The Physics of Stellar Collapse and Core-Collapse Supernovae

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Outline

• Onset of Collapse, Hydrodynamics, and Homologous Collapse.
• Collapse Microphysics and Neutrino Trapping.
• Core Bounce and Shock Formation.
• The Supernova Problem and its Energetics.
• Supernova simulations: 1D, 2D, and 3D and their ingredients.
• Candidate supernova mechanisms.
Situation at the Onset of Collapse

Hydrostatic Equilibrium

\[ \frac{dP}{dr} = -\frac{GM(r)\rho}{r^2} \]
The General Picture

Protoneutron Star, $R \sim 30$ km

Iron Core

$M \approx 1.3 - 2.2 \, M_{\text{SUN}}$

$M = M_{\text{CH,0}} + \text{corrections (thermal, GR, etc.)}$

“Core Bounce” at nuclear density.
The General Picture

Protoneutron Star, $R \sim 30$ km

Iron Core

Shock

Accretion

$M \approx 1.3 - 2.2 \, M_{\text{SUN}}$

$M = M_{\text{CH,0}} + \text{corrections (thermal, GR, etc.)}$
Blackboard/Lecture Notes
Studying Stellar Collapse: Essential Ingredients

Hydrodynamics

Gravity: Newton / General Relativity

Microphysics: Nuclear & Neutrino Physics

Transport Theory: Neutrino Transport

Fully coupled!
Hydrodynamics

Conservation Laws: Mass, Momentum, Energy
(Ideal fluid approximation -> No viscosity, radiation)

• Mass Conservation -> Equation of Continuity

\[ \frac{\partial}{\partial t} \int \rho dV = - \int \rho \vec{v} \cdot \vec{n} dS \]

Using Gauss’s theorem, we can rewrite this to:

\[ \frac{\partial}{\partial t} \int \rho dV = - \int \nabla (\rho \vec{v}) dV \]

Since this must hold for any volume V, the continuity equation follows:

\[ \frac{\partial}{\partial t} \rho + \nabla (\rho \vec{v}) = 0 \]

• Same approach: momentum and energy equations
The Equations of Newtonian Hydrodynamics

(Eulerian Formulation -> “laboratory frame”)

**Continuity**

\[
\frac{\partial}{\partial t} \rho + \nabla (\rho \vec{v}) = 0
\]

**Momentum**

\[
\frac{\partial}{\partial t} (\rho \vec{v}) + \nabla (\rho \vec{v} \vec{v}) + \nabla P = -\rho \nabla \Phi
\]

**Energy**

\[
\frac{\partial}{\partial t} (\rho e) + \nabla (\vec{v} (\rho e + P)) = -\rho \vec{v} \cdot \nabla \Phi
\]

\[
e = \epsilon + \frac{1}{2} v^2
\]

- Alternative: Lagrangian formulation (“comoving frame”).

Transformation:

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \nabla
\]
Stellar Collapse: Self Similarity

Self-Similarity in Stellar Collapse

- Separation into homologously \((v \propto r)\) collapsing inner core and supersonically collapsing outer core.

Analytic similarity solutions:
- Goldreich & Weber 1980
- Yahil & Lattimer 1982
- Yahil 1983
Important Collapse Microphysics

In collapse, pressure support is reduced by

- **Photodissociation** of heavy nuclei: \(~125 \text{ MeV/reaction}\)
  \[
  \gamma + ^{56}_{26}\text{Fe} \rightleftharpoons 13\alpha + 4n
  \]

- **Electron Capture**
  \[
  e^- + (Z, A) \xrightarrow{(W)} \nu_e + (Z - 1, A)
  \]
  \[
  e^- + p \xrightarrow{(W)} \nu_e + n.
  \]
  
  Capture rates:
  \[
  \frac{\partial}{\partial t} Y_e \propto \mu_e^5 \propto \rho^{5/3}
  \]

- Neutrinos stream off freely at densities below \(~10^{12} \text{ g/cm}^3\).
  -> core “deleptonizes” during collapse.

- Net entropy change very small,
  -> collapse proceeds practically adiabatic.
Neutrino Trapping

- Collapse phase: Neutrino opacity dominated by coherent neutrino-nucleus scattering: \( \nu + (A, Z) \leftrightarrow \nu + (A, Z) \)

  Neutrino mean-free path: 
  \[
  \lambda_\nu \approx 10^7 \text{ cm} \left( \frac{10^{12} \text{ g cm}^{-3}}{\rho} \right) \frac{A}{N^2} \left( \frac{10 \text{ MeV}}{E_\nu^2} \right)
  \]

- For \( \rho \geq 3 \times 10^{12} \text{ g/cm}^3 \), diffusion time \( \tau_{\text{diff}} \gg \tau_{\text{coll}} \) - neutrinos become trapped in the collapsing core.

- Consequences:
  
  Deleptonization stopped
  \[
  Y_{\text{lep}} = Y_e + Y_\nu = \text{const}.
  \]

  Beta Equilibrium
  \[
  e^- + p \leftrightarrow \nu_e + n
  \]
  \[
  \mu_e + \mu_p = \mu_\nu + \mu_n
  \]

  Detailed simulations: 
  \[
  Y_{\text{lep}} \approx 0.32
  \]
The Nuclear Equation of State (EOS)

Nuclear Statistical Equilibrium \((\rho > 10^7 \text{ g/cm}^3, T > 0.5 \text{ MeV})\)

\[ \rightarrow P = P(\rho, T, Y_e) \]

\[ s = 1.2 \text{ k}_B/\text{baryon} \]

\[ Y_e = 0.3 \]
The Nuclear Equation of State (EOS)

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![Graph showing the nuclear equation of state](image)
The Nuclear Equation of State (EOS)

Nuclear Statistical Equilibrium \((\rho > 10^7 \text{ g/cm}^3, T > 0.5 \text{ MeV})\)

\[ P = P(\rho, T, Y_e) \]

- Something happens near \(10^{14} \text{ g/cm}^3\)!

\[ s = 1.2 \text{ k}_B/\text{baryon} \]

\[ Y_e = 0.3 \]

Density (g/cm\(^3\))

Pressure (dyn/cm\(^2\))

Pressures dominated by electrons.
The Nuclear Equation of State (EOS)

Nuclear Physics:

\[ R_{\text{nuc}} = A^{1/3} r_0 \]
\[ r_0 = 1.25 \text{ fm} \]

Nuclear Density:

\[ \bar{\rho}_{\text{nuc}} = \frac{A m_b}{\frac{4}{3} \pi R_{\text{nuc}}^3} \]
\[ \bar{\rho}_{\text{nuc}} \approx 2 \times 10^{14} \text{ g/cm}^{-3} \]

\[ s = 1.2 \text{ k_B/baryon} \]
\[ Y_e = 0.3 \]

\[ \Gamma = \frac{d \ln P}{d \ln \rho} \]

recall:

\[ P \approx K \rho^\Gamma \]

“Stiffening” of the EOS
Nuclear EOS: What happens near $\rho_{\text{nuc}}$?

Nuclear Physics:

$$R_{\text{nuc}} = A^{1/3} r_0$$

$$r_0 = 1.25 \text{ fm}$$

Nuclear Density:

$$\bar{\rho}_{\text{nuc}} = \frac{A m_b}{\frac{4}{3} \pi R_{\text{nuc}}^3}$$

$$\bar{\rho}_{\text{nuc}} \approx 2 \times 10^{14} \text{ g/cm}^{-3}$$

- Above $\approx \rho_{\text{nuc}}$, n, p are so close that “repulsive core” of the strong force kicks in and leads to the stiffening of the EOS

\[ \Gamma = \frac{d \ln P}{d \ln \rho} \]

\[ P \approx K \rho^\Gamma \]

**Phase transition from inhomogeneous to homogeneous nuclear matter**

\[ s = 1.2 \text{ k_B/baryon} \]

\[ Y_e = 0.3 \]
Stellar Collapse: Nuclear EOS

Collapse and Bounce

- **Inner Core** reaches $\rho_{\text{nuc}}$, rebounds ("bounces") into still infalling outer core.
• Stiffening of EOS leads to sound wave that propagates through the inner core and steepens to a shock at the sonic point.
The Mass $M_{ic}$ of the inner core at bounce is determined by nuclear physics and weak interactions, is $\sim 0.5 \, M_{\text{SUN}}$, and is practically independent of progenitor star mass and structure.
Why worry about $M_{ic}$?

- $M_{ic}$ is the amount of matter dynamically relevant in bounce.
- $M_{ic}$ sets kinetic energy imparted to the shock.
- $M_{ic}$ (and IC radius) sets the angular momentum that can be dynamically relevant.
- Sets mass cut for material that the shock needs to go through.
- $M_{ic} \sim 0.5 \, M_{\text{SUN}}$ can easily stabilized by nuclear EOS. No “prompt” Black Hole formation.
- $M_{ic}$ sets the mass that must be accreted (before explosion?) to make a canonical $1.4 \, M_{\text{SUN}}$ neutron star.
The Supernova Problem

Radius (km)

Radial Velocity (v/c)

$v(r)$ at time -0.013621s (postbounce)
Stellar Collapse: Getting into trouble.

Why Does the Shock Stall

- Shock loses energy to:
  - Dissociation of infalling heavy nuclei: \(~8.8\,\text{MeV/baryon}\)
  - Neutrinos that stream away from behind the shock.

Inner core \(\rightarrow\) Core of the proto-neutron star (PNS)

*Janka et al. 2007*
Neutrino Burst

- **Optical depth**
  \[ \tau_\nu(r) = \int r \frac{1}{\lambda_\nu} dr' \]

- **Neutrinosphere:**
  \[ R_\nu = R \left( \frac{\tau_\nu}{\frac{2}{3}} \right) \]
  Depends on \((\varepsilon_\nu)^2\)

- **Postbounce neutrino burst:**
  Release of neutrinos created by \(e^-\) capture on free protons in shocked region when shock ‘breaks out’ of the \(\nu_e\) neutrinospheres.

Thompson et al. 2003
Postbounce Neutrino Emission

- Neutrinos and Anti-neutrinos of ALL species:
  \[ \nu_e, \bar{\nu}_e, \, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau \]
  Don’t participate in charged-current reactions. Can be treated as ‘one’.

- Emission:
  \[ e^- p \iff \nu_e n \]
  \[ e^+ n \iff \bar{\nu}_e p \]

- Pair processes: hot & dense environment needed
  \[ e^- e^+ \iff \nu_i \bar{\nu}_i \]
  \[ N N \iff \nu_i \bar{\nu}_i \]
  \[ \gamma \iff \nu_i \bar{\nu}_i \]

- Accretion luminosity and diffusive luminosity.
Putting Things Together: Supernova Energetics

- Supernova problem: What revives the shock?
- Precollapse iron core gravitational energy:
  \[ E_{\text{grav,Fe(1.5M}_\odot)} \approx 5 \times 10^{51} \text{erg} = 5 \text{B} \]
- Binding energy of a cold 1.5 M\(_\odot\) NS, R=12.5 km -> Energy Reservoir
  \[ E_{\text{grav,NS}} \approx -\frac{3}{5} G \frac{M^2}{R} \approx -3 \times 10^{53} \text{erg} = -3 \times 10^{46} \text{J} = -300 \text{Bethe} \]
- Initial shock energy: \[ E_{\text{shock,0}} = \frac{1}{2} M v^2 \approx 1.2 \times 10^{51} \text{erg} = 1.2 \text{B} \]
- Dissociation: \[ E_{\text{diss}} = 17 \left( \frac{M}{M_\odot} \right) \text{B} \] (Shock formation at \(\sim 0.55 \text{M}_\odot\), \(v \sim 0.05 \text{c}\))
  -> Shock stalls “after” \(\sim 0.1 \text{M}_\odot\).
- Neutrinos: initially up to \(L_\nu,\text{total} \sim 100 \text{B/s}\)
- Binding energy of the mantle (12-M\(_\odot\) star): \[ E_{\text{bind,0.6–12M}_\odot} = -3.7 \text{B} \]
- -> need multiple Bethes to blow up the star!
Protonutron Star, R ~30 km
The General Picture

Protoneutron Star, $R \sim 30$ km

Accretion

L$_{\nu}$

Shock

Shock is revived.

Supernova Explosion
The General Picture

Proton-electron star, \( R \sim 30 \text{ km} \)

Accretion

Shock

\( L_\nu \)

Supernova Explosion

Shock is revived.

Shock is not revived.

Collapse to Black Hole (Collapsar)
The Supernova Problem

Proton neutron Star, R ~30 km

Supernova Explosion

Accretion

Shock

What is the Mechanism of shock revival?

Shock is revived.

Shock is not revived.

Collapse to Black Hole (Collapsar)
The Essence of any Supernova Mechanism

- Collapse to neutron star:
  \[ \sim 3 \times 10^{53} \text{ erg} = 300 \text{ Bethe [B]} \text{ gravitational energy}. \]

- \[ \sim 10^{51} \text{ erg} = 1 \text{ B kinetic and internal energy of the ejecta.} \]
  (Extreme cases: \[ 10^{52} \text{ erg}; \text{“hypernova”} \])

- 99% of the energy is radiated as neutrinos over hundreds of seconds as the protoneutron star (PNS) cools.

Explosion mechanism must tap the gravitational energy reservoir and convert the necessary fraction into energy of the explosion.
Supernova Mechanism: First Simulations

• No supercomputers yet (Cray-I only in 1976!): Limited to spherical symmetry, low resolution, poor neutrino transport.

• Nevertheless: Very important discovery - Energy deposition by neutrinos may revive/drive the shock.
The Neutrino Mechanism

Neutrino cooling: \( Q_\nu^- \propto T^6 \)

Net heating where:

Neutrino heating: \( Q_\nu^+ \propto L_\nu r^{-2} \langle \epsilon_\nu^2 \rangle \quad Q_\nu^+ > Q_\nu^- \)

- **Neutrino-driven mechanism:** Based on subtle imbalance between neutrino heating and cooling in postshock region.

\[ Q_\nu^+ - Q_\nu^- [10^{19} \text{ erg/g s}] \]

- [Ott et al. 2008]
Does it work?

- **Yes!**
  BUT: Only for lowest-mass massive stars.
  (Kitaura et al. 2006, Burrows 1988, Burrows, Livne, Dessart 2007)

- **FAILS** in spherical symmetry (1D) for garden-variety massive stars (≈15 M\(_{\text{Sun}}\)) in simulations with best neutrino physics and neutrino transport

![Graph showing stellar collapse and neutrino mechanism](image)
A few Words on Neutrino Transport

\[
\frac{1}{c} \frac{\partial I(\vec{r}, \vec{n}, \epsilon_\nu)}{\partial t} + \vec{n} \cdot \vec{\nabla} I(\vec{r}, \vec{n}, \epsilon_\nu) = \Xi[I(\vec{r}, \vec{n}, \epsilon_\nu), \rho, T, Y_e]
\]

\[
J = \frac{1}{4\pi} \int I d\Omega \quad \vec{H} = \frac{1}{4\pi} \int \vec{n} I d\Omega \quad K = \frac{1}{4\pi} \int \vec{n} \cdot \vec{n} I d\Omega
\]

• 6D problem: 3D space, 3D (\(\epsilon\), \(\theta\), \(\phi\)) momentum space.

• Limiting cases – easy to handle:
  (1) Diffusion (isotropic radiation field)
  (2) Free streaming ("forward-peaked" radiation field)
Stellar Collapse: Neutrino Mechanism

Neutrino Transport in Core-Collapse SNe

- Main complication: Need to track radiation field from complete isotropy to full free streaming over many orders of magnitude of $\tau$.

- Neutrino heating depends on details of the radiation field:

$$Q^+_{\nu_e/\bar{\nu}_e} = 4\pi \int_0^{\infty} d\epsilon_\nu \frac{\kappa_{a,\nu_e/\bar{\nu}_e}}{\epsilon_\nu} J_\nu = \frac{1 + 3g_A^2}{4} \sigma_0 N_A X_{n/p} \frac{\epsilon^2_{\nu_e/\bar{\nu}_e}}{4\pi c r^2} \bar{L}_{\nu_e/\bar{\nu}_e} \langle 1 \rangle \langle \epsilon^2_{\nu_e/\bar{\nu}_e} \rangle$$

- Inverse Flux factor:

$$\langle \frac{1}{F_{\nu_i}} \rangle = c \frac{\int d\epsilon_\nu E(\epsilon_\nu, \nu_i)}{\int d\epsilon_\nu F_r(\epsilon_\nu, \nu_i)}$$
Anyway... What next?

- Why does the neutrino mechanism fail in 1D?
- Is dimensionality an issue? What is 1D missing?
  - Rotation and magnetohydrodynamics (MHD)
  - Convection/Turbulence
  - Other multi-D processes; e.g., pulsations
- First multi-D radiation-hydrodynamics simulations:
  - early to mid 1990s:
A Look at the Beast:

Specific Entropy \([k_B / \text{baryon}]\)

\[
\begin{array}{cccccc}
1.5 & 4.4 & 7.3 & 10.2 & 13.1 & 16.0 \\
\end{array}
\]

\[\vec{M} - \text{Gain Region} - \text{SASI} \]

\[\vec{L}_v - \text{Shock} \]

[Ott et al. 2008, Ott 2009 b]
Convection

- Ledoux criterion for instability:
  \[ C_L \equiv \left( \frac{\partial \rho}{\partial s} \right) \left|_{\gamma_{np}} \right. \frac{ds}{dr} + \left( \frac{\partial \rho}{\partial Y} \right) \left|_{s_{np}} \right. \frac{dY}{dr} < 0 \]
  
  Entropy Gradient

- \( C_L > 0 \) -> convective instability.

- Postbounce supernova cores:
  - Negative entropy gradient in postshock region
    -> convection
  - Negative entropy region inside the neutrinosphere in the PNS -> convection

- **Important effect of convection:**
  - “Dwell time” of material in the heating (“gain”) region is increased -> leads to more favorable ratio \( \tau_{\text{advect}} / \tau_{\text{heat}} \).
Standing Accretion Shock Instability

[Blondin et al. ‘03,’06; Foglizzo et al. ‘06, Scheck et al. ‘06, ‘07, Burrows et al. ‘06, ‘07]

Advective-acoustic cycle drives shock instability.

Seen in simulations by all groups!
Status of the Neutrino Mechanism

- Best simulations are still in 2D.
- Things look better in 2D, some models explode under special circumstances.
- No robust explosions.
- Crucial conditions (?): General relativity
  Soft nuclear EOS
- Robust explosions in 3D? -> ongoing research!

Marek & Janka 2009
Alternatives to the Neutrino Mechanism

(qualitative discussion!)

- **Magnetorotational Mechanism**

- **Acoustic Mechanism**
  - [proposed by Burrows et al. 2006, 2007; not (yet?) confirmed by other groups/codes]
Alternatives to the Neutrino Mechanism

(qualitative discussion!)

Magnetorotational Mechanism


Acoustic Mechanism

[proposed by Burrows et al. 2006, 2007; not (yet?) confirmed by other groups/codes]
MHD-driven Explosions


- **Rapid rotation**: $P_0 < 4-6$ s
  - -> millisecond PNS
- PNS rotational energy: $\sim 10^8$ B
- Amplification of B fields up to equipartition:
  - compression
  - dynamos
  - magneto-rotational instability (MRI)
- Jet-driven outflows.
- MHD-driven explosion may be GRB precursor.

MHD jet/explosion launched when $P_{\text{mag}} / P_{\text{gas}} \sim 1$
Newtonian Radiation-MHD Simulations with VULCAN/2D

Magnetic field lines in M15B11UP2A1H of Burrows, Dessart, Livne, Ott, Murphy ‘07.
Features/Limitations of the Magnetorotational Mechanism  

[Burrows et al. 2007]

- Jet powers up to 10 B/s ($10^{52}$ erg/s).
- Simultaneous explosion and accretion.
- Hypernova energies (> 10 B) attainable.
- MHD mechanism inefficient for cores with precollapse $P_0 > 4$ s, but stellar evolution + NS birth spin estimates: $P_0 > 30$ s in most cores.  
  
  [Heger et al. 2005, Ott et al. 2006]

- MHD explosion — a GRB precursor?
- Limitations: Resolution does not allow to capture Magnetorotational Instability; Simulations 2D and Newtonian.
Alternatives to the Neutrino Mechanism
(qualitative discussion!)

- **Magnetorotational Mechanism**

- **Acoustic Mechanism**
  - [proposed by Burrows et al. 2006, 2007; not (yet?) confirmed by other groups/codes]
Alternatives: The Acoustic Mechanism

Setting the Stage: SASI

[e.g., Burrows et al. 2006, 2007bc, Ott et al. 2006]
The Acoustic Mechanism
[e.g., Burrows et al. 2006, 2007bc, Ott et al. 2006]

SASI-modulated supersonic accretion streams and SASI generated turbulence excite lowest-order (l=1) buoyancy mode in the PNS.

- g-modes reach large amplitudes ~800—1000 ms after bounce.
- Damping by strong sound waves that steepen into shocks; deposit energy in the stalled shock.
- Drive ~1 B explosions at late times.
PNS core oscillations, Burrows et al. 2006, 2007; Ott et al. 2006

Time = -0.50 ms

Width = 50.00 km
Alternatives: The Acoustic Mechanism
Alternatives: The Acoustic Mechanism

SASI-modulated supersonic accretion streams and SASI generated turbulence excite lowest-order (l=1) g-mode in the PNS. $f \approx 300$ Hz.

- g-modes reach large amplitudes $\sim 500$ ms —1 s after bounce.
- Damping by strong sound waves that steepen into shocks; deposit energy in the stalled shock.
- $\sim 1$ B explosions at late times.
- (1) hard to simulate; unconfirmed,
  (2) possible parametric instability, limiting mode amplitudes.
Summary

• Core-Collapse Supernovae are “Gravity Bombs”.

• **The Supernova Problem:**
  The supernova shock always stalls and must be revived.

• There are multiple possible supernova mechanisms: Neutrino, magnetorotational, and acoustic mechanism.

• **None of the mechanisms is robust** (generic & reproducible)

• Current best simulations in 2D, work towards 3D underway. But will 3D provide the solution to the supernova problem?

• How can we gain observational insight?
  -> neutrinos and gravitational waves! (see Ott 2009b)