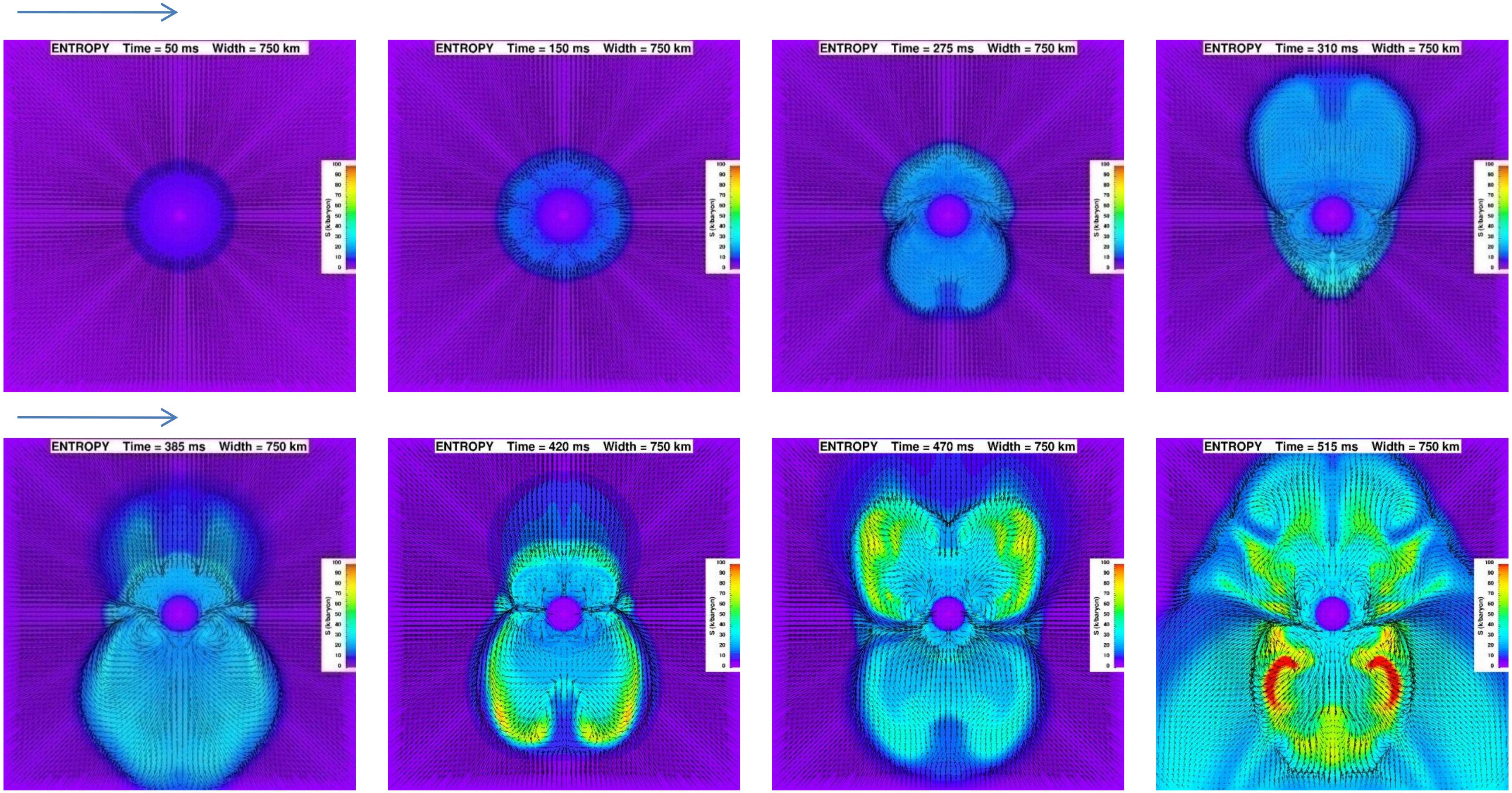
[LeBlanc & Wilson 1970, Bisnovatyi-Kogan et al. 1976, Meier et al. 1976, Symbalisty 1984]

Acoustic Mechanism

[proposed by Burrows et al. 2006, 2007; not (yet?) confirmed by other groups/codes]

Setting the Stage: SASI

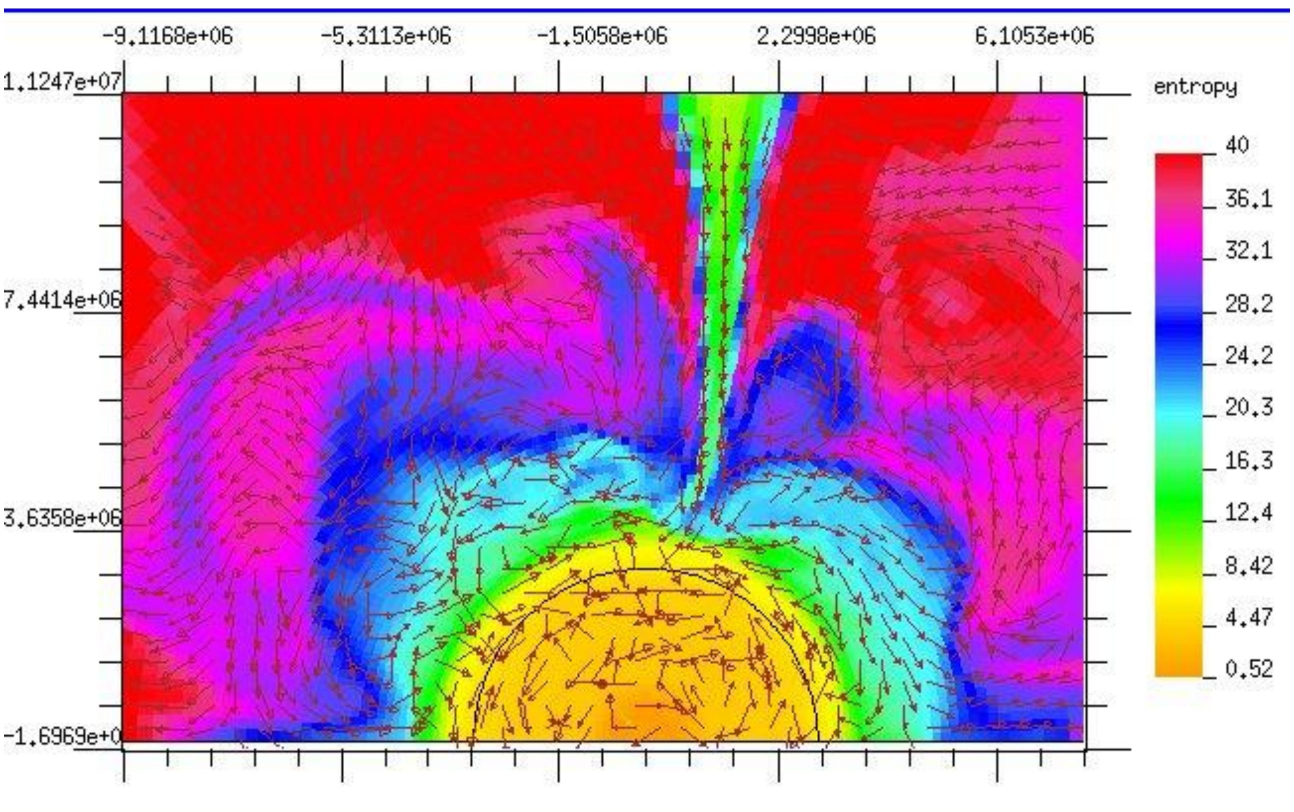
[e.g., Burrows et al. 2006, 2007bc, Ott et al. 2006]



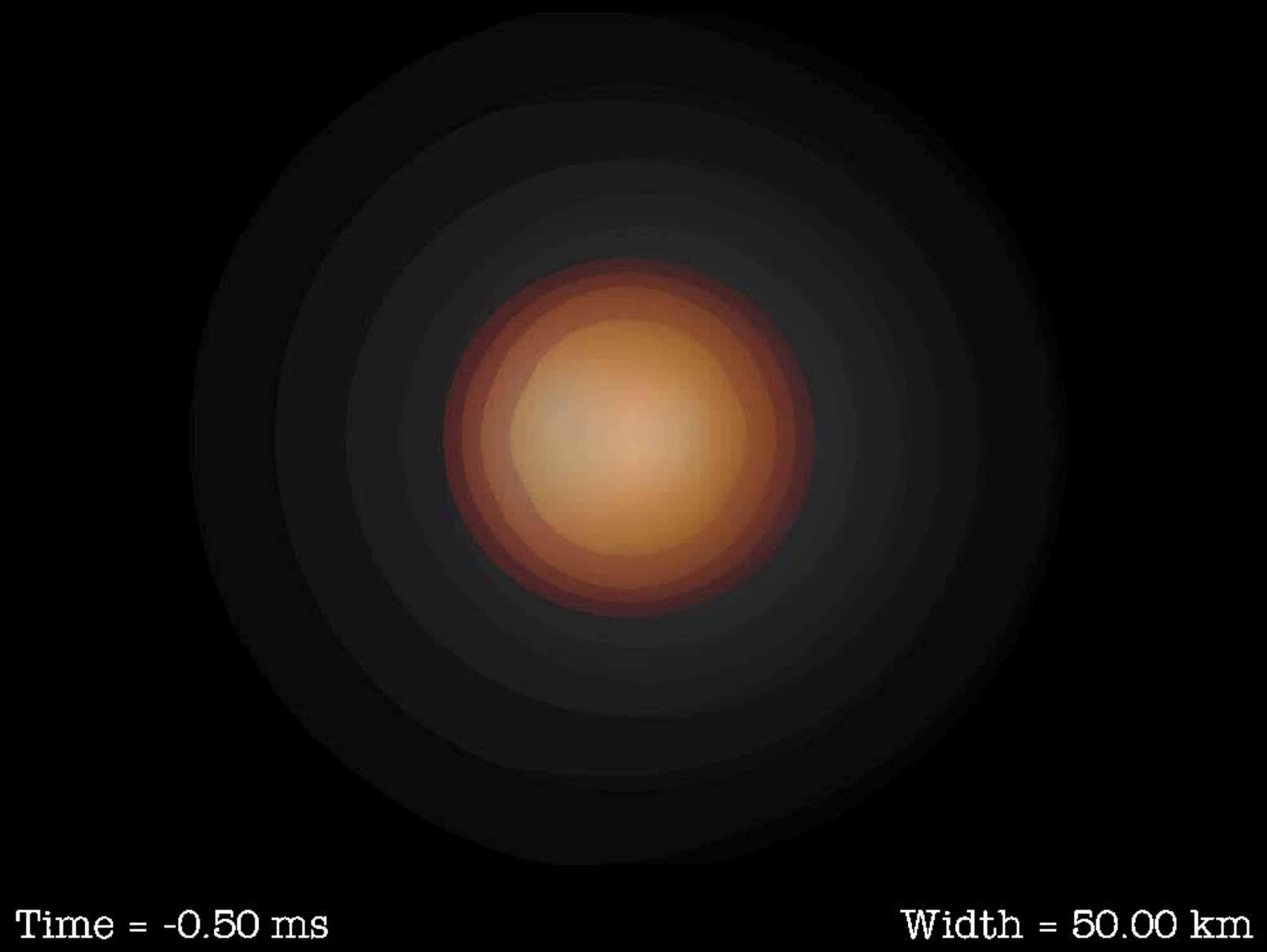
The Acoustic Mechanism

[e.g., Burrows et al. 2006, 2007bc, Ott et al. 2006]

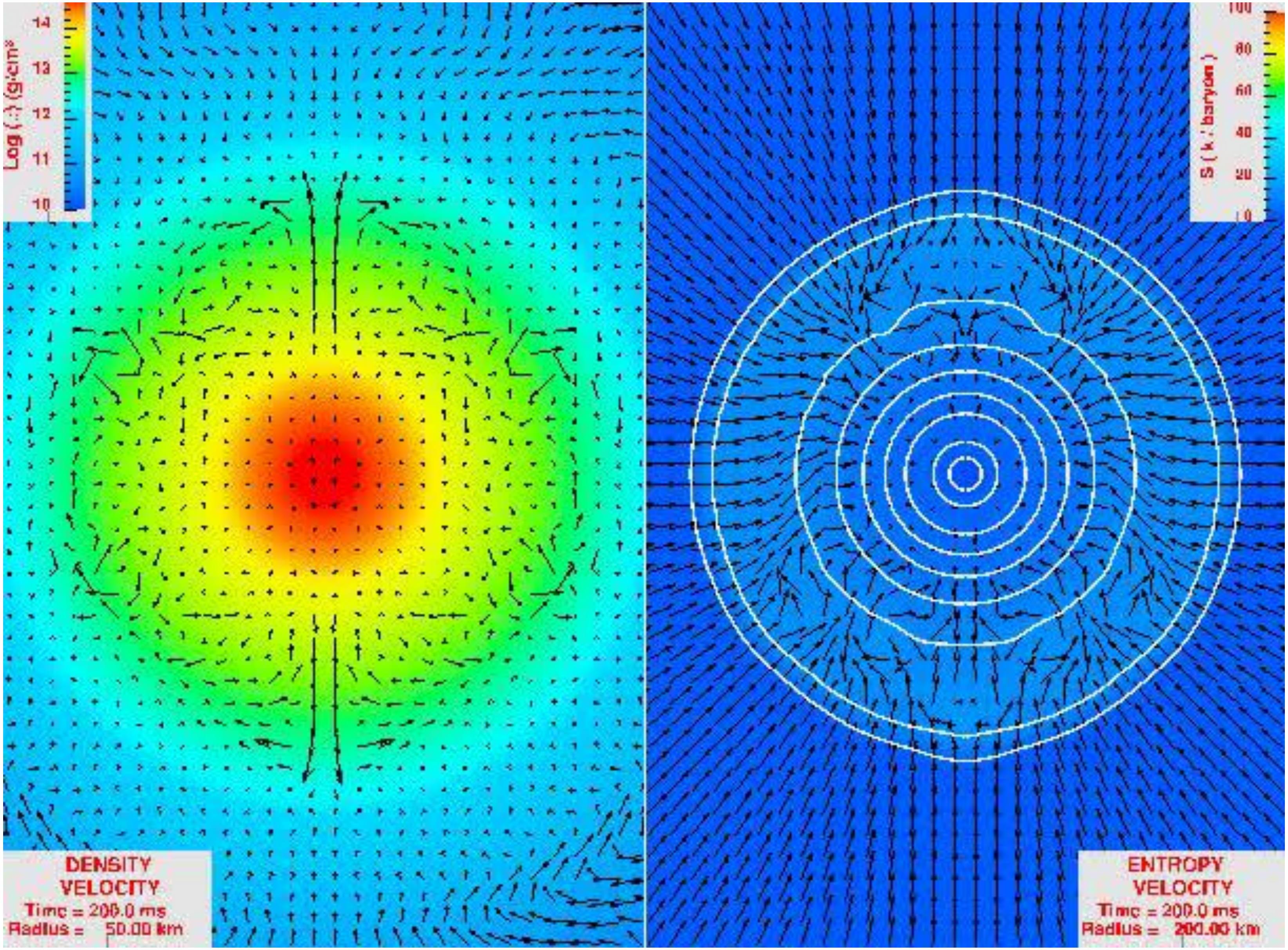
SASI-modulated **supersonic accretion streams** and SASI generated **turbulence** excite lowest-order (l=1) buoyancy mode in the PNS.



- g-modes reach large amplitudes ~800—1000 ms after bounce.
- Damping by strong sound waves that steepen into shocks; deposit energy in the stalled shock.
- **Drive ~1 B explosions at late times.**

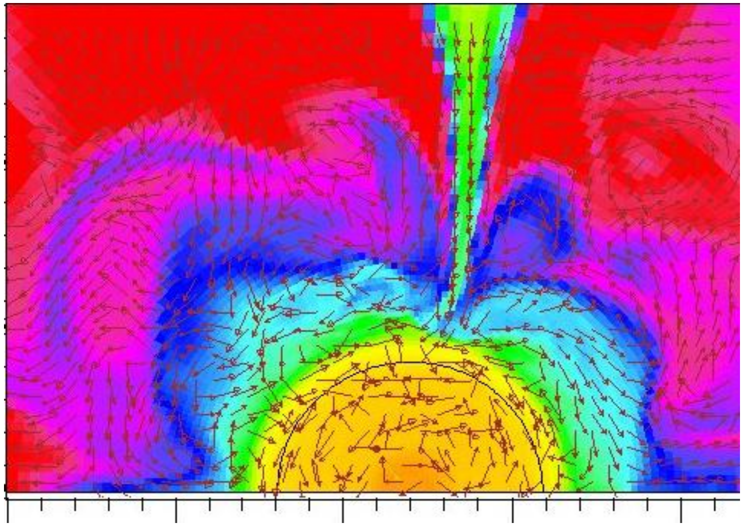


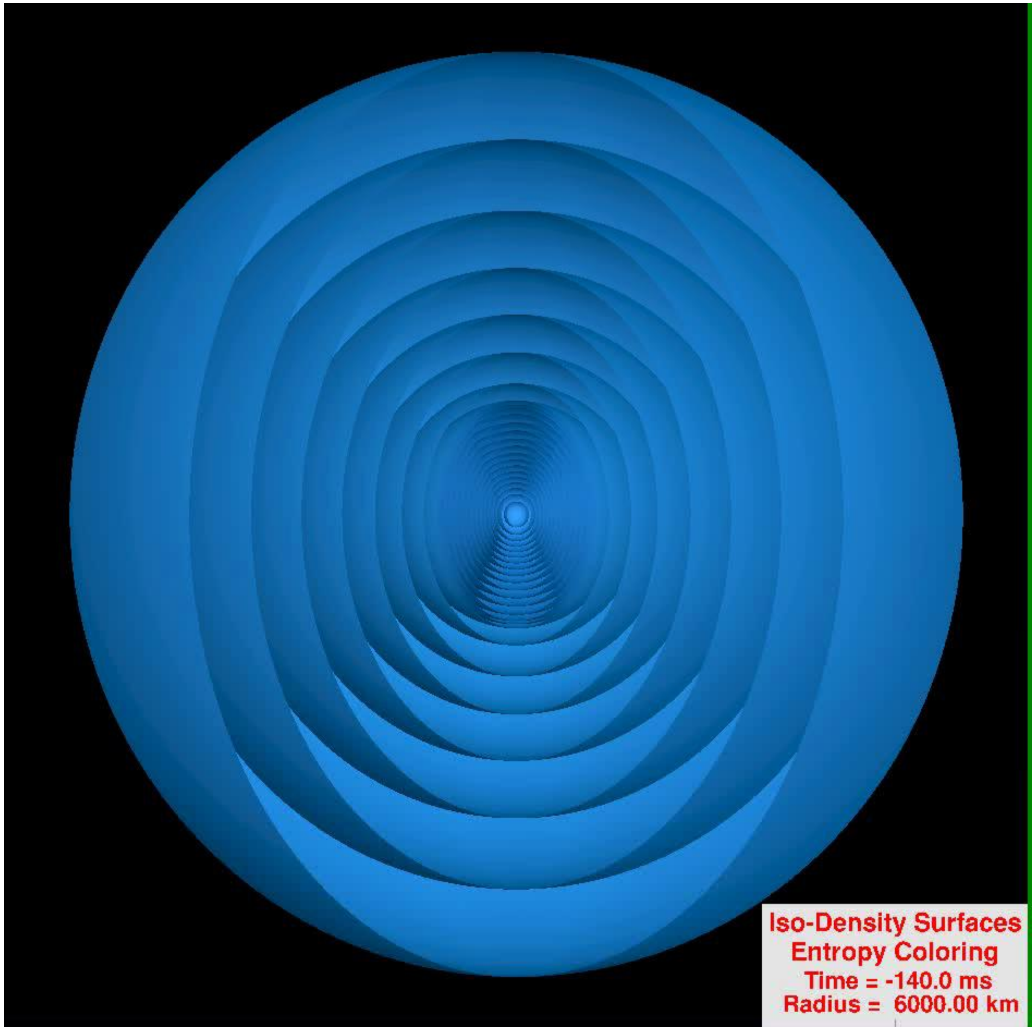
PNS core oscillations, Burrows et al. 2006, 2007; Ott et al. 2006


14
13
12
11
10
DENSITY
VELOCITY
Time = 200.0 ms
Radius = 50.00 km
100
80
60
40
20
0
ENTROPY
VELOCITY
Time = 200.0 ms
Radius = 200.00 km

Alternatives: The Acoustic Mechanism

SASI-modulated supersonic accretion streams and SASI generated turbulence excite lowest-order (l=1) g-mode in the PNS. f ≈ 300 Hz.





- g-modes reach large amplitudes ~500 ms —1 s after bounce.
- Damping by strong sound waves that steepen into shocks; deposit energy in the stalled shock.
- ~1 B explosions at late times.
- (1) hard to simulate; unconfirmed, (2) **possible parametric instability, limiting mode amplitudes**.

Summary

- Core-Collapse Supernovae are "Gravity Bombs".
- **The Supernova Problem**:
  The supernova shock always stalls and must be revived.
- There are multiple possible supernova mechanisms:
  Neutrino, magnetorotational, and acoustic mechanism.
- **None of the mechanisms is robust** (generic & reproducible)
- Current best simulations in 2D, work towards 3D underway.
  But will 3D provide the solution to the
  supernova problem?
- How can we gain observational insight?
  -> neutrinos and gravitational waves! (see Ott 2009b)